

THE
ELEMENTS
OF
NAVIGATION.

VOLUME THE FIRST.

THE
ELEMENTS
OF
NAVIGATION;

CONTAINING THE
THEORY and PRACTICE.

With the necessary TABLES,
And COMPENDIUMS for finding
The LATITUDE and LONGITUDE at SEA.

To which is added,

A TREATISE
OF
MARINE FORTIFICATION.

Composed for the Use of
The ROYAL MATHEMATICAL School at CHRIST'S HOSPITAL,
The ROYAL ACADEMY at PORTSMOUTH,
And the GENTLEMEN of the NAVY.

IN TWO VOLUMES.

VOL. I.

By J. ROBERTSON,

Late Librarian to the Royal Society, and formerly Head-Master
of the Royal Academy, at Portsmouth.

The SIXTH EDITION, with ADDITIONS.

Carefully revised and corrected by

WILLIAM WALES,

Master of the Royal Mathematical School, Christ's Hospital, London.

L O N D O N :

Printed for F. WINGRAVE, Successor to Mr. NOURSE, in the Strand;
G. G. and J. ROBINSON, and T. N. LONGMAN, Pater-noster-Row;
W. RICHARDSON, Royal Exchange; and J. SCATCHERD, Ave-Maria-Lane

M. DCC. XCVI.



'64-312

TO
JOHN EARL OF
SANDWICH,

&c. &c. &c.

FIRST COMMISSIONER

OF THE

Boards of Admiralty and Longitude.

MY LORD,

WHILE the public voice is unanimous in applauding your humanity toward the Artificers, in general, of His Majesty's Dock Yards, and your attention to restore the Royal Navy of Britain to the respectable state from which it had been suffered to decline since the last War, Philosophers not only admire these noble acts, but likewise, your generous encouragement to improve Geographical, Nautical, and Natural Knowledge.

EPD. U

DEDICATION.

Such exertions of your Lordship's extraordinary Mental and Official Abilities, will undoubtedly be transmitted with honour to the latest posterity: and your laudable example must inspire a regard for Works intended to promote public utility.

The Author of *The Elements of Navigation*, notwithstanding the favourable reception which the former impressions have met with from British Mariners, thinks himself extremely happy that this improved Edition is permitted to appear under your Lordship's Patronage.

That you may long enjoy the Opportunity as well as Inclination of promoting useful Arts and Learning, is a hope sincerely entertained by,

My LORD,

Your LORDSHIP'S

most obedient

and humble Servant,

Nov. 1, 1772.

John Robertson.

TO THE
RIGHT WORSHIPFUL
Sir ROBERT LADBROKE, Knt. Alderman,
PRESIDENT;

THE
Worshipful THOMAS BURFOOT, Esq.
TREASURER;

And the Rest of the
WORSHIPFUL GOVERNORS
OF

Christ's Hospital, London :

This Book, containing the Elements of Navigation,
and a Treatise on Marine Fortification, first published for the Use of the Children in the Royal Mathematical School, when they were under my Care, is, as a grateful acknowledgement for past favours, addressed by

Your WORSHIPS' most humble Servant,

Nov. 1, 1772.

John Robertson.

ADVERTISEMENT.

IN this Edition, the Editor has carefully corrected the errors which had crept into the former, and computed new Tables of the Sun's Declination, Book V. Art. 308, 309, and 310. He has also revised, as far as his materials extended, the Geographical Table, and added the names of such places as his own observations, or those of other persons, have furnished him with; so that he flatters himself it is the most extensive and correct of any extant. On the whole, he presumes, this Edition will be found as worthy of the approbation of the public in general, and of seamen in particular, as those which were printed under the Author's inspection.

ERRATA.

VOL. II. p. 355, log. of 1509, *for*, 17860 *read*, 17869.

———— p. 356, log. of 1653, *for*, 21807 *read*, 21827.

———— p. 359, log. of 3403, *for*, 53180 *read*, 53186.

———— p. 371, at 3° 30', *read*, in the line, 8,78568.

THE P R E F A C E.

IT having been part of my employment for many years past, to instruct youth in the theoretical and practical parts of Navigation, I was naturally led to draw up rules and examples fitted to the years and capacity of the scholar: some of the precepts, from time to time were altered, according as I had observed how they were comprehended by the majority of my pupils; until at length I had put together a set of materials, which I found sufficient for teaching this Art.

Upon my being intrusted by the governors of Christ's Hospital (in the beginning of the year 1748) with the care of the Royal Mathematical School there, founded by King Charles the Second, I had a great opportunity of experiencing the method I had before used; and finding it fully answered my expectation, I determined to print it for the use of that school: but as those children are to be instructed in the mathematical sciences, on which the art of Navigation is founded, I judged it proper, on their account, to introduce the subjects of Arithmetic, Geometry, Trigonometry, &c. for which reason, this treatise is distinguished by the title of Elements of Navigation.

After my appointment (in the year 1755) to be head master of the Royal Marine Academy at Portsmouth, founded by King George the Second, I also found that this book was sufficiently intelligible to beginners of middling capacities; and therefore, in the second edition, in the year 1764, the manner in which it was first composed was continued, except the removing of the book of Astronomy, from being the 8th, into the place of the 5th; whereby the books of Plane Sailing, Globular Sailing, and Days Works, which together nearly comprehend the art of Navigation, follow in succession. There were indeed some variations in the modes of expression in a few places; but the additions in every book were made rather to extend the notions of learners, than to supply any deficiency wanting in the former edition, except some of the additions in the 9th book, which were not so well known at the time of the first impression.

The work is divided into ten parts or books, each being a distinct treatise; the preceding ones contain the necessary elements which are wanted in those that follow. The demonstration of the several propositions are given as concisely as I could contrive, to carry with them a sufficient degree of evidence: Throughout the whole of the elementary parts, brevity and perspicuity were considered; but the practical parts are more fully treated on, and intended to include every useful particular, worthy of the mariner's notice.

In the elementary parts, where it is not easy to introduce new matter, there will be found the common principles treated in a manner, which, it is apprehended, is better adapted to beginners; and such new lights thrown on several particulars, as will render them more obvious than in the view wherein they have been commonly seen.

The treatise of Fortification annexed, is the result of many years application; and is delivered in a very different mode from what other writers have

have taken; for, among the multitude which I have seen, they generally begin with the fortifying of a town, the most difficult part of the art, and end with works the most easy to contrive and execute: Herein the works of the simplest construction are begun with, and the learner gradually advanced to the fortifying of a town: Indeed the limits chosen for this tract have caused some articles to be briefly mentioned, and others to be totally omitted; nevertheless, it is conceived that, in its present state, it may be of considerable use to Marine Officers, and even furnish some hints not altogether unworthy the notice of the Gentlemen of the Army.

The Maritime parts of these Elements, contained in the viiith, viiith, and ixth books, are also delivered in a manner somewhat different from what is seen in other writers; who, for the most part, copying from one another, have not much contributed toward perfecting the art of Navigation; the writers indeed have been many, but the improvers have been very few; Wright, Norwood, and Halley having done the most of what has been discovered since a little before the beginning of the 17th century: However, in the method here taken, it is apprehended that the proper judges will find some few improvements, as well in the art itself as in the manner of communicating it to learners.

The common treatises of Navigation, which, on account of their small bulk and easy price, are vended among the British Mariners, seem not to be written with an intention to excite in their readers a desire to pursue the Sciences, farther than they are handled in those books; so that it is no wonder our seamen in general had so little mathematical knowledge; for the person who could keep a trite journal, formed on the most easy occurrences, has been reckoned a good artist; but whenever those occurrences have not happened, the journalist has been at a loss, and unable to find the ship's place with any tolerable degree of precision; and such accidents have probably contributed to the distress which many ships crews have experienced, and which a little more knowledge among them might have prevented, or at least have lessened.

About the middle of the 16th century, Navigation began to be considered as an art, in a great measure dependent on the Mathematical sciences; and on such a plan has it been cultivated by the labours of the most judicious, who have applied themselves toward its perfection; and although the art has been enriched by the observations of some learned men in different nations, yet it has so happened, that the chief of the improvements, and particularly the mathematical ones*, were first published in Britain.

Into this work are collected most, if not all, of the useful and curious particulars relating to the art of Navigation; there are also interspersed historical remarks of inventions, with the names of many eminent men, and their works; these were intended as incentives to inspire learners and our seamen with a desire not only of knowing the things herein treated of from their foundations, but of pushing their inquiries into such other parts of the sciences as may procure to themselves pleasure, profit, and respect, and

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* See the following Dissertation.

render them more useful to their country by the skill resulting from such acquisitions.

I have always thought that the chief motives which ought to induce a person to appear as a writer should be, either that he has something new to publish, or that he has arranged the parts of a known subject, in a method more regular and useful than had been done before; in either of these cases he cannot be a proper judge, unless he has seen the pieces extant on that subject, or, at least, those of the most eminent authors already published: On these principles I was led to examine what had been done by the different writers on Navigation; and having perused most of their Books, of which I could get information, I had an opportunity of discovering the steps by which this art has risen to its present perfection, and consequently of knowing the most material parts of the history of its progress: Among other things, I could not avoid remarking a mistake which has crept into many of the modern Books of Navigation; which is, that Wright's invention of making a true sea-chart was stolen by Mercator, and published as his own. I suspect this story had its rise in a Book printed in the year 1675 by Edward Sherburne, intitled, "*The Sphere of Marcus Manilius made an English Poem, with Annotations and an Astronomical Appendix.*"

My enquiries into these matters, induced the late learned Dr. James Wilson to review and complete his observations on the same subject, and produced his *Dissertation on the History of the Art of Navigation*; which he was pleased to give me leave to publish with the second edition of this work.

There are few persons, however knowing and careful, who may not commit, and overlook, inadvertencies in their own compositions, which may be discovered by others: therefore, at my request, the greatest part of the manuscript for the first edition was read and examined by two of my friends*, well acquainted with the theory and practice of Navigation; who, by their judicious observations, enabled me to improve several articles: Some part of the additions to the second Edition, received much elegance and perspicuity through the friendly advice and communications of the late learned Dr. Henry Pemberton, F. R. S.

The second Edition of these *Elements* having also been well received by the Public; Dr. Wilson took the pains to revise his *Dissertation*, which he improved in many particulars: And I have also endeavoured to retain their favourable opinions of my labours, by giving *Compendiums* for performing the operations of the new methods of finding the latitude and longitude of a ship at sea; and some other alterations and additions which I conceived would render this third Edition more generally useful.

Nov. 1. 1772.

* William Mountaine, Esq. F. R. S. and Mr. William Payne.

T H E

C O N T E N T S.

THIS treatise in two volumes contains ten books; each is divided into several sections, and numbered with the Roman numerals.

The particular articles are numbered by the common figures, each book beginning with the number 1.

The references made from one article of the treatise to another is of two kinds.

First. When in the same book. Then the number of the article referred to, is put in a parenthesis. Thus (27), refers to the article numbered 27 in the same book.

Second. When in another book. Then the number of the book in Roman figures, and of the article in common figures, is put in a parenthesis. Thus (II. 160) refers to the 160th article of the second book.

In the following contents, the sections refer to the number of the page: the particulars to the number of the article.

VOL. I. BOOK I. ARITHMETICK.

From Page 1 to Page 42.

SECTION I. Definitions and Principles	p. 1	bers	p. 44
Notation, numeration, and fractions	articles 1 to 21	<i>The Rule of Three</i>	46
Signs, and tables of coin, weights, &c.	22, 23	<i>A collection of 26 questions</i>	47
SECTION II. Addition	p. 6	SECTION VIII. Of powers and roots	p. 26
Of whole numbers and fractions	24	<i>To raise numbers to a proposed power</i>	art. 49
SECTION III. Subtraction	p. 8	<i>To extract the square root</i>	54
Of whole numbers and fractions	26	<i>To extract the cube root</i>	56
Questions to exercise Addition and Subtraction	27	SECTION IX. Of numeral series	p. 30
SECTION IV. Multiplication	p. 10	<i>Of arithmetic progression</i>	art. 59
Multiplication table	art. 29	<i>Of geometric progression</i>	60
Multiplication of whole numbers and fractions	art. 30	<i>Some properties of such progression numbers</i>	from art. 62 to 67
SECTION V. Division	p. 12	<i>Four problems in progressions</i>	68 to 71
Of whole numbers and fractions	31	SECTION X. Of logarithms	p. 34
SECTION VI. Reduction	p. 14	<i>Of the nature of logarithms</i>	73
Of several names to one name	35	<i>To make logarithms</i>	77
Of inferior name to sup. name	36	<i>Of the tables of logarithms</i>	80
Vulgar fractions to decimals	38	<i>Of the indices of logarithms</i>	83
Inferior names to a decimal	40	<i>Multiplication by logarithms</i>	85
To value decimal fractions	42	<i>Division by logarithms</i>	86
Practical questions	43	<i>Of proportion by logarithms</i>	87
SECTION VII. Of proportion	p. 20	<i>Of the arith. comp. of logarithms</i>	88
Of the nature of proportional num-		<i>Of powers and roots by logarithms</i>	89

BOOK

C O N T E N T S.

BOOK II. GEOMETRY.

From Page 43 to Page 88.

SECTION I. Definitions and Principles p. 43 <i>Definitions</i> art. 1 to 42 <i>Postulates</i> 43 <i>Axioms</i> 46 SECT. II. Geometrical Problems p. 48 <i>Of right lined figures</i> 55 to 69 <i>Of the circle</i> 70 to 74 <i>Of proportional lines</i> 75 to 80 <i>Scales of equal parts and chords</i> 81 to 84 <i>Of polygons</i> 85 SECTION III. Geom. Theorems p. 58 <i>Of lines and triangles</i> art. 91 to 106 <i>Of quadrangles and the circle</i> 107 to 137	SECTION IV. Of Proportion p. 68 <i>Definitions and Principles</i> 138 to 147 <i>General Properties</i> 148 to 164 <i>In similar triangles</i> 165 <i>In the circle and regular polygons</i> 169 <i>Quadrature of the circle</i> 191 SECT. V. Of planes and solids p. 82 <i>Definitions and Principles</i> art. 198 <i>Properties of cutting planes</i> 209 <i>Section of pyramids and cones</i> 211 SECTION VI. Of the spiral p. 86 <i>Of the common spiral</i> art. 221 <i>Of the proportional spiral</i> 222 <i>Logarithmic spiral</i> 226 <i>Of Napier's Logarithms</i> 229
---	--

BOOK III. PLANE TRIGONOMETRY.

From Page 89 to Page 124.

SECTION I. Definitions and Principles p. 89 <i>Definitions</i> art. 1 to 16 SECTION II. Triangular canon p. 92 <i>Construction of the plane scale</i> art. 17 <i>Of sines of small arcs</i> 25 <i>Of sines above 60 degrees</i> 29 <i>Of the relation of tangents, &c.</i> 31 <i>To compute the sines</i> 38 <i>Of the tables of sines, tangents, &c.</i> 39 SECT. III. Solution of triangles p. 100 <i>By opposite sides and angles</i> art. 45 <i>Two sides and included right angle</i> 46 <i>Two sides and included oblique angle</i> 48 <i>By three sides</i> 49, 50	<i>Synopsis of the rules</i> art. 52 <i>Twelve numeral examples</i> 53 to 64 SECTION IV. Of the Gunter's scale p. 114 <i>Of natural and logarithmic scales</i> art. 65, 66, 67 <i>Of the line of numbers</i> 68 <i>Of the line of sines</i> 69 <i>Of tangents and versed sines</i> 70, 71 <i>Demonstration of their construction</i> 72 SECT. V. Use of Gunter's scale p. 118 <i>Precepts</i> art. 76 to 78 <i>Exemplifications</i> 79 to 86 SECTION VI. Properties of plane triangles p. 122 <i>In oblique angled triangles</i> art. 88 <i>Rules for finding an angle</i> 90 to 98
--	--

BOOK IV. OF SPHERICS.

From Page 125 to Page 193.

SECTION I. Principles p. 125 <i>Definitions</i> art. 1. <i>Axioms</i> 16 SECTION II. Stereographic projections p. 128 <i>Principal properties</i> art. 30 to 59	SECT. III. Spheric Geometry p. 135 <i>The construction and demonstration of 37 cases</i> art. 60 to 81 SECTION IV. Spheric Trigonometry p. 145 <i>Definitions and Principles</i> art. 82 SECTION
--	--

C O N T E N T S.

SECTION V. Spherical Theorems

p. 146

Fifteen propositions art. 90 to 110

SECTION VI. Solutions of right

angled spheric triangles p. 152

By two new rules art. 111

By Napier's general rule 119

SECTION VII. Solution of oblique

angled spheric triangles p. 157

With a perpendicular 120 to 124

Given three sides or three angles 125

Synopsis of right angled spher. 127

Synopsis of oblique angled spher. 144

SECTION VIII. Construction and

calculation of the cases of right

angled spheric triangles p. 156

Numeral examples art. 156 to 161

Exam. of a quadrantal triangle 162

SECTION IX. Construction and

calculation of the cases of oblique

angled spheric triangles p. 173

Numeral examples art. 163 to 168

SECTION X. Goniometric lines p. 179

Properties deduced art. 169

Table of goniometric relations

171 to 210

Variety of theorems 211 to 224

Relation between the sides and an-

gles of spheric triangles 225 to 236

Rules when two sides and the in-

cluded angle are known, to find

the other angles 237

Rules when three sides are known 254

To find the natural sines of given

arcs; and the contrary 256, 257

BOOK V. ASTRONOMY.

From Page 195 to Page 328.

SECTION I. Solar Astronomy p. 195

Of the Solar System art. 1

Number of constellations and stars 6

Primary and secondary planets 12

Annual and diurnal motions 17

Planets orbits and nodes 21

Elliptic orbits of the planets 23

Kepler's laws 29

Conjunctions and oppositions 34

Table of the solar system 37

Of the secondary planets 38

Of the figure of the planets 40

SECTION II. Terrestrial Astronomy

p. 204

Of the direct, stationary, and re-

trograde appearances art. 45

Phenomena of the inferior planets 46

Phenomena of the superior planets 49

Apparent motions of the Sun 50

Obliquity of the ecliptic 55

Of the different seasons 56

Rising and setting of stars 65

Of parallaxes 66

The measure of the Earth 73

Of the Moon 74

Of solar and lunar eclipses 79

SECTION III. Astronomy of the

sphere p. 214

Definitions and Principles art. 89

Of astronomical tables 132

To convert time into degrees, and

degrees into time p. 133

Of the culminating of the stars 134

SECTION IV. Of the projection of

the sphere p. 221

Four projections on the four prin-

cipal circles art. 135

SECT. V. Problems of the sphere p. 225

The construction and numeral so-

lution of 22 problems 139 to 169

SECT. VI. To find the latitude p. 242

Stereographic solution of fifteen

problems of latitude 170 to 184

Fourteen other problems for find-

ing the latitude 185 to 203

Nonagesimal degree 204

SECTION VII. Practical Astrono-

my p. 268

Of astronomical instruments art. 205

Of the Clock 206

Of the Telescope 207

Of the Micrometer 208

Of the Quadrant 209

Of the Transit Instrument 210

Of the Astronomical Sector 212

Of the equal altitude instrument 214

To adjust the clock 215

Of the change in the Sun's declina-

tion 216

Of Vernier's dividing plate 219

SECTION

C O N T E N T S.

SECTION VIII. Elements of the <i>Earth's motion</i> p. 280 <i>Of mean motion and anomaly</i> art. 222 <i>Of the different years</i> 229 <i>Twelve problems relating to the</i> <i>Earth's motion</i> 234 to 255 SECT. IX. Equation of time p. 291 <i>Of the syderial and equatorial day</i> art. 261	<i>Of mean and apparent time</i> p. 263 <i>To calculate the equation of time</i> 268 SECT. X. To make solar tables p. 293 <i>For finding the Sun's place</i> art. 269 to 280 <i>Of declinat. and right ascen.</i> to 289 <i>Equation of time</i> 299 <i>Nineteen astronomical tables</i> 301 to 319
--	--

BOOK VI. GEOGRAPHY, &c.

From Page 329 to Page 400.

SECTION I. Definitions and Prin- <i>ciples</i> p. 329 <i>Of the poles and circles</i> art. 3 <i>Of latitude and longitude</i> 6, 9 <i>Division of the Earth by zones</i> 13 <i>Division of the Earth by climates</i> 17 SECTION II. Natural division of <i>the Earth</i> p. 332 <i>Definitions</i> art. 18 to 31 <i>Of continents and oceans</i> 32, 33 SECTION III. Of the political di- <i>vision of the Earth</i> p. 334 <i>Europe and its chief parts</i> art. 35 <i>Asia</i> 38 <i>Africa</i> 42 <i>America</i> 46 SEC. IV. Geographical problems p. 342 <i>Concerning latitude and longitude</i> 50 to 53	SECT. V. The use of the globes p. 346 <i>Twenty-two problems on the celestial</i> <i>and terrestrial globes</i> art. 54 to 76 SECTION VI. Of Winds p. 353 <i>Air and Atmosphere</i> art. 78, 79 <i>Trade-winds and Monsoons</i> to 83 <i>Dr. Halley's observations</i> 84 SECTION VII. Of Tides p. 358 <i>Of gravitation and attraction</i> to 89 <i>Cause of the general tides</i> art. 90 <i>The chief propositions on tides</i> to 101 <i>Of the tides about Britain</i> 103 SECTION VIII. Chronology p. 364 <i>Of eras or epochs</i> art. 111 <i>Julian account or old stile</i> 114 <i>Gregorian account, or new stile</i> 115 <i>Of the Kalendar</i> 116 <i>Chronological problems</i> to 126 <i>Geographical and tide table</i> 137
---	--

VOL. II. BOOK VII. OF PLANE SAILING.

From Page 1 to Page 130.

SECTION I. Of Principles p. 1 <i>Definitions</i> art. 1 <i>Necessary elements</i> 2 <i>Nature of maps</i> 3 SECT. II. Of the plane chart p. 3 <i>Construction of the plane chart</i> art. 5 <i>Its use, shewn in 7 cases</i> 6 to 12 SECTION III. Plane sailing p. 6 <i>Definitions and Principles</i> art. 13 to 21 <i>Rules for constructions</i> 22 <i>Of the Traverse table</i> 24 <i>Proportions of sides and angles,</i> <i>making either side the radius</i> 25 <i>Canons of plane sailing</i> 26 <i>Table of rhumbs</i> 27	SECT. IV. Of single courses p. 11 <i>The six cases of plane sailing</i> to art. 33 <i>Questions in plane sailing</i> 34 SECTION V. Compound courses p. 21 <i>To work a traverse</i> art. 36 <i>To construct a traverse</i> 37 <i>To construct a compound course</i> 39 <i>Twenty questions in comp. courses</i> 40 SECTION VI. Oblique sailing p. 35 <i>Bearing and setting of objects</i> art. 41 <i>Twenty questions and their solu-</i> <i>tions</i> ibid. SECT. VII. Sailing to windward p. 42 <i>Definitions</i> art. 42 <i>General construction</i> 46 <i>Twenty questions and their solutions</i> ibid. SECT. VIII.
--	--

C O N T E N T S.

SECT. VIII. Current sailing	p. 51	To survey on ship-board	art. 52
Definitions and Principles	art. 47	To survey on shore	53
General construction	49	Of colours and their use	55
Twenty questions and their solutions	ibid.	SECT. XI. To estimate distances	p. 76
SECTION IX. Miscellaneous questions	p. 60	By the motion of sound	art. 58
Twenty complicated questions with their solutions	art. 51	By the curvature of the earth	61
SECTION X. Surveying of coasts and harbours	p. 70	Table of distances seen at sea	65
		Traverse tables	art. 66, 67
			from p. 81 to 130

BOOK VIII. OF GLOBULAR SAILING.

From Page 131 to Page 224.

SECTION I. Definitions and Principles	p. 131	SEC. V. Compound courses	p. 174
Of a ship's true place at sea	art. 3	Principles and rules	to art. 76
Of Mercator's chart	4	Four examples wrought by the several rules	to art. 81
Of Wright's construction	6	SECT. VI. Of Mercator's chart	p. 182
SECTION II. Reckoning of longitude, in 14 propositions	p. 134	Construction	art. 82
Meridional parts by the secants	art. 12	Of the lines of longitude and latitude on Gunter's scale	83
To construct a true sea chart	13	Thirteen Problems on the use of Mercator's chart	to art. 99
Truth of sailing by the Mercator's chart	15	SECT. VII. Great circle sailing	p. 191
Of the spiral rhumbs on the globe	21	The use	art. 102
Meridional parts analogous to the logarithmic tangents	24	The six cases in a single course	to 108
SECTION III. Parallel sailing	p. 146	Concerning compound courses and the principles	to art. 110
Rules of computation	art. 31	Three examples	to 114
Merid. dist. of meridians	34	SECTION VIII. Of the figure of the Earth	p. 203
The cases of parallel sailing	37	Of the experiments to find the figure of the Earth	to art. 121
Other examples	42	Of the ratio and magnitude of the axes of the Earth.	to art. 126
SECTION IV. Middle latitude and Mercator's sailing	p. 152	Of the parallels of lat. degrees of lat. and merid. parts on the spheroid	to 142
Principles for Mid-latitude	art. 44	Tables of merid. parts	p. 215 to 224
Principles for Mercator's	49		
Thirteen Problems of globular sailing, by Mid-lat. Mercator and logarith. tangents	to art. 73		

BOOK IX. OF DAYS WORKS.

From Page 225 to Page 341.

SECTION I. Of a ship's run	p. 226	Of the azimuth compass	17
Of the log. line and half min. glass	art. 3	To make artificial magnets	18
Of their corrections	4	To observe by the azimuth compass	21
Of M. Bouguar's log.	7	SECTION III. Of amplitude	p. 236
SECT. II. Of the sea compass	p. 231	To work them	{ by logarithms art. 25
Construction and properties	art. 14		{ by Traverse table 26
Imperfections and amendments	15	SECTION IV. Of azimuths	p. 239
Discovery of the variation	16	To work them	{ by logarithms art. 28
			{ by Gunter's scale 29
		SECTION V.	

C O N T E N T S.

SECTION V. Of the variation of the compass p. 242 <i>What the variation is</i> art. 30 <i>To find the variation</i> 31 <i>To correct courses by variation</i> 32 SECTION VI. Of leeway p. 244 <i>What, and how allowed for</i> art. 33 <i>To correct the courses</i> 35 SECTION VII. Of quadrants p. 245 <i>Of Davis's quadrant</i> art. 36 <i>Of the fore-staff</i> 40 <i>Of Hadley's quadrant</i> 41 <i>To rectify and observe by it</i> 42 <i>Improvements</i> 50 <i>Of an artificial horizon</i> 51 <i>Corrections, by the dip</i> 52 <i>By refraction</i> 53 <i>By parallax</i> 54 <i>Tables for corrections of alts.</i> p. 255 SECTION VIII. Of comparing and correcting time p. 256 <i>To find the time, and correct the watch</i> 258 SECTION IX. To find the latitude at sea p. 261 1st. <i>By decl. and zen. dist.</i> art. 60 2d. <i>To correct the alts.</i> 63 3d. <i>By 3 zen. dist. at equal intervals, near noon</i> 64 4th. <i>By 3 zen. dist. at unequal intervals, near noon</i> 67 5th. <i>By 3 zen. dist. at equal intervals, not near noon</i> 68 6th. <i>By 3 alts. at unequal intervals, not near noon</i> 69 7th. <i>By 2 altitudes, the interval of time, declination, and latitude by account</i> 70 8th. <i>By declination, two altitudes, the interval of time, lat. by account, and course and distance run between the observation</i> 75 SECTION X. To find the longitude at sea p. 289 <i>Methods of correcting the long.</i> 290	METHOD I. By a current art. 79 II. <i>By the course and dist.</i> 80 III. <i>By the variation chart</i> 85 IV. <i>By a perfect time-keeper</i> 89 V. <i>By the sun's declination</i> 92 VI. <i>By the moon's culminating</i> 93 VII. <i>By eclipses of Jupiter's satellites</i> 94 VIII. <i>By eclipses of the moon</i> 97 IX. <i>By occultation of stars</i> 98 <i>To observe the angular distance of celestial objects</i> 102 <i>To correct the observed angular distance</i> 107 <i>Compendium for obtaining the true angular distance, with examples</i> 108 <i>Mr. Witchell's method</i> 109 <i>To find the Greenwich time answering to a given distance</i> 110 METHOD X. Longitude by observations of dist. and alts. 111 XI. <i>Longitudes by observations of the distance only</i> 112 SECT. XI. Of a ship's reckoning p. 317 <i>Of dead reckoning and the log-book</i> art. 113 <i>General precepts</i> 114 <i>Four independent days works to</i> 120 <i>Journal of a voyage from England to Madeira in 27 days, with each day's work</i> p. 324 to 341 APPENDIX p. 342 <i>Theory of Davis's quadrant</i> art. 124 <i>Theory of Hadley's quadrant</i> 126 <i>Dip of the horizon</i> 128 <i>Of refraction</i> 130 <i>Of latitude</i> 131 <i>Fccio's problem</i> 133 <i>De la Caille on longitude</i> 135 <i>Of the moon's phases</i> 136 <i>Of the time of high-water</i> 137 <i>Of the moon's rising</i> 141 <i>Tables of logarithms of numbers, sines and tangents</i> from p. 353 to 392
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F O R T I F I C A T I O N.

PART I. OF LAND FORTIFICATIONS.

From Page 1 to Page 51.

SECTION I. Of lines p. 2 <i>What, and where used</i> art. 6 to 9 <i>Dimensions and construction</i> to 17 VOL. I.	SECTION II. Of batteries p. 7 <i>Several kinds</i> art. 18 <i>Of the ditch and parapet</i> 22, 23 a <i>Platform</i>
---	--

C O N T E N T S.

<i>Platform and magazine</i>	28, 32	<i>Orillon and retired flank</i>	art. 114
<i>A facine battery</i>	34	<i>Double and casemated flanks</i>	116
<i>A coffer battery</i>	38	SECTION V. <i>Of works for the de-</i>	
<i>To construct embrasures</i>	46	<i>fence of the foss</i>	p. 38
<i>Profiles of batteries</i>	50	<i>Of tenails, caponiers, and rams-</i>	
SECTION III. <i>Additional works</i>	p. 16	<i>horns</i>	art. 117 to 120
<i>Of ramps and barbets</i>	51, 56	SECTION VI. <i>Of outworks</i>	p. 39
<i>Of cavaliers and traverses</i>	58, 59	<i>Necessary observations</i>	122
<i>Of pallisades and barriers</i>	61, 62	<i>Of the ravelin</i>	123
<i>Of redoubts and redan</i>	63, 71	<i>Of Coehorn's ravelin</i>	129
SECTION IV. <i>Fortifying of towns</i>		<i>Of counterguards</i>	131
	p. 23	<i>Of the tenaillon</i>	132
<i>Definitions</i>	art. 73 to 84	<i>Of horn and crown-works</i>	135
<i>General principles</i>	85	<i>Of the covered-way, glacis, and</i>	
<i>Table for regular works</i>	86	<i>places of arms</i>	139
<i>To draw the master-line</i>	89	<i>Of traverses and sally-ways</i>	142
<i>To make the rampart</i>	93	<i>Of communications</i>	144
<i>To make the foss and esplanade</i>	98	<i>Of works beyond the glacis</i>	145
<i>Of gates</i>	104	<i>Of profiles</i>	148
<i>Of streets and buildings</i>	109	<i>Construction of profiles</i>	152
<i>Of bridges</i>	112	<i>Of irregular fortifications</i>	160

PART II. OF MARINE FORTIFICATIONS.

From Page 52 to Page 76.

SECTION I. <i>Of harbours</i>	p. 52	<i>Of mooring in regard to the stream</i>	
<i>Artificial harbours</i>	art. 170		art. 200
<i>Natural harbours</i>	171	<i>Of mooring in regard to the wind</i>	
SECTION II. <i>Of the fortifying of</i>			204
<i>roads and harbours</i>	p. 53	SECTION V. <i>Of gallant actions</i>	
<i>Exemplified in five sorts</i>			p. 65
	from art. 174 to 185	<i>Battle of Santa Cruz</i>	1657 art. 207
<i>Of the figure and size of forts</i>	186	<i>Action at Londonderry</i>	1689 208
<i>Of proper forts and their disposi-</i>		<i>Action at Gibraltar</i>	1693 art. 200
<i>tion</i>	187	<i>Disposition at Cadiz</i>	210
SECTION III. <i>Of booms</i>	p. 58	<i>Disposition at St. John's</i>	1697 211
<i>To lay a boom</i>	art. 191	<i>Battle of Vigo</i>	1702 212
<i>In a straight channel</i>	193	<i>Action at Carthagen</i>	1741 213
<i>In the bend of a river</i>	195	<i>Action in river Hughly</i>	1757 214
<i>General maxims</i>	198	<i>Siege of Louisburg</i>	1758 215
SECTION IV. <i>Of mooring ships</i>	p. 63	<i>Taking of Quebec</i>	1759 216

In this Treatise are XVI Plates.

VOL. I.			VOL. II.					
	Plate	Page		Plate.	Page		Plate	Page
<i>Spherics</i>	I.	125		VII.	I	<i>Globular</i>	XII.	131
	II. III.	195		VIII.	35	<i>Sailing</i>	XIII.	202
<i>Astronomy</i>	IV.	224	<i>Plane</i>	IX.	43	<i>Days Works</i>	XIV.	352
	V.	250	<i>sailing</i>	X.	51	<i>Fortifica-</i>	XV.	22
<i>Geography</i>	VI.	329		XI.	59	<i>tion</i>	XVI.	76

A
D I S S E R T A T I O N
ON THE
R I S E A N D P R O G R E S S
OF THE
Modern A R T OF N A V I G A T I O N.

IT has been much disputed to whom the world was obliged for the mariner's compass. A late *Italian* writer indeed contends, after many*, that the honour of the invention is due to *Flavio Gioja* of *Amalfi* in *Campania*, who lived about the beginning of the 14th century†, though others say it came from the East, and was earlier known in *Europe*‡. However that may be, it is certain, this wonderful discovery gave rise to the present art of navigation; which seems to have made some progress during the voyages that were begun in the year 1420, by *Henry Duke of Viséu*§. This learned Prince, brother to *Edward King of Portugal*, was particularly knowing in cosmography, and sent for one master *James* from the island of *Majorca*, to teach navigation, and make instruments and charts for the sea¶.

These voyages being greatly extended, the art was improved under the succeeding monarchs of that nation. For *Roderic* and *Joseph*, physicians to King *John the Second*, together with one *Martin de Bohemia*, a *Portuguese* native of the island of *Fayal*, scholar to *Regiomontanus*, about the year 1485, calculated tables of the Sun's declination, for the use of the sailors, and recommended the astrolabe for taking observations at sea¶.

The famous *Christopher Columbus* is said, before he attempted the discovery of *America*, to have consulted *Martin de Bohemia*, with others, and during the course of his voyage to have instructed the *Spaniards* in

* Suitable to that verse of *Pannormitana*,
Prima dedit nautis usum magneti Amalphis.

† See Signor *Gregorio Grimaldi's* Dissertation on this subject in the *Memoirs* of the *Etruscan Academy of Certona*, tom. iii. p. 193. printed at Rome in 1742.

‡ *Histoire des Mathematiques*, par *M. Montucla*, à Paris, 1758.

§ *Marianæ Hist. Hispan.* lib. xx. cap. 11. and lib. xxvi. cap. 17. *Meguntis*, 1605.

¶ *Decados d'Asia* par *J. di Barros*, lib. xvi. 1552.

¶ *Maffei Hist. Indic.* lib. i. p. 6. printed at Florence in 1588.



iv DISSERTATION ON THE RISE, &c.

navigation *; for the improvement of which art, the Emperor *Charles* the Fifth afterward founded a lecture at *Seville* †.

The variation of the sea-compass could not be long a secret. *Columbus*, on the 14th of September 1492, observed it, as his son *Ferdinand* asserts †, though others seem to attribute that discovery to *Sebastion Cabot* ‖. And as this variation differs in different places, *Gonzales d' Oviedi* found there was none at the *Azores* §; where some geographers have thought fit in their maps to make their first meridian to pass through one of those islands; it not being then known, that the variation altered in time.

The use of the *Cross-Staff* now began to be introduced amongst the sailors. This very ancient instrument being described by *John Werner* of *Nuremberg*, in his *Annotations* on the first book of *Ptolemy's Geography*, printed in 1514; he recommends it for observing the distance between the Moon and some star, in order thence to determine the longitude. *Werner* seems to have been the greatest geometer, as well as astronomer, of the time. In 1522, he published a tract ¶, containing a specimen of the conics, with some solid problems: he also there determined the precession of the equinox more exactly than it had been done.

But the art of navigation still remained very imperfect, from the constant use of the plane-chart, the gross errors of which must have often misled the mariner, especially in voyages far distant from the equator. Its precepts were probably at first only set down on the earliest sea-charts, as that custom is continued to this day; and larger directions have been usually premised by the *Dutch*, to collections of their charts called *Wagoner's*, from the name of the publisher: The *Dutch* call these collections also by many other affected titles, such as *Fiery-Columns*, *Sea-Beacons*, *Mirrors*, *Atlases*, &c.

At length there were published in *Spanish* two treatises, containing a system of the art, which were in great vogue; the first by *Pedro de Medina*, at *Valladolid*, 1545, called, *Arte de Nauegar*; the other at *Seville*, in 1556, by *Martin Cortes*, with this title, *Breve Compendio de la Sphera, y de la Arte de Nauegar con nuevos Instrumentos y Reglas*. The author of this last tract says, he composed it at *Cadiz* in 1545.

These seem to have been the oldest writers, who had fully handled this subject; for *Medina*, in his dedication to *Philip* Prince of *Spain*, laments, that multitudes of ships daily perished at sea, because there were

* La Historia general y natural de las Indias par Gonzalles de Miedo, en Sevilla, 1535. And Descriptione de las Indias Occidentale, de Antonio de Herrera, en Madrid, 1601.

† Hackluyt, in the dedication of his first volume of Voyages, printed in 1599.

‡ In *Columbus's* life written in *Spanish*, which is very scarce, but it was printed in *Italian* at *Venice* in 1571.

‖ See *Livio Sanuto's Geographia*, at the same place in 1585. Dr. *William Gilbert*, de Magnete, London, 1600; and *Purchas's Pilgrim*, in 1625, vol. I.

§ *Cabot*, a *Venetian* by birth, first served our King *Henry* the Seventh, then the King of *Spain*, and lastly, returning to *England*, he was constituted grand pilot by King *Edward* the Sixth, with an annual salary of above 160 pounds. Of this famous navigator and his expeditions, many writers have made mention, both foreigners and *English*, as *Peter Martyr*, *Ramusio*, *Herrera*, *Holinshed*, Lord *Bacon*, and particularly *Hackluyt* and *Purchas*, in their Collections of Voyages.

¶ Opera Mathematica at *Nuremberg*, in qua to.

neither teachers of the art, nor books by which it might be learnt; and *Cortes*, in his dedication, boasts to the Emperor, that he was the first who had reduced navigation into a *compendium*, enlarging much on what he had performed*.

Medina gave ridiculous directions, how to guess at the place of the horizon, when it could not be seen; as also he defended the errors of the plane-chart, and advanced against the variation of the magnetic needle such absurd arguments, as *Aristotle* and his followers had done to prove the impossibility of the Earth's motion. But *Cortes* briefly and clearly made out the errors of the plane-chart, and seemed to reflect on what had been said against the variation of the compass, when he advised the mariner rather to be guided by experience, than to mind subtle reasonings. Besides he endeavoured to account for this variation, in imagining the needle to be influenced by a magnetic pole (which he called the point attractive) different from that of the world, and this notion has been farther prosecuted by others.

However, *Medina's* book, being perhaps the first of its kind, was soon translated into *Italian*, *French*, and *Flemish* †, serving for a long time as a guide to navigators of foreign countries.

But *Cortes* was our favourite author, a translation of whose work by Mr. *Richard Eden* was, on the recommendation of that great navigator Mr. *Stephen Burrough*, and the encouragement of the Society for making discoveries at sea, published at *London* in 1561; which underwent various impressions ‡, whilst that of *Medina*, though translated within twenty years after the other, seems to have been neglected, notwithstanding the encomiums bestowed on it by Mr. *John Frampton*, the translator.

A system of navigation at that time consisted of some such particulars as these: An account of the *Ptolemaic* hypothesis, and the circles of the sphere; of the roundness of the Earth, its longitudes, latitudes, climates, and eclipses of the luminaries; a kalendar; how to find the prime, epact, &c. and by the last the Moon's age, and thence the tides; a description of the sea-compass, not forgetting the loadstone, with something about the variation, called its north-easting and north-westing, for the discovering of which, by night as well as by day, *Cortes* said, an instrument might be easily contrived; tables of the Sun's declination for four years §, in order to find the latitude, from his meridian altitude; to do the same thing by those called the guard-stars in the north, and the crossiers in the south; of the course of the Sun and Moon; the length of the days; of time and its divisions; to find the hour of the day, and by the nocturnal

* The learned Don *Nicolo Antonio*, in his *Bibliotheca Hispanica*, printed at *Rome* in 1672, tom. i. p. 323. puts down a book, intitled, *Tratado de la Sphera y del marear con el regimento de las alturas*, written by *Francisco Faleiro*, a Portuguese, and printed at *Seville* in 1535; but perhaps there is a mistake in the date. He also mentions an edition of *Cortes* in 1551.

† The *Italian* and *French* translations were printed in 1554, the first at *Venice*, the other at *Lyons*; the *Flemish* edition, I have seen, was at *Antwerp* in 1580; perhaps it had been printed before.

‡ In the latter editions some mistakes in the translation are corrected.

§ *Cortes* sets down the places of the Sun for a twelvemonth, with an equation-table to correct those places, serving for many years to come; and also another table to find the Sun's declination from his longitude being given.

vi DISSERTATION ON THE RISE, &c.

that of the night; and lastly, a description of the sea-chart, on which to discover where the ship is, they made use of a small table, that shewed, upon an alteration of one degree in the latitude, how many leagues were run on each rhumb, together with the departure from the meridian. Besides some instruments were described, especially by *Cortes*; as one to find the place and declination of the Sun, with the days and place of the Moon; certain dials, the astrolabe and cross-staff, with a complex machine to discover the hour and latitude at once.

And after this manner the art continued to be treated, though from time to time improvements were made by the following authors.

As *Werner* had proposed to find the longitude by observations on the Moon; so *Gemma Frisius*, in a tract, intitled, *De Principiis Astronomiæ et Cosmographiæ*, printed at *Antwerp* in 1530, advised the keeping of the time by means of small clocks or watches for the same purpose, then, as he says, lately invented. He also contrived a new sort of cross-staff, which he describes in his treatise *De Radio Astronomico et Geometrico*, printed at the same place 1545, and in his additions to *Peter Apian's* Cosmography, gives the figure of an instrument, he calls a *Nautical Quadrant*, as very useful in navigation, promising to write largely on the subject; accordingly, in an edition he made anno 1553, of his above-mentioned book *De Principiis Astronomiæ*, &c. he delivers several *nautical axioms*, as he calls them, which with some alterations were repeated by his son *Cornelius Gemma*, in a posthumous piece of his father on the *Universal Astrolabe*, published in 1556. *Gemma Frisius* died in 1555, aged 45 years.

With us *Dr. William Cunningham*, in his *Cosmographical Glass*, printed in 1559, amongst other things briefly treats of navigation, especially shewing the use of the *Nautical Quadrant*, much praising that instrument.

But a greater genius than these undertook this subject; for the famous mathematician, *Pedro Nunez*, or *Nomius*, having so early as 1537 published a book, written in the *Portuguese* language, to explain a difficulty in navigation proposed to him by the commander *Don Martin Alphonso de Sufa*; which was thirty years after printed at *Basil*, in *Latin*, with the addition of a second book, the whole intitled *de Arte et Ratione Navigandi*; where he exposes, both truly and learnedly, the errors of the plane-chart; and besides gives the solution of several curious Astronomical Problems, amongst which is that of determining the latitude from two observations of the Sun's altitude, and the intermediate azimuth being given. He also delivers many useful advices about the art of navigation, particularly how to perform its operations on the globe. He observed, that though the oblique rhumbs are spiral lines, yet the direct course of a ship will always be the arch of some great circle, whereby the angle with the meridians will continually change; all that the steersman can here do for the preserving of the original rhumb, is to correct these deviations as soon as they appear sensible. But thus the ship will in reality describe a course without the rhumb-line intended; and therefore his calculations for assigning the latitude, where any rhumb-line crosses the several meridians, will be in some measure erroneous. He also again sets down his method of division of a quadrant by concentric circles *, which he had described

* The admirable division, now so much in use, is a very great improvement of this; so that when the famous *Dr. Edmund Halley*, the Royal Astronomer, revived

described in his ingenious treatise *de Crepusculis*, printed in 1542, imagining it had been practised by *Ptolemy*. There were also added other tracts of his, but the completest edition of his *Latin* works was made by himself at *Coimbra* in 1573. His treatise of *Algebra*, written in *Spanish*, was printed at *Antwerp* six years before.

In 1577 Mr. *William Bourne* published his treatise *, intitled, *A Regiment for the Sea*, which he designed as a supplement to *Cortes*, whom he frequently quotes. Besides many things common with others, *Bourne* gives a table of the places and declinations of thirty-two principal stars, in order to find the latitude and hour; as also a larger tide-table than that published by Mr. *Leonard Digges*, in 1556 †. He shews, by considering the irregularities in the Moon's motion, the errors of the sailors in finding her age by the epact, and also in their determining the hour from observing upon what point of the compass the Sun and Moon appeared. He advises in sailing toward high latitudes to keep the reckoning by the globe, as there the plane-chart errs most. He despairs of our ever being able to find the longitude by any instrument, unless the variation of the compass should be caused by some such attractive point, as *Cortes* had imagined. Though of this he doubts, and as he had shewn how to find the variation of the compass at all times, he advises to keep an account of the observations, as useful to discover thereby the place of a ship; which advice the famous *Simon Stevin* prosecuted at large in a treatise published at *Leyden* in 1599, intitled, *Portuum investigandorum Ratio Metaphrasto Hugone Grotio*; the substance of which was the same year printed at *London* in *English*, by Mr. *Edward Wright*, intitled, *The Haven-finding Art*.

But the most remarkable thing in this ancient tract is, the describing of the way by which our sailors estimated the rate a ship made in her course, by an instrument called the *log*. This was so named from the piece of wood or log that floats in the water, while the time is reckoned during which the line that is fastened to it is veering out. The author of this device is not known, and I find no farther mention of it till 1607, in an *East-India* voyage, published by *Purchas*; but from that time its name occurs in other voyages, that are amongst his collections. And henceforward it became famous, being taken notice of, both by our own authors, and by foreigners; as by *Gunter* in 1623, *Snellius* in 1624, *Mettius* in 1631, *Oughtred* in 1633, *Herigone* in 1634, *Saltonstall* in 1636, *Norwood* in 1637, *Fournier* in 1643; and indeed by almost all the succeeding writers on navigation, of every country. And it continues to be still in use as at first, though attempts have been often made to improve it, and other contrivances proposed to supply its place. Many of these have succeeded in quiet water, but proved useless in a troubled sea.

A following edition of this book was revised by the author, where, in the preface, he sets forth the gross ignorance of the old ship-masters, repeating some of the insipid jests they made use of to justify their want of

revived that by adapting it to his *Mural Arith*, some body here named it *A Nonius*, in which he has been followed by many.

* He had ten years before published what he calls *Rules of Navigation*, as appears from his *Almanac*, printed in 1571.

† In his treatise, intitled, *A Prognostication everlasting*, fol. 23.

viii DISSERTATION ON THE RISE, &c.

knowledge in their art. Amongst the additions, he enlarges on the account of the log-line. And at the end subjoins an *Hydrographical Discourse touching the five several Passages into Cathay*.

Bourne published other tracts, as one called *Inventions or Devises*, where he describes a method by *wheel-work* of measuring the velocity of a ship at sea, which artifice he attributes to one Mr. *Humfrey Cole*.

At *Antwerp*, in 1581, *Michael Coignet*, a native of the place, published a small treatise, intitled, *Instruction nouvelle des Points plus excellents & necessaires touchant l'Art de Naviger* *. This served as a supplement to *Medina*, whose mistakes *Coignet* well exposed. He there shewed, that as the rhumbs are spirals, making endless revolutions about the poles, numerous errors must arise from their being represented by straight lines on the sea-charts; and expressed his hopes of discovering a rule to remedy those errors; saying, that most of the speculations, delivered by the great mathematician, *Peter Nonius*, for that purpose, were scarce practicable; and therefore in a manner useless to sailors. In treating of the Sun's declination, he took notice of the gradual decrease in the obliquity of the ecliptic, a point long disputed, but now settled from the theory of attraction. He also described the cross-staff with three transverse pieces, as it is at present made, which he acknowledged to be then in common use amongst the mariners; but he preferred that of *Gemma Frisius*. He likewise gave some instruments of his own invention, which are now quite laid aside, except perhaps his nocturnal. As the old sea-table, mentioned above, erred more and more in advancing toward the poles; he set down another to be used by such as sailed beyond the 60th degree of latitude. At the end of the book is delivered a method of sailing on a parallel of latitude by means of a ring dial, and a 24 hour glass; on which the author very much values himself.

The same year Mr. *Robert Norman* † published a discovery, he had long before made, of the dipping of the magnetic needle, in a small pamphlet called *The Newe Attraction*, where he shews how to determine its quantity; and in speaking of the loadstone, he disputes against *Cortes's* notion, that the variation of the compass was caused by a point fixed in the heavens, contending that it should be sought for in the earth, and proposes how to discover its place. He also treats of the various sorts of compasses, setting forth at large the dangers that must arise from the then prevailing practice of not fixing, on account of the variation, the wire directly under the *flower-de-luce*; as compasses made in different countries have it placed differently. *Bourne* indeed had warned against this abuse, and there are many things common to both authors.

To *Norman's* piece is always subjoined, Mr. *William Burrough's Discourse of the Variation of the Compass or Magneticall Needle*. The author had been a famous navigator, having used the sea from fifteen years of

* It had been published in *Flemish*, but the *French* edition is the fullest. *Coignet* died in 1623, leaving many mathematical manuscripts, see *Valerii Andreae Bibliotheca Belgica*, printed at *Louvain* in 1643.

† He is commended for an excellent Artist by our authors, as *Bourne*, *Burrough*, Sir *Humfrey Gilbert*, *Huss*, *Potter*, *Blundeville*, *Wright*, and Dr. *Gilbert*.

age, and for his merit promoted to be Controller of the navy by Queen Elizabeth *. He shews how to determine the variation several ways, setting down many observations of it made by an azimuth compass of Norman's invention, but improved by himself. He demonstrates the falshood of the rules commonly used, to find the latitude by the guard-stars. He particularizes many errors in the then sea-charts, occasioned by the neglect of the variation; adding, *But of these coastes* (toward the North), *and of the inwarde partes of the countries of Russia, Muscovia, &c. I have made a perfect plat and description, by myne owne experience in sundrie voiajes and travailes, bothe by sea and lande to and fro in those partes, which I gave to her Majestie in anno 1578.* And, lastly, he justly finds fault with Coignet's instrument, called a *Nautical Hemisphere*; but speaks too severely against the writers of navigation, concluding thus:

But as I have already sufficientlie declared, the cumpas sheweth not alwaies the pole of the worlde, but uarieth from the same diversly, and in sayling describeth circles accordyngly. Which thing, if Petrus Nonius, and the rest that have written of Navigation, had jointlie considered in the tractation of their rules and instruments, then might they have been more availeable to the use of Navigation; but they perceivynge the difficultie of the thyng, and that if they had dealt therewith, it would have utterly overwhelmed their former plaussible conceits, with Pedro de Medina (who, as it appeareth, hauyng some small suspicion of the matter, reasoneth very clerkly, that it is not necessary that such an absurdity as the Variation, should be admitted in such an excellent art as Navigation is) they have all thought best to passe it over with silence.

The Spaniards too continued to publish treatises of the art; particularly at Seville was printed in 1585 an excellent *Compendium* by Roderico Zamorano; which is written clearly and with brevity, not being incumbered with such idle speculations as abound in *Medina* and *Cortés*. The author was Royal Lecturer at Seville, and contributed much to the reforming the sea-charts; as we are told by his successor, *Andres Garcia de Cespedes*, who also published a *Treatise of Navigation* at Madrid, in 1606.

As globes may be very serviceable for the mariner, Mr. Edward Mullineux set forth in 1592, at the charges of Mr. William Sanderson, merchant †, a pair much larger than those the famous geographer Gerard Mercator published in 1541. On the terrestrial one were described many new discovered countries, and traced out the respective voyages round the world by Sir Francis Drake in 1577, and Mr. Thomas Candish in 1586, with the progress Sir Martin Frobisher had made toward the north in 1576, to a place called his Straits.

These globes were accompanied with a tract containing their uses written in *English*; but in 1594 Mr. Robert Hues published a more elaborate one in *Latin*; wherein, amongst others, he solves by the globe the problem of determining the latitude from two heights of the Sun ob-

* Hackluyt's Voyages, vol. I. p. 417. printed in 1599.

† Mr. Sanderson was commended for his knowledge as well as generosity to ingenious men.

DISSERTATION ON THE RISE, &c.

served with the intermediate time being given *; and in the last part of his book, he performs the usual questions in navigation, premising a discourse on the rhumb-lines, where he attempts to refute what *Gemma Frisius* had asserted, who says, that they meet in the poles. At the conclusion he highly praises a treatise of Mr. *Thomas Hariot*, hoping it would be soon published, in which that author had treated of this subject upon geometrical principles, with great sagacity and judgment. But all the manuscripts of that great mathematician were lost, except his *Artis Analyticae Praxis*, which was published long after his death in 1631; wherein is first advanced that idea of algebraic equations, which has been ever since followed.

Hues † was a person of letters, and besides had been far at sea. Amongst other curious particulars, he gives a good account of the attempts that had been made at various times to measure the Earth. In the *Epistle* to Sir *Walter Rawley*, he takes an occasion to enumerate the many discoveries of our mariners in very different parts of the world. His book was received with great applause, and has been indeed a pattern for such as afterward handled the same subject. It has been often printed abroad, particularly in 1617 with the notes of *John Isaac Pontanus*, who omitted the *Epistle* and the mentioning of *Hariot*. However, from this mutilated edition it was translated into *English* by one Mr. *John Chilmead*, and published in 1639.

Amongst our sailors none were more famous than Captain *John Davis* ‡, who gave name to the straits which he discovered; and great matters were expected from his long experience and skill. In 1594 he published a small treatise, intitled, *The Seaman's Secrets*. This is written with brevity, though somewhat pedantically, and was esteemed in its time, an eighth edition being printed in 1657; so that it seems to have supplanted *Cortes*. *Davis* treats of plane sailing, calling it *horizontal*, and sets down the form of keeping a reckoning at sea. He likewise shews how to sail by the globe, and boasts of what he intended to do; much commending great circle sailing, without describing it, as also what he calls *paradoxal*; that is, by a projection on the plane of the equator with spiral rhumbs, saying, he will publish a chart for that purpose. But, above all, he extols the use of calculations in the cases of navigation, and promises to handle that subject.

* This problem has been discussed by Dr. *Henry Pemberton*, in the *Philosophical Transactions*, vol. li. part. 2d. 1760, p. 910, where he has also given some improvements in trigonometry.

† There is an account of *Hues* and *Hariot* in *Anthony Wood's Athen. Oxon.* vol. i. printed in 1721, as being both members of that university.

‡ Several of his voyages are in *Hackluyt's* and *Purchas's* collections. He and Captain *Abraham Kendal* are greatly praised by Sir *Robert Dudley*, in his *Arcano del Mare*, as keeping a perfect reckoning by the way of longitude and latitude, where are given two of their *Journals*. This *Dudley* was a natural son of the great Earl of *Leicester*, and had commanded, in 1594, a fleet against the *Spaniards*; but retiring to *Florence*, he assumed the titles of Duke of *Northumberland* and Earl of *Warwick*. His *Arcano* was printed at that place in two volumes in 1646 and 1647.

At the end of the book is given the figure of a staff of his contrivance, to make a back observation. Of this the author is so vain as to say, *Than which instrument (in my opinion) the seamen shall not finde any so good, and in all clymates of so great certaintie, the invention and demonstration whereof, I may boldly challenge to appertain unto my selfe (as a portion of the talent which God hath bestowed upon me) I hope without abuse or offence to any.*

This instrument seems to have for some time been in use; for *Adrian Metius*, in his treatise, intituled *Astronomiæ Institutio*, printed in 1605, gives a figure of it from an original, in the possession of *M. Frederic Hautman*, governor of *Amboyna*. But it soon yielded to one of a more commodious form, which is now commonly called *Davis's Quadrant**; as if it was also of his invention, and that perhaps only because a back observation is made by both instruments, so the quadrant itself was at first styled a *Staff* and *Back-Staff*.

The famous traveller *Signor Pietro della Valle* passing, in 1623, from *Ormuz* to *Surat* aboard an *English* vessel, where observing this quadrant much practised by the seamen, as it was quite new to him, takes an occasion to shew its use very distinctly, and says, they told him, that it had been lately invented and called *David's Staff*† from its author. Also Captain *Charles Saltonstall*, in his *Navigation*, describes it under the name of a *Back-Staff*; and in Captain *Thomas James's* famous voyage for discovering a north-west passage, begun in 1631, amongst the many instruments he carried along with him, are mentioned two of *Mr. Davis's Back-staves*, which were doubtless these quadrants.

Contemporary with *Davis* was *Mr. Richard Polter*, who, it is said, had been a principal master aboard the *Royal Navy*. He wrote a very small book, intituled, *The Pathway to Perfect Sailing*, where, from an observation he made in 1586, he would infer‡, that different loadstones communicated different degrees of variation to the magnetic needle, and therefore despises the publishing observations of that kind, as needless. His book was not printed till 1644, nor did it deserve to be published at all, as it abounds with mistakes, and is written fantastically, obscurely, and arrogantly.

But all this while the plane-chart, notwithstanding its errors were frequently complained of, continued to be followed; as its use is easy, and serves tolerably well in short voyages, especially near the equator.

However, a way to remedy these errors had, for some time, been inquired after. And *Gerrard Mercator* seems to be the first, who conceived the means of effecting this, in a manner convenient for seamen, by continuing to represent both the meridians and parallels of latitude by parallel straight lines, as in the plane-chart, but gradually augmenting the distances between the degrees of latitude in advancing from the equator toward

* It is called by the *French*, *Quartier Anglois*.

† *David Staff*, che in lingua *Inglese* vale à dir *legnodi David Viaggi*; Part 3. Letter 1. à *Roma*. This author not only praises the Captain *Nicholas Woodcock*, and other officers, but also the common sailors, for their care and skill; and says, the *Portuguese* lose a great number of ships for not being so exact in their observations as the *English*.

‡ Perhaps he should have thence concluded the variation altered, as was discovered afterward.

xii DISSERTATION ON THE RISE, &c.

either pole, that the rhumbs also might be extended into straight lines, so that a straight line drawn between any two places, laid down in this chart by their longitudes and latitudes, should make an angle with the meridians, expressing the rhumb leading from one to the other. But though *Mercator*, in 1569, set forth an universal map thus constructed *, it does not appear upon what principles he proceeded; probably, by observing in a globe furnished with rhumbs, what meridians the rhumbs passed at each degree of latitude. That he knew not the genuine principles, I shall make evident; our countryman, Mr. *Edward Wright*, was certainly the first who discovered them.

Wright insinuates, but without sufficient grounds, that this enlargement of the intervals between the parallels had been suggested before by *Cortes* †, and even by *Ptolemy* himself.

As to *Cortes*, he speaks of the number of the degrees of latitude, and not the extent of them; for his expression amounts to no more than this, that the degrees of latitude are to be numbered from the equator, and consequently both northward and southward from that line the numbers affixed to them must continually increase; and from any place having latitude (suppose *Cape St. Vincent* in *Spain*, which is his instance) the degrees of latitude will be denoted by numbers increasing toward the pole, and decreasing toward the equator. He had before expressly directed, that they should be all equal by measurement on a scale of leagues adapted to the map ‡.

The passage in *Ptolemy* §, referred to by *Wright* ||, does indeed relate to the proportion between the distances of the parallels and meridians, but contains no shadow of *Mercator's* scheme: for instead of proposing any gradual enlargement of the distances of the parallels in a general chart; that passage relates only to particular maps, and is more distinctly explained in the first chapter of his last book; where he advises explicitly not to confine a system of such maps to one and the same scale, but to plan them out by a different measure, as occasion shall require, with this only caution; that the degrees of longitude should in each bear in some measure that proportion to the degrees of latitude, which the magnitude of the respective parallels bear to a great circle of the sphere; and subjoins, that in particular maps, if this proportion be observed in regard to the middle parallel, the inconvenience will not be great, though the meridians should be straight parallels to each other; wherein his design is plainly no other, than that the maps should in some sort represent the figures of the countries they are drawn for. *Mercator*, who drew maps for *Ptolemy's* tables ¶, understood him in no other sense, thinking it an

* See his life, written by his intimate friend, *Gaulterus Gbymmius*, which was prefixed to an enlarged edition of his *Atlas*, published at *Duisburg*, in 1593, by *Rumoldus* his son, a year after his father's death. *Gerrard Mercator* was born in 1512.

† See the 2d chapter of *Wright's* book.

‡ *Part* 3d, cap. 2d. fol. 58.

§ *Geograph. lib. ii. cap. 1.*

|| In an advertisement set down on his universal map, at the end of his second edition of his book; and in this mistake he has been followed by others.

¶ In an edition he made of *Ptolemy's Geography*, in 1584.

improvement not to regulate the meridians by one parallel, but by two; one distant from the northern, and the other from the southern extremity of the map by a fourth part of the whole depth; whereby in his maps, though the meridians are straight lines, they are generally drawn inclining to each other toward the pole.

But *Mercator's* universal map, mentioned above, though the author designed it for the benefit of sailors, was so far from being readily adopted, that some of the most skilful amongst them objected to its usefulness. Thus Mr. *Burrough* says of it—By augmenting his degrees of latitude toward the poles, the same is more fitte for suche to beholde, as studie in cosmographie, by readyng authours upon the lande, then to bee used in Navigation at the sea.

And Mr. *Thomas Blundeville*, in his *Briefe Description of Universal Mappes and Cardes*, first printed in 1589, gives an account of this map, observing that *Barnardus Puteanus* of *Bruges* had published, in 1579, one altogether like it. And though *Blundeville* is so particular, as to set down numbers expressing the distances between each parallel of latitude in those maps; yet he seems to slight them, by saying, that no better rules than those given by *Ptolemy* can be devised. But what is delivered by this geographer about the construction of a general map, is a very indifferent performance, altogether unworthy the author of the *Almagest*, and not in the least corresponding with the sagacity shewn in two treatises on the *Planisphere* and *Analemma*, which the *Arabians* have handed down to us as *Ptolemy's* *.

Maginus also, at the end of his *Geographia Universa*, (the former part of which is a translation of *Ptolemy's*) first printed at *Venice*, in 1596, mentions this map of *Mercator*, and gives even a sketch of it; but seems to have no distinct conception of the author's design.

That *Mercator's* map was not rightly described, is manifest from the numbers given by *Blundeville*; and that he was ignorant of a genuine method of dividing the meridian, appears from a passage in his life, where the writer says *Mercator* often assured him, that this extending a sphere into a plane answered to the quadrature of the circle, as that nothing seemed to be wanting but the demonstration.

However, our authors now began to entertain favourable thoughts of it, perhaps from the report that Mr. *Wright* was about to treat on that subject. Dr. *Thomas Hood*, to the first edition which he gave of *Bourne's Regiment* in 1592, added a *Dialogue* of his own, called *The Mariner's Guide*, written only to shew the use of the plane chart, where he acknowledges and sets forth its errors, and highly praises *Mercator's*, saying, he had composed a treatise concerning it; but the indistinct account he gives of it, shews it would not be this author's lot to render it fit for the use of navigation. And Mr. *Blundeville*, in the following editions of his above-mentioned tract, omitted the commendation he had given of *Ptolemy's* method of delineating an universal map.

Mercator's scheme was not indeed contrived for representing the parts of a country in a just proportion to each other; but is appropriated to the use of mariners, who sail upon rhumbs by the guidance of the compass;

* These were published by *Fed. Commandinus*, one at *Venice*, in 1518, the other at *Rome*, in 1562.

xiv DISSERTATION ON THE RISE, &c.

which our countryman, Mr. *Edward Wright*, perfected *, by discovering a true way of dividing the meridian. An account of this he sent from *Caius* college, in *Cambridge*, where he was then a fellow. to his friend the above-mentioned Mr. *Blundeville*, containing a short table for that purpose, with a specimen of a chart so divided, together with the manner of dividing it. All which *Blundeville* published, in 1594, amongst his *Exercises*, in that part which treats of navigation †; where he has well delivered what had been before written on that art; insomuch that his book was long in great repute, a seventh edition having been printed in 1636. To the second edition, *Anno* 1606, and following ones, was added his former discourse of univereal maps.

In 1597, the Reverend Mr. *William Barlowe*, in his *Navigator's Supply*, gave a demonstration of this division, as communicated by a friend; saying, *This manner of carde has been publicly extant in print these thirtie yeares at least ‡, but a cloude (as it were) and thicke miste of ignorance doth keepe it hitherto concealed: And so much the more, because some who were reckoned for men of good knowledge, have by glauncing speeches (but never by any one reason of moment) gone about what they could to disgrace it.*

This book of *Barlowe's* contains descriptions of several instruments for the use of navigation, the principal of which is an *azimuth compass*, with two upright sights §; and as the author was very curious in making experiments on the loadstone, he discourses well and largely on the sea-compass; and still farther handles that subject in a tract he published some years after, intitled, *Magnetical Advertisements*.

At length, in 1599, Mr. *Wright* himself printed his famous treatise, intitled, *The Correction of certain Errors in Navigation*, which had been written many years before; where he shews the reason of this division ||, the manner of constructing his table, and its uses in navigation, with other improvements: A book, as Dr. *Halley* says, *well deserving the perusal of all such as design to use the sea ¶.*

In the preface, *Wright* complains of the obstinacy of our mariners,

* Some of our modern writers have said, *Mercator* took the hint from *Wright*, but that is a mistake; for *Mercator's* map was published thirty years before *Wright's* book, who frequently refers to it. See *Edward Sherburn's* translation of the first book of *Manilius*, in 1675, p. 86.

† Chap. 29.

‡ He should have said 28 only.

§ Many of these instruments are in the *Arcano del Mare*, together with the demonstration above mentioned.

|| Maps with their meridians thus divided had been published at *Amsterdam* by *Jodocus Hondius*, who, when in *London*, working as an engraver, learnt the manner of doing it from Mr. *Wright's Manuscript*; the fourth chapter of which he had transcribed into one of his maps. *Hondius* afterward in his letters, both to Mr. *Briggs*, and also to Mr. *Wright*, begged pardon for not having acknowledged the obligation. See *Wright's* preface, where he complains of *Hondius's* proceeding; and farther relates, how his book, a copy of which having been presented to the Earl of *Cumberland*, had liked to have come out under the name of a famous navigator, whom, from some circumstances there mentioned, I imagine to have been *Abrabam Kendal*.

¶ *Philosophical Transactions*, for 1696, N^o 219.

for not liking an improvement in their art, saying, that they were like those whose ignorance Master *Bourne* had exposed, repeating *Bourne's* very words:

"Though this great improvement in navigation by *Wright* has been embraced and followed by all proper judges; yet some undiscerning persons have of late, even amongst us, found fault with it, particularly *Henry Wilson*, author of a *Treatise on Navigation*, by a proposal for a *curvilinear sea-chart*, in 1720; and the Rev. Mr. *West*, of *Exeter*, in a posthumous piece, printed in 1762. But their *cavils* were sufficiently obviated; those of the first by Mr. *Hafelden*, in his *Mercator's Chart*, and in his *Reply*, both printed in 1722; and of the second, by Mr. *William Mountaine*, in the *Philosophical Transactions*, vol. LIII. p. 69, Anno 1763."

In 1610 a second edition of Mr. *Wright's* book was published, and dedicated to Prince *Henry*, his royal pupil*, where the author inserted farther improvements; particularly, he proposed an excellent way of determining the magnitude of the Earth; at the same time recommending, very judiciously, the making our common measures in some settled proportion to that of a degree on its surface, that they might not depend on the uncertain length of a barley-corn.

Some of his other improvements were, The Table of Latitudes for dividing the meridian, computed to minutes; whereas before it was but to every tenth minute, and the short table sent by him to *Blundeville* to degrees only: An instrument, he calls the Sea-rings, by which the variation of the compass, altitude of the Sun, and time of the day, may be determined readily at once in any place, provided the latitude be known: The correcting of the errors arising from the eccentricity of the eye in observing by the cross-staff: A total amendment in the Tables of the declinations and places of the Sun and stars, from his own observations, made with a six-foot quadrant, in the years 1594, 95, 96, and 97: A sea-quadrant, to take altitudes by a forward or backward observation, and likewise with a contrivance for the ready finding the latitude by the height of the pole-star, when not upon the meridian. And that his book might be the better understood by beginners, in this edition is subjoined a translation of the above-mentioned *Zamorano's Compendium*; he correcting some mistakes in the original, and adding a large table of the variation of the compass observed in very different parts of the world, to shew it is not occasioned by any magnetical pole.

This excellent person was allowed fifty pounds a year (no inconsiderable sum at that time) by the *East India Company*, for reading a lecture of navigation; he also projected the conveying water to *London*, but was prevented from executing his scheme by designing men, which is frequently the case. Whilst he led a studious and retired life, his reputation was so far known, that Queen *Elizabeth* granted, in 1589, a *dispensation* for his absence from the university, in order to accompany the Earl of *Cumberland* in the expedition to the *Azores*; as I am informed by

* In 1657 a 3d edition was published by Mr. *Joseph Moxon*, where the dedication is unadvisedly left out, and at the end is added by the editor the above mentioned *Haven finding Art*, as also *Wright's* universal map, improved by the discoveries made since his time.

xvi DISSERTATION ON THE RISE, &c.

Sir James Burrough, Master of Caius College, whose fine taste in architecture, part of the new buildings in Cambridge shew, they rendering the rest of those buildings a disgrace to that famous seat of learning, which has produced many great men, as, (to mention here only mathematicians) Wright, Briggs, Oughtred, Dr. Pell, Foster, Horrox, Bainbridge, Bishop Ward, Dr. Wallis, Dr. Barrow, Rooke, Sir Isaac Newton, Cotes, and Dr. Brook Taylor.

Wright's improvements on Mercator's chart became soon known abroad.

In 1608 were published the *Hypomnemata Mathematica* of the above-mentioned Simon Stevin, composed for the use of Prince Maurice. In the part concerning navigation, the author, having treated of sailing on a great circle, and shewn how to draw mechanically the rhumbs on a globe, sets down Wright's two tables of latitudes and of rhumbs, in order to describe those lines more accurately; and in an Appendix he commends Hues, shews a mistake committed by Nonius in relation to the rhumbs, and pretends to have discovered an error in Wright's latter table; but Wright himself, in the second edition of his book, has fully answered all Stevin's objections, demonstrating that they arose from his gross way of calculating.

And in 1624 the learned Willebrordus Snellius, Professor of the Mathematics at Leyden, published his *Typhis Batavus**, a treatise of navigation on Wright's plan, written somewhat obscurely. In the introduction are praised Nonius, Mercator, Stevin, Hues, and Wright. But since what had been performed by our artists on this subject, is not there particularly declared, as are the improvements made by the others; it has happened that some have attributed Wright's principal discovery to this author. Thus Albert Girard, who, in 1634, published a French translation of Stevin's Works with notes, in one of them observes, that Snellius had calculated, what he calls, *Tabulæ Canonicae Parallelorum*, to minutes as far as 70 degrees; whereas Wright had set forth in 1610 such a table so calculated to 89 degrees 59 minutes; notwithstanding which M. de Lagni, in the Memoirs of the Royal Academy of Sciences at Paris for 1703, treating of the *Corrected Chart*, says, *c'est Willebrord Snellius qui en est l'inventeur*. But the French writers now acknowledge our countryman to have been its author †.

Snellius was followed in Holland by Adrian Metius, in a treatise, intitled, *Primum Mobile*, printed at Amsterdam, in 1631; and in France by the learned Peter Herigone, in his *Cursus Mathematicus*, where, in the dedication of the fourth tome to the Marshal Bassompierre, the author says, *Artem navigandi in censu Mathematicis non reposuere plerique nostrum, neque sanè in hunc ordinem ascribi meruit, quandiu cæcâ tantum nautarum praxi celebrata est; nunc verò cum inventis tabulis loxodromicis (quas nos primum Gallis exhibemus) formam certam firmasque leges acceperit sine injuriâ omitti non potest*. But to return to our countrymen.

* In 1617 had been published his *Eratosthenes Batavus*, where is given an account of his measuring the earth.

† — *c'est qu' on appelle les cartes réduites, invention admirable, de la quelle on est redevable à Edouard Wright, quoiqu'on l'ait souvent attribuée à Mercator.* Hist. de l'Acad. Royale des Sciences, An. 1753, p. 275.

Mr. *Wright*, in the 12th chapter, having shewn how to find the place of a ship on his chart, observed, the same might be performed more accurately by calculation; but considering, as he says, that the latitudes, and especially the courses at sea, could not be determined so precisely, he forbore setting down particular examples; as the mariner may be allowed to save himself this trouble, and only mark out upon his chart, when truly constructed, the ship's way after the manner then usually practised.

However, in 1614*, Mr. *Raphe Handson*, among his nautical questions subjoined to a translation of *Pitiscus's Trigonometry*, solved very distinctly every case of navigation, by applying arithmetical calculations to *Wright's* table of latitudes, or of meridional parts, as it hath since been called.

And besides, though the method *Wright* discovered for determining the change of longitude by a ship sailing on a rhumb, is the adequate means of performing it; *Handson* proposed two ways of approximation for that purpose, without the assistance of *Wright's* division of the meridian line. The first was computed by the arithmetical mean between the co-sines of both latitudes; the other by the same mean between their secants, as an alternative, when *Wright's* book was not at hand, though this latter is wider from the truth than the first; and farther he shewed by the afore-said calculations, how much each of these *compendiums* deviates from the truth, and also how erroneously the computations on the principles of the plain chart differ from them all.

There is another method of approximation, by what is called The Middle Latitude †, which, though it errs more than that by the arithmetical mean between the co-sines; yet being less operose, is that generally used by our sailors; notwithstanding the arithmetical mean between the logarithmic co-sines, equivalent to the geometrical mean between the co-sines themselves, had been since proposed by Mr. *John Bassat* ‡, which in high latitudes is somewhat preferable.

The computation by the middle latitude, will always fall short of the true change of longitude; that, by the geometrical mean, will always exceed; but that, by the arithmetical mean, fall short in latitudes above 45 degrees, and exceed in lesser latitudes. However, none of these methods, when the change in latitude is sufficiently small, will deviate greatly from the real change in longitude.

About this time logarithms || began to be introduced into the practice of the mathematics; and as they are of excellent use in the art of navigation, we shall here say something about their original.

These were invented by *John Napier*, Baron of *Marchistoun* in Scot-

* It was reprinted in 1630.

† *Gunter's* works, first printed in 1623.

‡ About 1630, in a dialogue which was published after the author's death, in an Appendix to the *Pathway to perfect Sailing*. *Bassat* had been a teacher of navigation at *Chatham*, and well made out what he undertook, that a ship would return to the place it departed from, by sailing on the same rhumb, contrary to what *Fuller* and others had maintained. At the end of this discourse, he applies his compendium to the three principal problems in sailing.

|| The foundation of logarithms is a property of two series of numbers, one in arithmetical, the other in geometrical proportion; which property is declared by *Archimedes* in his *Arenarius*.

xviii DISSERTATION ON THE RISE, &c.

land, as appears from his treatise, intitled, *Mirifici Logarithmorum Canonis Descriptio*, first printed in 1614*. Soon after, the author communicated to Mr. Henry Briggs, Professor of geometry at Gresham college in London†, another form of logarithms; with which Mr. Briggs was so well pleased, that he immediately set about computing a very large table of them, which he published in 1624, with his *Arithmetica Logarithmica*‡. But in the mean time, as a specimen, he printed in 1617 a few copies for his own use and that of his friends, of a very small one, not exceeding a thousand natural numbers.

From this table Mr. Edmund Gunter, Mr. Briggs's colleague in Astronomy, computed one of artificial sines and tangents to every minute of the quadrant, which he published in 1620, being the first of its kind||. And when he made an edition of his works three years after, both these tables were subjoined to his book.

There he applied to navigation, according to Wright's table of meridional parts, as well as to other branches of the mathematics, his admirable Ruler§, on which were inscribed the logarithmic lines for numbers, and for sines and tangents of arches. He also greatly improved the Sector¶ for the same purposes. And he shewed how to take a back observation by the cross-staff, whereby the error, arising from the eccentricity of the eye, is avoided; describing likewise an instrument of his invention, named by him a Cross-Bow, for taking altitudes of the Sun or stars, with some contrivances for the more ready collecting the latitude from the observation**.

The discoveries relating to the logarithms were carried to France by Mr. Edmund Wingate, who, going to Paris in 1624, published in that

* In 1619 was made, after the author's death, a second edition, with his farther improvements in Spherical Trigonometry.

† He was in 1619 appointed by Sir Henry Saville, his professor of geometry at Oxford.

‡ Adrian Vlacq made an edition of this book at Tergou, in 1628, where the table of logarithms was continued by him to one hundred thousand numbers, though the logarithms themselves are but to ten places, whereas in Briggs's book they were to fourteen. Some copies of Vlacq's tables were purchased by our booksellers, and published at London, with an English explanation premised, dated 1631.

|| Vlacq also published, at the same place, in 1633, his *Trigonometria Artificialis*, with tables of logarithmic sines and tangents to every tenth second of the quadrant. Vlacq's tables have a great reputation for their exactness, as Sherwin's first edition in 1706, and Gardiner's in 1742, have amongst us. M. de Fontenelle, in the History of the Academy of Sciences for 1717, commends an edition of Vlacq's smaller tables, made at Lyons, in 1670, as does M. de la Lande, in his Astronomy, printed at Paris, in 1764, tables published there in 1760.

§ This Ruler is so constantly in the practice of our artists, that it has got the name of *The Gunter*.

¶ The uses of a Sector had been shewn by Dr. Robert Hood, in a tract he published in 1598.

** This ingenious person died 1626, aged 45 years. His works have been several times reprinted with successive additions; the second edition was made in 1636 from his own manuscript; then from those of Mr. Samuel Forster, Professor of Astronomy at Gresham College; again by Mr. Henry Bond, and Mr. William Leybourn. The fullest and last, being the fifth, was in 1673.

city two small tracts in *French**, and dedicated them both to *Gaston*, the King's only brother. In the first he teaches the use of *Gunter's Ruler*, and in the other, of the tables of logarithms and artificial sines and tangents, as modelled according to *Napier's* last form, attributed by *Wingate* to *Briggs*, which is a mistake; as appears from the dedication of *Napier's Rabbologia*, printed in 1616, and from what Mr. *Briggs* himself said in the preface of his *Arithmetica Logarithmica*.

The Reverend Mr. *William Oughtred* projected this Ruler into a circular arch, shewing fully its uses in a treatise first printed in 1633, intituled, *The Circles of Proportion*; where, in an Appendix, are well handled several important points in navigation. It has been made in the form of a Sliding Ruler. See *Seth Partridge's* use of the double scale in 1662.

As by the logarithmic tables all trigonometrical calculations are greatly facilitated; so the first author, who, I find, has applied them to the cases of sailing, was Mr. *Thomas Addison*, in his treatise, intituled, *Arithmetical Navigation*, printed in 1625. He also gives two traverse tables with their uses, the one to quarter points of the compass, the other to degrees.

Mr. *Henry Gellibrand*, Mr. *Gunter's* successor at *Gresham College*, published his discovery of the changes in the variation of the compass in a small quarto pamphlet, intituled, *A Discourse Mathematical on the Variation of the Magnetical Needle*, printed in 1635. This extraordinary phenomenon he found out by comparing the observations made at different times near the same place by Mr. *Burrough*, Mr. *Gunter*, and himself, all persons of great skill and experience in these matters. And this discovery was soon known abroad †, for father *Athanasius Kircher*, in his treatise intituled *Magnes*, first printed at *Rome* in 1641, says our countryman Mr. *John Greaves* had informed him of it, and then gives a letter of the famous *Marinus Mercennus*, containing a very distinct account thereof. *Gellibrand* had been famous for the part he bore in the *Trigonometria Britannica* of his deceased friend Mr. *Briggs*, which was printed in 1633, at *Tergou*, under the care of *Adrian Vlacq*. *Gellibrand* also, in 1635, published in *English* an *Institution Trigonometricall*.

In 1631 Mr. *Richard Norwood* had published an excellent treatise of *Trigonometry*, adapted to the invention of logarithms, particularly in applying *Napier's* general canons ‡. The author having, as he says, acquired his knowledge in the mathematics at sea §, especially shewed the use of trigonometry in the three principal kinds of navigation. And toward the farther improvement of that art, he undertook a laborious work for examining the division of the log-line.

* These were afterwards printed at *London*, in *English*, with improvements.

† In the History of the Royal Academy of Sciences at *Paris* for 1712, p. 19, it is said by M. *de Fontenelle*, that the learned *Peter Gassendi* was the principal discoverer of this property; but *Gassendi* himself acknowledged that he had before received information of *Gellibrand's* discoveries. *Gassend. Oper.* vol. ii. p. 152, *Ludg.* 1658.

‡ A very advantageous report of it was made by M. *Mariotte*, at a meeting, in 1668, of the Academy. *Du Hamel, Hist. Acad. Scient.* p. 51. 1701.

§ From a sailor he became a teacher, styling himself before his books, A Reader of the Mathematics in *London*.

XX DISSERTATION ON THE RISE, &c.

As altitudes of the Sun are taken on ship-board, by observing his elevation above the visible horizon; to collect from thence the Sun's true altitude with correctness, *Wright* observes it to be necessary, that the dip of the horizon below the observer's eye should be brought into the account, which cannot be calculated without knowing the magnitude of the earth. Hence he was led to propose different methods for finding this; but complains, that the most effectual was out of his power to execute; and therefore contented himself with a rude attempt, in some measure sufficient for his purpose: and the dimensions of the Earth deduced by him corresponded so well with the usual divisions of the log-line, that as he wrote not an express treatise on navigation, but only for the correcting such errors as prevailed in general practice, the log-line did not fall under his notice. But Mr. *Norwood*, for regulating this instrument upon genuine principles, put in execution the method Mr. *Wright* recommends, as the most perfect for measuring the dimension of the Earth, with the true length of the degrees of a great circle upon it; and, in 1635, actually measured the distance between *London* and *York*; from whence, and the summer-solstitial altitudes of the Sun observed on the meridian at both places, he found a degree on a great circle of the Earth to contain 367196 *English* feet, equal to 57300 *French* fathoms or toises, which is very exact; as appears from many measures, that have been made since that time.

Of this affair Mr. *Norwood* gives a full and clear account in his treatise, called *The Seaman's Practice*, first published in 1637. There, with unaffected modesty, he apologizes for the hardiness of a private person's undertaking so difficult a task; and very cautiously points out the true reason, how so great a mathematician as *Snellius* had failed in his attempt. He also shews various uses of his discovery, particularly for correcting the gross errors hitherto committed in the divisions of the log-line. But such necessary amendments have been little attended to by the sailors, whose obstinacy in adhering to inveterate mistakes has been always complained of by the best writers on navigation. This improvement has at length, however, made its way into practice: few navigators of reputation using now the old measure of 42 feet to a knot.

Farther, Mr. *Norwood* likewise there describes his own excellent method of *setting down and perfecting a Sea-Reckoning*, using a traverse-table, which method he had followed and taught for many years; and besides, shews how to rectify the course, by the variation of the compass being considered; as also how to discover currents, and to make proper allowance on their account.

This treatise, and that of *Trigonometry*, were continually reprinted, as the principal books for learning scientifically the art of Navigation. What he had delivered, especially in the latter of them, concerning this subject, was contracted as a manual for sailors, in a very small piece, called his *Epitome*, which useful performance has gone through numberless editions.

No alterations were ever made in *The Seaman's Practice*, till in the 12th edition, printed in 1676, after the author's decease, there began to be inserted, at page 59, the following paragraph in a smaller character [*About the year 1672, Monsieur Picart has published an account in French, concerning the measure of the Earth, a breviate whereof may be seen in the Philosophical*

Philosophical Transactions, N^o 112, wherein he concludes one degree to contain 365184 English feet, nearly agreeing to Mr. Norwood's experiment.] And this advertisement is continued in the subsequent editions, as I find it in one printed so lately as 1732.

Norwood's measure, therefore, though it was not known to the great Sir Isaac Newton in his youth, was not buried in oblivion, on account of the confusions occasioned by our civil wars, as M. de Voltaire has been pleased to say *; on the contrary, it has been constantly commended by our writers on navigation: as by Mr. Henry Bond, soon after its publication, in a note at page 107 of the *Seaman's Kalendar*, which ancient book he reprinted and improved, whose use, through numberless editions, is continued amongst our sailors to this day; by Mr. Henry Phillips in his *Geometrical Seaman* in 1652, and in his *Advancement of Navigation* in 1657; by Mr. John Collins in his *Navigation by the Plane Scale*, in 1659; by the reverend Dr. John Newton in his *Mathematical Elements*, in 1660; Mr. John Seller in his *Practical Navigation*, in 1669; Mr. John Brown in his *Triangular Quadrant*, in 1671.

And in the *Philosophical Transactions* for 1676, N^o 126, there is given a very particular account of it. Nor had it escaped the royal notice; for when King James, in 1690, honoured the observatory at Paris with a visit, he informed the gentlemen, then present, of this measure of the Earth; and upon their acquainting his Majesty how that had been determined by Mr. Picard, the King wished the two measures might be compared together †.

But that it was not commonly known in France is no wonder, seeing our books were not then so much inquired after as at present by that polite and learned people.

In the *Journal des Sçavans* for December 1666, it was observed of Dr. Hooke's *Micrographia*, qu'il est écrit en une langue que peu de personnes entendent; but long after, in the same *Journal* for February 1750, it is said of the English tongue, that it was une langue que tous les vrais sçavans devoient savoir. And now, as Norwood is taken notice of in the latter editions of Sir Isaac Newton's *Principia*, his name and merit indeed are become universally known. Inasmuch that a particular account of his measure is given by M. de Maupertuis, in the preface to his *Treatise of the figure of the Earth*, printed at Paris in 1738; wherein he describes his method of determining the length of a degree on the Earth in Lapland; and Norwood is mentioned by two learned Spanish sea officers, D. Jorge Juan, and D. Antonio d'Ulloa, in their *Voyage* printed at Madrid in 1748, which was undertaken, as they were appointed to accompany the French mathematicians, sent to measure a degree near the equator.

About the year 1645 Mr. Bond published in Norwood's *Epitome* a very great improvement in Wright's method, by a property in his meridian line, whereby its divisions are more scientifically assigned, than the author himself was able to effect; which was from this theorem, That these divisions are analogous to the excesses of the logarithmic tangents of half the respective latitudes augmented by 45 degrees above the logarithm of the radius.

* *Elemens de la Philosophie de Newton*, chap. xviii. printed at Paris in 1738.

† Du Hamel, *Hist. Academ. Regal. Scient.* p. 285.

xxii DISSERTATION ON THE RISE, &c.

This he afterwards explained somewhat more fully in the third edition of *Gunter's* works, printed in 1653, where, after observing that the logarithmic tangents from 45° upward increase in the same manner (as he expresses it) that the secants added together do, if every half degree be accounted as one whole degree of *Mercator's* meridional line; his rule for computing the meridional parts appertaining to any two latitudes (supposed on the same side of the equator) is laid down to this effect; To take the logarithmic tangent, rejecting the radius, of half each latitude augmented by 45 degrees, and dividing the difference of those numbers by the logarithmic tangent of $45^\circ 30'$, the radius being likewise rejected, and the quotient will be the meridional parts required, expressed in degrees. And this rule is the immediate consequence from the general theorem, That the degrees of latitude bear to 1 degree (or 60 minutes, which in *Wright's* table stands as the meridional parts for 1 degree) the same proportion as the logarithmic tangent of half any latitude augmented by 45 degrees, and the radius neglected, to the like tangent of half a degree augmented by 45 degrees, with the radius likewise rejected.

But here was farther wanting the demonstration of this general theorem, which was at length supplied by that great mathematician, Mr. *James Gregory* of *Aberdeen*, in his *Exercitationes Geometricæ*, printed at *London* in 1668; and since more concisely demonstrated, together with a scientific determination of the divisor, by Dr. *Halley*, in the *Philosophical Transactions* for 1695, N^o 219, from the consideration of the spirals into which the rhumbs are transformed in the stereographic projection of the sphere upon the plane of the equinoctial; which the excellent Mr. *Roger Cotes* has rendered still more simple, in his *Logometrio*, first published in the *Philosophical Transactions* for 1714, N^o 388.

It is moreover added in *Gunter's* book, that if $\frac{1}{26}$ of this divisor (which does not sensibly differ from the logarithmic tangent of $45^\circ 1' 30''$ curtailed of the radius) be used, the quotient will exhibit the meridional parts expressed in leagues: and this is the divisor set down in *Norwood's Epitome*.

After the same manner the meridional parts will be found in minutes, if the like logarithmic tangent of $45^\circ 0' 30''$ diminished by the radius be taken, that is, the number used by others * being 12633, when the logarithmic tables consist of eight places besides the index.

This Mr. *Bond*, who introduced so useful a discovery into the art, was a teacher of the mathematics in *London*, and employed to take care of and improve the impressions of the current treatises of navigation. In an edition of the *Seaman's Kalendar*, p. 103, he declared, he had discovered the longitude, by having found out the true theory of the magnetic variation; and to gain credit to his assertion, he foretold, that at *London* in 1657 there would be no variation of the compass, and from that time it would gradually increase the other way, which happened accordingly.

* See Mr. *Perkins's Treatise of Navigation* in Vol. I. of Sir *Jonas Moore's New System of the Mathematicks*, p. 208, printed at *London* in 1681. *Perkins's* book was published by itself the year following, under the title of the *Seaman's Tutor*.

Again,

Again, in the *Philosophical Transactions* for 1668, N^o 40, he published a table of the variations for 49 years to come.

This joyful news to all sailors acquired Mr. Bond a great reputation; insomuch that the treatise he had composed, called *The Longitude found*, was in 1676 published by the special command of King Charles the Second, and ushered into the world with the approbation of several of the most eminent mathematicians of that time *.

But it was soon opposed, there being published at London a book in 1678, called *The Longitude not found*, written by Peter Blackborrow. And indeed as Bond's hypothesis did not in any wise answer its author's sanguine expectations, the famous Dr. Halley again undertook this affair; and from a multitude of observations he would conclude, that the magnetic needle was influenced by four poles. His speculations on this subject are delivered in the *Philosophical Transactions* for 1683, N^o 148, and for 1692, N^o 195. But this wonderful phenomenon seems to have hitherto eluded all our researches.

However, that excellent person in 1700 published a general map, on which were delineated curve lines expressing the paths, where the magnetic needle had the same variation. This was received with universal applause †, as it may lead to some discovery in so abstruse an affair, and at present be useful on many occasions in determining the longitude. The positions of these curves will indeed continually suffer alterations; but then they should be corrected from time to time; as they have been for the year 1744, and 1756, by two ingenious persons, Mr. William Mountaine and M. James Dodson, Fellows of the Royal Society. The latter died not long after he had been chosen, for his merit, mathematical master, at *Christ's Hospital*, in London.

Dr. Halley also gave, in the *Philosophical Transactions* for 1690, N^o 183, a dissertation on the monsoons, containing many observations very useful for all such as sail to places that are subject to those winds.

The true principles of navigation having been settled by Wright, Norwood, and Bond, many authors amongst us trod in their steps, making some little improvements. It would be impossible to enumerate each particular. Of the writers already mentioned, Phillips and Collins, in the title pages of their books, declare what they aimed at; Phillips also, in his tract called the *Advancement of Navigation*, recommends a pendulum instead of a half minute glass, to estimate the time the log-line is running out. He also proposes to do the same thing by wheel-work. Besides, in the *Philosophical Transactions* for 1668, N^o 34, he delivers a better method to determine the tides by than what was commonly practised; for which purpose Mr. John Flamsteed, the Royal Astronomer, still gave more perfect directions in the same *Transactions* for 1683, N^o 143; as

* In the *Philosophical Transactions* for the same year, N^o 130, it is said, the Lord Brouncker's name was inserted by mistake.

† It is particularly commended in the History and Memoirs of the Royal Academy of Sciences at Paris, for the year 1701, 1705, 1706, 1708, and 1710. See also Mr. Robins's Reflections in his Introduction to Lord Anson's Voyage round the World, made in 1743, &c. as also in the ninth chapter of the first book, and eighth of the third.

xxiv DISSERTATION ON THE RISE, &c.

likewise he first ordered a glass lens to be fixed on the shade vane, in what is called *Davis's quadrant* *, which contrivance Dr. *Robert Hook*, Professor of Geometry at *Gresham College*, had before thought of †.

Seller's Practical Navigation, though without demonstrations, has the rules of sailing in the different kinds, as performed by calculation, by the plane scale, by the *Gunter*, and by the sinical quadrant, with various other matters relative to the art; as also the use of the azimuth-compass as now modelled, the ring-dial, the sea-ring, cross-staff, *Davis's quadrant*, plough, nocturnal, inclinatory needle and globe, together with all the necessary tables; the whole being delivered in a manner so well adapted to the general humour of mariners, that it has undergone numberless editions; the last, I have seen, was in 1739; but some late writers seem to have abated the run of this book.

As in sailing especial regard ought to be had to the lee-way a ship makes, so many authors have touched upon this point; but the allowances usually made on that account are very particularly set down by Mr. *John Buckler*, and published in a small tract first printed in 1702, intitled, *A New Compendium of the whole Art of Navigation*, written by Mr. *William Jones*.

We ought not here to pass over in silence the very useful invention of Dr. *Gowin Knight*, which is the making *artificial magnets*, that are of greater efficacy than the natural ones. Though the Doctor has not thought fit to reveal his secret; yet others have found it out, who have made it public, particularly the Rev. *John Mitchel*, and Mr. *John Canton*; the first in a treatise of *Artificial Magnets*, printed in 1750; the other in the *Philosophical Transactions*, vol. XLVII. Ann. 1751.

The Earth being now universally agreed to be not a perfect globe, but a spheriod, whose diameter at the poles is shorter than any other; the Rev. Dr. *Patrick Murdoch* published a tract in 1741, where he accommodated *Wright's* sailing to such a figure; and Mr. *Colin Maclaurin*, the same year, in the *Philosophical Transactions*, N° 461, gave a rule to determine the meridional parts of a spheriod, which speculation he farther treats of in his book of *Fluxions*, printed at *Edinburgh*, in 1742.

Though Sir *Isaac Newton* in his *Principia*, first printed in 1686, had demonstrated from the theory of gravity, that this must be the real form of the Earth, as it revolved about an axis; yet in the year 1718 M. *Cassini* again ‡ undertook from observations to shew the contrary, and that the Earth was a spheroid, having its longest diameter passing through its poles §; and in 1720 M. *de Mairan* advanced arguments, supposed to be strengthened by geometrical demonstrations, to confirm farther M. *Cassini's* assertion. But in the *Philosophical Transactions* for 1725, N° 386,

* See the above-mentioned *Perkins's Navigation*, page 250.

† See Bishop *Thomas Sprat's* excellent History of the Royal Society in 1667, page 246; and *Hook's* Posthumous Works, published by *Richard Waller*, Esq. in 1705, p. 557.

‡ In the Memoirs of the Royal Academy of Sciences at *Paris*, his father in 1701, and he in 1713, attempted to prove the Earth was an oblong spheroid.

§ M. *John Bernoulli* in his *Essai d'une Nouvelle Physique Celeste*, printed at *Paris* in 1735, triumphs over Sir *Isaac Newton*; vainly imagining these precarious observations could invalidate what Sir *Isaac* had demonstrated.

387, 388, Dr. *Desaguliers* published a dissertation, wherein he made appear the weakness of the reasoning, and the insufficiency of the observations, as they were managed, to settle so nice an affair. He there also proposed a proper method for adjusting this point, when he says, *If any consequence of this kind could be drawn from actual measuring, a degree of latitude should be measured at the equator, and a degree of longitude likewise measured there; and a degree very northerly; as, for example, a whole degree might be actually measured upon the Baltic sea, when frozen, in the latitude of sixty degrees.* There, according to M. *Cassini's* last supposition, a degree would be 56653 toises; whereas at the equator it would be of 58019 toises, the difference being 1366 toises, about the two-and-fortieth part of a degree, which must be sensible; and likewise the degree of longitude would, according to him, be of 56817 toises, less by 1202, or the forty-eighth part, than a degree of latitude at the same place.

On this admonition, in 1735, there were sent from *France* two sets of mathematicians, members of the Royal Academy of Sciences; one towards the pole, the other to the equator, in order to measure, at each place, the length of a degree on the meridian. The report they brought home, quite overfet what had been urged in favour of the oblong figure; a degree towards the north, in the latitude of $66^{\circ} 20'$, being found to contain about 57438 toises, and near the equator but 56750.

This unwelcome news caused a degree to be again measured in *France*, which at length came out to be consonant with those which had been brought from very distant parts of the world. Thus these mathematicians confirmed by painful observations, what Sir *Izaak Newton* had, as M. de *Maupeflus* used to say, determined in his elbow-chair; Sir *Izaak* making the length of a degree under the pole to be 57382, and at the equator 56637 toises. And perhaps no observations can be exact enough to determine this matter more precisely.

But let us mention some of the foreign writers on navigation.

At *Rome*, in 1607, came forth a treatise, intituled, *Nautica Mediterranea*, written in *Italian* by *Bartolemew Crescenti*, the Pope's engineer. The author misses no opportunity of exposing the errors of *Medina*; but scarce gives any thing of his own, except a machine for measuring the way a ship made.

As the *Jesuits* have treated of most branches of learning, so this art has not been beneath their consideration; the three following authors having been of their society.

At *Paris*, in 1633, Father *George Fournier*, published an *Hydrography*, principally relating to navigation. The author would persuade us, that one of *Dieppe* had corrected the plane chart; and that the *Hollanders* learnt of the *French* the making charts so corrected; whereas this had been engraved long before at *Amsterdam*, by *Isaac Hondius*, and others.

John Baptist Riccioli, in his *Geographia & Hydrographia Reformata*, printed at *Bologna* in 1661, inserts a treatise of navigation, collecting his materials from almost every writer, as he does in his *Almagest* and *Chronology*, which is indeed the chief merit of his works.

Father *Millet Dechalles* wrote on this subject after a more masterly manner, both in his *Cursus Mathematicus*, first printed at *Lyons* in 1674, and in a *French* treatise, published in 1677, intituled *L'Art de Naviger démontré par Principes*.

These three authors, besides treating of the different kinds of sailing, abound in methods for taking of altitudes, finding the variation, and estimating the way a ship makes, &c. They also describe a machine resembling that of *Crescenti*. *Riccioli* gives a very faulty measure of the Earth, made by himself; and *Dechalles* advises the use of a pendulum in reckoning by the log-line, as also of wheel-work for the same purpose, as *Phillips* and *Cole* had done.

But there were writers in *France* between *Fournier* and *Dechalles*. For in 1666, and the following years, there were printed at *Dieppe* several tracts handling different parts of navigation, composed by M. G. *Denys*, which have been often reprinted.

And in 1671 the *Sieur Blondel S. Aubin* published a book called, *L'Art de Naviger par le Quartier de Reduction*, describing an instrument * much in use amongst the *French* sailors, by which may be performed, as by the sinical quadrant, the operations of navigation, though not much more speedily than by the traverse table, and not at all so accurately. He also published in 1673 his *Tresor de la Navigation*, where the art is well treated of, particularly by calculations.

M. *Saverien*, in his *Marine Dictionary*, printed at *Paris* in 1758, says, that M. *Dassier* seems to have been the first of the *French* writers that shewed the use of *Gunter's* scales [*échelles Angloises*] in his *Pilote expert*, printed in 1683.

At *Paris*, ten years after, was published the first part of a pompous work, intitled, *Le Neptune François*, by order of the *French* king, consisting of sea-charts, according to *Wright's* scheme, made from the latest observations, and reviewed by Mess. *Pene*, *Cassini*, and others. As this contained the charts of *Europe* only, there were added others of different parts of the world, printed the same year at *Amsterdam*. The whole was preceded by a discourse of M. *Sauveur*, who had formed some of the charts, where he shews how to perform the problems of astronomy and navigation by scales; which discourse had been published by itself at *Paris*, in 1692.

M. *John Bouguer* composed, by authority, his *Traité Complet de la Navigation*, first printed in 1698, which was well received, as containing most of the practices then known; and Father *Pézenas*, Jesuit and Royal Professor of Hydrography at *Marseilles*, published there, in 1733, a tract, called, *Elemens de Pilotage*; and at *Avignon*, in 1741, a larger work, intitled, *Practique du Pilotage*. This author shews how to find the meridional parts by the *Artificial Tangents*, an old discovery amongst us, declared so long ago as 1645, in *Norwood's Epitome*: he also has been industrious in translating several of our mathematical books into *French*.

But in 1753 M. *Peter Bouguer*, son of the former, published a very elaborate treatise on this subject, intitled, *Nouveau Traité de Navigation*, which is written sensibly, the author being an excellent mathematician, and famous for other productions. He there gives a variation-compass †

* It is only a kind of skeleton of *Wright's* universal map.

† Many of these sorts of compasses have been proposed at different times, as by M. *Buache*, in the *Memoirs* of the *French* Academy of Sciences for 1752, page 377; Captain *Christopher Middleton*, in the *Philosophical Transactions*, N^o 450, Ann. 1738; and Dr. *Knight*, as improved by the ingenious Mr. *John Smeaton*, *ibid.* N^o 495, Ann. 1750.

of his own invention, and attempts to reform the log, as he had done in the Memoirs of the Academy of Sciences for 1747. He is also very particular in determining the lunations more accurately than by the common methods, and in describing the corrections of the dead reckoning.

The excellent astronomer, M. de la Caille, in 1760, made an edition of M. Bouguer's book, which he somewhat abridged and improved.

In 1766, came out at Paris, a treatise, with this title, *Abregé du Pilotage divise en deux parties, ou on traite principalement des Amplitudes, des Loxodromies, dans l'hypothese de la Sphere et de Spherode, des marées, des variations de l'aiman.*

The former part of this book was first published in 1693. Here the whole is improved by M. le Monnier.

Though the Spaniards were the earliest writers on navigation, yet they were very backward to adopt its improvements. Indeed Antonio de Naiera published at Lisbon, in 1628, a treatise, intitled, *Navegacion especulativa y practica*; where, though the author rectifies the tables of the Sun and fixed stars, from Tycho Brahe's observations, he proceeds no farther in the theory of navigation than what had been advanced by Nonius, as followed by Cespedes. But of late, in 1712, was printed at the same place, *Arte de Navegar por Manuel Pimental*; where is shewn the use of Wright's chart, which, in imitation of the French, the author calls *Charta Reduzida*. He likewise describes Davis's quadrant, and mentions Norwood and Picard's measures of the Earth. In 1757 a treatise was printed at Cadiz, intitled, *Compendio de Navegacion para el uso de los Cavalleros Guardias Marinas*, written by the ingenious gentleman mentioned above, Don Jorge Juan. This is a good performance, delivering very distinctly the several parts of the art, as now improved. Some things are here omitted, that usually occur in books on this subject; but for the knowledge of such particulars, references are made to tracts composed expressly for the use of the society of gentlemen, destined for the sea-service.

Bouguer and Jorge Juan, describe and commend the method of dividing instruments for taking of angles, published by Peter Vernier, in a treatise, intitled, *La Construction, &c. du quadrant nouveau*, printed at Brussels, in 1631. This division is an improvement of that of Curtius, as that of Fernelius is of the division by diagonals*, and readily follows from the first Lemma of Clavius's treatise on the *Astrolabe*†, as has been observed by Pézenas, in a book he published at Avignon in 1765, intitled, *Astronomie des Marins*.

As to those who have treated of Wright's chart, I mentioned above Snel-lius and Metius. To an edition, in 1665, of Vlacq's small tables of logarithms, &c. is added, by Abraham de Gruel, one of the meridional parts, whose use he shews, with other parts of navigation, in his *Course of Mathematics*, written in Dutch, and printed at Amsterdam in 1676: the same was done by John Viret, in his *Flambeau reluisant ou Thresor de la Navigation*, at the same place, in 1677.

The Dutch are great navigators, and have been famous for their At-

* See Mr. Robins's Mathematical Tracts, where these divisions are largely treated of.

† First printed at Rome in 1693.

xxviii DISSERTATION ON THE RISE, &c.

lasses, before which are premised treatises of navigation, as has been already observed. The oldest I have seen of these, was published at *Leyden* in 1584, intitled, *Spiegel der Zee-Vaert* (or *Mirror of Navigation*;) by *Lucas Jansz Waghenae*r. In their later *Atlases* there is described an instrument to be used after the manner of *Davis's* quadrant, but where instead of circular arches are substituted straight lines.

Notwithstanding all the improvements hitherto mentioned, the *sea-reckonings*, though kept by such as were deemed very skilful mariners, are often found widely different from the truth. But this often happens through negligence, as I have heard *Dr. Halley*, who had used the sea, say.

These errors would be avoided, if from time to time the latitude and longitude could be determined. The first is generally obtained by the meridian altitude and declination of the Sun being given. The declination is got by the help of tables of the Sun, with an easy trigonometrical operation.

But even the latitude could not be very exact, before the famous *Kepler* had determined the true form of the Earth's orbit*. Hence were fabricated his *Tabulae Rudolphinae*. Next, those of *Mr. Thomas Street* were in great request†. But they, in their turn, yielded to *Dr. Halley's*, and his again to those of the accurate and elaborate *Mayer*; which, however, will want to be corrected hereafter: For, as *Sir Isaac Newton* has shewn, that all bodies mutually attract one another, the Earth will be disturbed in its motion by the actions of some of the other planets.

To find the longitude is a much more difficult affair. For this end, at present, the societies of learned men in *Europe* offer from time to time rewards to such as shall best treat of particular subjects in mathematics or physicks. Some of these have been relating to navigation, when *Polleni*, *Bernouilli*, *Bouguer*, and others, have obtained the prizes. And it is hoped, this institution may contribute to the advancement of the art.

Eclipses of the Moon were used of old; and *Kepler* recommended those of the Sun as preferable‡.

The satellites of *Jupiter* were no sooner discovered by the great *Galileo*§, than the frequency of their eclipses recommended them for this purpose; and amongst those who attempted this subject, none were more successful than *Signor Dominic Cassini*.

This great astronomer in 1688 published at *Bologna* tables for calculating the appearances of their eclipses, with directions for finding thence the longitudes of places; and being invited to *France* by *Lewis the Fourteenth*, he there published correcter tables in 1693. But the mutual attractions of the satellites on one another rendering their motions excessively irregular, the tables soon run out; insomuch that they require to be renewed from time to time, which has been performed by ingenious

* In his treatise *de Motu Martis*, in 1609.

† In his *Astronomia Carolina*, in 1661.

‡ *Tabulae Rudolph.* printed at *Ulm* in 1627, cap. xvi. & xxxii.

§ In his *Syderens Nuncius*, first printed at *Venice* in 1610.

persons, as Dr. *James Pound*, Dr. *James Bradley**, M. *Cassini* the son, and M. *Peter Wargentin*†; so that now many of the common *Almanacs* set down, when these eclipses happen throughout the year.

The Rev. *Nevil Maskelyne*, D. D. our present Royal Astronomer, has published annually, since the year 1767, by order of the Commissioners of Longitude, a work entitled, *The Nautical Almanac and Astronomical Ephemeris*, containing not only the eclipses of the satellites, but also many other tables, to enable the mariner to determine the longitude at sea; particularly tables of the distances which the Moon's center will have from that of the Sun, and from fixed stars, at every three hours, under the meridian of the Royal Observatory at *Greenwich*, and which have since been copied into the *Connoissance des Temps* for these latter years by the editor of that work.

The large reward granted by the Parliament for a practical way of discovering the longitude at sea, has put many upon the search: inso-much that several idle and absurd schemes have been offered by ignorant and wrong-headed men. But the perfecting the methods proposed long ago by *John Werner* and *Gemma Frisius*, seems at present to engage the attention of the public.

The theory of the Moon, though much amended by the noble *Tycho Brahe* and Mr. *Jeremy Horrox*‡, was found to be insufficient to answer this end. But the causes of her various irregularities having been discovered by Sir *Isaac Newton*, and her theory thence improved beyond expectation, gave great hopes of success: which have since been happily fulfilled by means of the improvements which have since been made in the methods of computing the several quantities of these inequalities by M. *Euler*, and *Tobias Mayer* of *Gottingen*§: The former of these gentlemen having been happy in reducing Sir *Isaac Newton*'s theory into neat analytical expressions, of which the latter availing himself, was, by a very singular address of his own, enabled to bring out the greatest quantities of the equations with ease and exactness, and thence to construct tables

* He succeeded Dr. *Halley* at *Greenwich*, where he made a great number of Astronomical Observations, which, as they are most accurate, it is hoped will not be lost. He became famous on observing and accounting for an apparent motion in the fixed stars, and called their aberration, which was immediately exhibited by the great mathematician Dr. *Brook Taylor* according to the exact theory of the Earth's motion. See Mr. *Robins*'s Mathematical Tracts, vol. II. page 276.

† *Wargentin*'s tables are much esteemed; they were first published at *Stockholm* in the *Acta Societatis Regii Scientiarum Upsalensis* for the year 1741, but since more correct from a new copy of the author's at *Paris* in 1759, by M. *de la Lande*. The ingenious author has rendered them yet more correct, and his labours on this head may be seen in the *Connoissance des Temps* for 1766, and the *Nautical Almanacs* for 1771, and 1779.

‡ This great genius died in 1641, scarce 23 years old. See his *Opera Posthuma*, published by the famous Dr. *John Wallis* at *London*, in 1673. *Horrox* first observed the Transit of *Venus* over the Sun in 1639. He wrote an account of this Phenomenon, which was published by the great astronomer *Hevelius*, at *Dantzic*, in 1661.

§ *Com. Societ. Reg. Gottingenf.* tom. II. page 283.

agreeing

xxx DISSERTATION ON THE RISE, &c.

agreeing to the Moon's motion in every part of her orbit, with very surprising exactness. And this ingenious person has left behind him tables still more exact *, for which the *British* Parliament have rewarded his widow with £. 3000, as also Mr. Euler with £. 300. These tables were published in 1770, by Dr. Maskelyne.

As to the method of *Gemma Frisius*, M. Huygens was persuaded it might be accomplished by his inventions of pendulum clocks and watches; a description of the first he published in a small tract, printed at the *Hague*, in 1658; and of the second, as improved, in the *Journal des Sçavans* for the month of *February*, 1675. And great expectations of success had been raised from some trials made in a voyage with these watches of the first construction, by Major *Holmes*; an account whereof is given in the *Philosophical Transactions*, *Ann.* 1669. But the various accidents those movements are liable to, soon caused that way to be laid aside.

Notwithstanding which, the ingenious Mr. *John Harrison* has for many years past employed himself in contriving a machine, that shall be free from all imaginable inconveniencies; and his endeavours were so well approved of by gentlemen of the greatest knowledge in these subjects, that the commissioners for the longitude thought fit to allow him some gratifications for his pains. He was afterwards farther considered, upon disclosing the internal structure of his machine, and the whole reward has since been given him by Parliament.

The difficulty of making observations at sea with sufficient exactness for finding the longitude, was feared to be insurmountable; but attempts have not been wanting to overcome it. In the *History of the Royal Society*, at page 246, we meet with the first mention of an invention in these words: *A new instrument for taking angles by reflection, by which means the eye at the same time sees the two objects both as touching the same point, though distant almost to a semicircle; which is of great use for making exact observations at sea.* A figure of this instrument, drawn by Dr. *Hook*, the inventor, is given in the Doctor's posthumous works, with a description, at page 503. But here, as one reflection only was made use of, it would not answer the purpose. However, this was at last effected by Sir *Isaac Newton*, who communicated to Dr. *Halley*, about the year 1700, a paper of his own writing, containing a description of an instrument with two reflections, which soon after the Doctor's death was found among his papers by Mr. *Jones*, who communicated it to the *Royal Society*, and it was published in the *Philosophical Transactions*, N° 465, *Ann.* 1742.

How it happened that Dr. *Halley* never mentioned this in his lifetime, is very extraordinary; seeing *John Hadley*, Esq. † had described,

* See his *Elogium* in the *Nova Acta Eruditorum*, for March 1762.

† Mr. *Hadley* being well acquainted with Sir *Isaac Newton*, might have heard him say, *Hook's* proposal could be perfected by means of a double reflection. However, Mr. *Hadley*, being a very ingenious person, might have hit on the same thought; as well as Mr. *Godfrey* of *Pennsylvania*; to whom the invention of this admirable instrument has been ascribed by some gentlemen of that colony: this is not the only case, wherein different persons have produced similar inventions.

in N° 430, *Ann.* 1731, an instrument grounded on the same principles which is so well esteemed, that our shops abound with them, accommodated with *Vernier's* division, as they are made by our most skilful workmen; and are now in general use amongst the skilful seamen of most of the maritime nations.

Though *Medina's* method for finding the place of the horizon was absurd; yet, for this end, several plausible ones have been proposed by ingenious persons, as Mess. *Elton*, *Hadley*, *Godfrey*, and *Leigh*; and that chiefly by applying a level to *Davis's* quadrant. Their devices are described in the *Philosophical Transactions* for 1732, 33, 34, and 37. And, lastly, an *Horizontal Top*, invented by the late Mr. *Serfon*, who was unfortunately lost at sea aboard the *Victory* man of war, has been approved of, and published by Mr. *Smeaton* in the *Philosophical Transactions*, vol. XLVII. for 1752, part ii. page 352.

Some methods used for obtaining the place of the horizon, and of observing with Mr. *Hadley's Reflecting Sector*, are described by Mr. *Robertson*, in his *Elements of Navigation*; which treatise has deservedly met with the approbation of the public.

Thus have I endeavoured to trace out the principal steps by which the art of navigation has advanced to its present height; nor without hopes that the attempt may not prove altogether unacceptable to those whose business or curiosity lead them to be acquainted with this very useful branch of the mathematics: on the successful practising of which depends, in an especial manner, the flourishing state of our country.

This Dissertation, written at first by desire, is now reprinted with alterations. Though I may be thought to have dwelt too long on some particulars not directly relating to the subject; yet I hope that what is so delivered, will not be altogether unentertaining to the candid reader. As to any apology for having handled a matter quite foreign to my way of life, I shall only plead, that very young, living in a sea-port town, I was eager to be acquainted with an art that could enable the Mariner to arrive across the wide and pathless ocean at his desired harbour.

London.

JAMES WILSON.

ADVERTISEMENT.

AS it may be expected that four kinds of readers will look into this book, it was thought convenient to point out to some of them, the places where they may meet with what they more particularly want.

FIRST. Those who having made a proficiency in the mathematics, will, it is likely, examine in what manner the subjects are here treated, and whether any thing new is contained therein: it is conceived that such readers will find some things which may recompence them for their trouble, in almost every one of the books.

SECONDLY. Those learners, who are desirous of being instructed in the art of Navigation in a scientific manner, and would chuse to see the reason of the several steps they must take to acquire it: To such persons, it is recommended that they read the whole book in the order they find it; or, if the learner is very young, he may omit the IVth and Vth books till after he is master of the VIth and VIIth.

THIRDLY. That class of readers, which, with too much truth may be said, comprehends most of our mariners, who want to learn both the elements and the art itself by rote, and never trouble themselves about the reason of the rules they work by: As it is probable there ever will be many readers of this kind, they may be well accommodated in this work; thus, if they are not already acquainted with Arithmetic and Geometry, let them read the five first rules of Arithmetic, to page 20; thence proceed to the definitions and problems in Geometry, from page 43 to 58. In the book of Trigonometry, read pages 89, 90, 91, 92, 98, 99, and from 104 to 114: the whole of book VI. In book the VIIth they may read to page 35, and as much more as they please. In book VIII, let them read the sections III, IV, V, VI, from page 146 to page 190. In book V, they may read section III, and as many problems in the Vth and VIth sections as they can; and let them read the whole of the ninth book.

FOURTHLY. That set of readers who will not be at the pains of learning any thing more than how to perform a day's work; such may herein meet with the practice almost independent of other knowledge. Let such persons make themselves acquainted with section IV. of book VI, and the use of the table at page 374; then learn the use of the Traverse Table at the end of book VII, which they will find exemplified between pages 8 and 35, Vol. II; also they must learn the use of the Table of Meridional parts at the end of book VIII. After which, they may proceed to book IX, where they will find ample instructions in all the particulars which enter into a day's work. But with this scanty knowledge of things, they will be obliged to omit some parts, which it is well worth their pains to be acquainted with.



THE
ELEMENTS
OF
NAVIGATION.

BOOK I.
OF ARITHMETICK.

SECTION I.
Definitions and Principles.

1. **A**RITHMETICK is a science which teaches the properties of numbers; and how to compute or estimate the value of things.
2. An UNIT or UNITY, is any thing considered as one.

3. NUMBER, in general, is many units.

4. DIGITS or FIGURES are the marks by which numbers are denoted or expressed, and are the nine following.

Digits, 1. 2. 3. 4. 5. 6. 7. 8. 9.

Names, One. Two. Three. Four. Five. Six. Seven. Eight. Nine.

And with these is used the mark 0, called cypher, which of itself stands for nothing; but being annexed to a digit, alters its value.

Thus 40 signifies forty; and 400 stand for four hundred, &c.

5. INTEGER, or WHOLE, NUMBERS, are such as express a number of things, each of which is considered as an unit.

Thus four pounds, twelve miles, thirty-four gallons, one hundred days, &c. are, in each case, called an integer number, or whole number.

6. FRACTIONAL NUMBERS, are those which express the value of some part or parts of an unit.

Thus one half, one quarter, three quarters, &c. are each the fractional parts of an unit.

VOL. I.

B

7. NOTA-

7. **NOTATION** is the expressing by digits or figures any number proposed in words; and the reading of any number that is expressed by figures, is called **NUMERATION**.

8. **DECIMAL NOTATION** is that kind of numbering in which ten units of any inferior name are equal in value to an unit of the next superior.

9. Every number is said to consist of as many places as it contains figures.

10. The value of every digit in any number is changed according to the place it stands in; and the reading of any number consists in giving to each figure its right name and value.

11. The right hand place of an integer number is called the place of units; and from this place all numbers begin, whether *whole* or *fractional*; the integers increasing in order from the unit place toward the left; and the fractions decreasing in order from the unit place toward the right: and to distinguish decimal fractions from integers, there is always a point or comma (,) set on the left hand side of the fractional number; so that the integers stand on the left hand side of the mark, and the fractions on the right hand.

12. For the more convenient reading of numbers, they are divided into periods of six places each, beginning at the unit place; and each period into two degrees of three places each, the names and order of which are as follow: where X. stands for the word tens, C. for hundreds, and Th. for thousands.

13.

13.	Integers												Decimal fractions											
	Second period						First period						First period						Second period					
	Degree						Degree						Degree						Degree					
	Billions						Thousands						Hundredths						Billionths					
	Th. Millions						Thousands						Th. Millionths						Th. Millionths					
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14. RULE. 1st. Suppose the number parted into as many sets or degrees of three places each, beginning at the unit's place, as it will admit of; and if one or two places remain, they will be the units and tens of the next degree.

2d. Beginning at the left hand, read in each degree, as many hundreds, tens, and units, as the figures in those places of the degree express, adding the name thousands, if in the second degree of a period; and adding the name of the period, after reading the hundreds, tens, and units in its first degree.

Thus the integer number in the preceding table will be read :

Five billions, four hundred thirty two thousand, one hundred twenty three millions, four hundred fifty six thousand, seven hundred eighty nine.

15. All fractional numbers consist of two parts, which are usually written one above the other, with a line drawn between them: the number below the line, called the *denominator*, shews how many equal parts the unit is divided into: the number above the line, called the *numerator*, shews by how many of these equal parts the value of that fraction is expressed.

Thus 9 pence, is 9 parts in twelve of a shilling; and may be written thus, $\frac{9}{12}$, when a shilling is the unit.

16. Those fractions, the denominators of which are 10, or 100, or 1000, or 10000, or 100000, &c. are called *decimal fractions*: but fractions with any other denominators are called *vulgar fractions*.

The vulgar fractions that most frequently occur, are these :

$\frac{1}{4}$, which is read one fourth, or one quarter.

$\frac{1}{3}$ - - - - - one third.

$\frac{1}{2}$ - - - - - one half.

$\frac{2}{3}$ - - - - - two thirds.

$\frac{3}{4}$ - - - - - three fourths, or three quarters.

17. As decimal fractions are parts of an unit divided into either 10, 100, 1000, 10000, &c. parts, according to the places in the fractional number; therefore they are read like whole numbers, only calling them so many parts of 10, or of 100, or of 1000, &c.

Thus a decimal fraction of	{	one	} places, will be so	{	10, Ten.
		two			100, Hundred.
		three			1000, Thousand.
		four			10000, Ten Thousand,
		&c.			&c.

18. Cyphers on the right hand of integers increase their value; on the left hand of a decimal fraction diminish its value: but on the left hand of integers, or on the right hand of fractions, do not alter their value.

Thus $\left\{ \begin{array}{l} 8 \text{ is } 8 \text{ units.} \\ 80 \text{ } 8 \text{ tens.} \\ 800 \text{ } 8 \text{ hundreds.} \end{array} \right. \quad \left| \quad \text{And } \left\{ \begin{array}{l} ,8 \text{ is } 8 \text{ parts in } 10 \\ ,08 \text{ } 8 \text{ parts in } 100 \\ ,008 \text{ } 8 \text{ parts in } 1000 \end{array} \right\} \text{ of an unit } \right. \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{ so divided.}$

When a fraction has no integer prefixed, it is convenient to put 0 in the place of units.

19. A MIXED NUMBER, is when a fraction is annexed to a whole number.

Thus five and a half is called a mixed number, and is written $5\frac{1}{2}$, or thus, 5,5; which is thus read, five and five tenths.

20. Like names in different numbers are such figures as stand equally distant from the place of units; or have the same denomination annexed to them.

Thus all numbers of pounds sterling are like names, and so are all numbers of shillings; the like of any numbers of miles, &c.

21. Besides the decimal notation explained in article 8, there are other kinds in common use; such as the *duodecimal*, in which every superior name contains 12 units of its next inferior name: the *sexagenary*, or *sexagesimal*, in which sixty of an inferior name make one of its next superior. The former is used by workmen in the measuring of artificers works in building; and the latter is used in the division of a Circle, and of Time.

22. The following characters or marks are frequently used in Arithmetical computations, briefly to express the manner of operation.

The mark + (more) belongs to addition; and shews that the numbers it stands between are to be added together.

Thus $12 + 3$ expresses the sum of 12 and 3; or that 3 is to be added to 12, and is thus read, 12 more 3.

The mark — (less) is for subtraction; and shews that the number following it, or on the right hand, is to be taken from the number preceding it, or on the left hand.

Thus $12 - 3$, expresses the difference between 12 and 3; or that 3 is to be subtracted from 12, and is thus read, 12 less 3, or 12 lessened by 3.

This mark $\times$ (into) for multiplication, shews that the numbers on each side of it are to be multiplied the one by the other.

Thus 12×3 , denotes the product of 12 into 3; or that 12 is to be multiplied by 3.

Division is expressed by setting the divisor under the dividend with a line drawn between them, like a fraction.

Thus $\frac{12}{3}$, expresses the quotient of 12 by 3; or that 12 is to be divided by 3.

This sign = (equal) shews that the result of the operation by the numbers or quantities on one side of it, is equal either to the numbers or quantities on the other side, or to the result of the operation by these numbers or quantities.

Thus $12 + 3 = 15$; and $12 - 3 = 9$; and $12 \times 3 = 36$; and $\frac{12}{3} = 4$; severally shews the value of the preceding expressions.

23. TABLES of ENGLISH MONEY, WEIGHTS, and MEASURES.

MONEY.				TROY WEIGHT.			
Farthings	Pence	Shill.	Pound	Grains	Pennyw ^{ts}	Ounces	Pound
960	= 240	= 20	= 1 £.	5760	= 240	= 12	= 1 lb.
48	= 12	= 1 s.		480	= 20	= 1 oz.	
4	= 1 d.			gr. 24	= 1 dwt.		
Note, 1, 2, 3 farthings, are thus written, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$.				Note, Gold and Silver are weighed by Troy Weight.			
AVOIRDUPOISE WEIGHT.				WINE MEASURE.			
Drams	Ounces	Pounds	Hund. Ton	Solid inch.	Pints	Gal.	Hogsh. Pipe Tun
573440	= 35840	= 2240	= 20 = 1	58212	= 2016	= 252	= 4 = 2 = 1
28672	= 1792	= 112	= 1 C.	29106	= 1008	= 126	= 2 = 1 P.
256	= 16	= 1 lb.		14553	= 504	= 63	= 1 Hhd.
16	= 1 oz.			231	= 8	= 1	
Note, Provisions, Stores, &c. are weighed by the Avoirdupoise, or great weight.				42 = 1 Tierce. 84 = 1 Puncheon.			
DRY MEASURE.				CLOTH MEASURE.			
Pints	Gall.	Pecks	Bush. Quarter	4 Nails	= 1 Quarter of a Yard.		
512	= 64	= 32	= 8 = 1	4 Quarters	= 1 Yard.		
64	= 8	= 4	= 1	5 Quarters	= 1 English ell.		
16	= 2	= 1		3 Quarters	= 1 Flemish ell.		
8	= 1			6 Quarters	= 1 French ell.		
Note, 4 bush. = 1 Comb: 10 qrs. = 1 a Span = 9 inches.				a Hand = 4 inches.			
Wey: 12 Weys = Last of Corn.							
36 Bushels = 1 Chaldron of Coals.							
LONG MEASURE.							
Barley-corns	Inches	Fect	Yards	Poles	Furl.	Mile.	
190080	= 63360	= 5280	= 1760	= 320	= 8	= 1	
23760	= 7920	= 660	= 220	= 40	= 1		
594	= 198	= 16½	= 5½	= 1			
108	= 36	= 3	= 1				
36	= 12	= 1					
3	= 1						
Also 3 miles make 1 league.							
And 20 leagues or 60 sea miles make a degree.							
But a degree contains about 69½ miles of statute measure.							
A fathom = 6 feet = 2 yards.							
TIME.							
Seconds	Minutes	Hours	Days	Year			
31556937	= 525948	= 8766	= 365½	= 1.			
86400	= 1440	= 24	= 1 day				
3600	= 60	= 1 hour.					
60	= 1						
Pence Table.				Even parts of a Pound Sterling.			
Pence.	s.	d.	Pence	s.	d.	is	of a Pound Sterling.
20	= 1.	8	70	= 5.	10	10 . 0	½ of a Pound Sterling.
36	= 2.	6	80	= 6.	8	6 . 8	⅓ of a Pound Sterling.
40	= 3.	4	90	= 7.	6	5 . 0	¼ of a Pound Sterling.
50	= 4.	2	100	= 8.	4	4 . 0	⅕ of a Pound Sterling.
60	= 5.	0	110	= 9.	2	3 . 4	⅙ of a Pound Sterling.
						2 . 6	⅓ of a Shilling.
						2 . 0	⅔ of a Shilling.
						10	of a Pound.

24. SECTION II. ADDITION.

ADDITION is the method of collecting several numbers into one sum.

RULE 1st. Write the given numbers under each other, so that like names stand under like names; that is units under units, tens under tens, &c. and under these draw a line.

2^d. Add up the first or right hand upright row, under which write the overplus of the units of the second row, contained in that sum.

3^d. Add these units to the sum of the second row, under which write the overplus of the units of the third row, contained in that sum.

And thus proceed until all the rows are added together,

EXAMPLES.

Ex. I. Add 28—76—47—18 and 12 together.

These numbers being written under each other will stand thus:

28
76
47
18
12
—
181
—

Say 2 and 8 is 10, and 7 is 17, and 6 is 23, and 8 is 31; then, because 10 units in the right hand row make an unit in the next row; therefore in 31 there are 3 units of the second row, and an overplus of 1: write down the 1, and add the 3 to the second row, saying, 3 that is carried and 1 is 4, and 1 is 5, and 4 is 9, and 7 is 16, and 2 is 18, in which is one unit of the third row (had there been a third) and an overplus of 8; write down the 8, and add the 1 to the third row: but as there is no third row, the 1 carried must be written on the left hand of the 8; and 181 will be the sum of the five given numbers.

Ex. II. Add 476—3784—18329—290—75—7638—and 46 together.

476
3784
18329
290
75
7638
46
—
The Sum 30638
—

The given numbers
set in order will stand
thus }

Ex. III. Add the numbers, 10768—3489—28764—289—6438—19—and 438 together.

10768
3489
28764
289
6438
19
438
—
The Sum 50205
—

The given numbers
placed as the rule directs, stand thus }

Ex. IV. Add these numbers together.

3720,45
25,0036
4179,802
3,6284
—
Sum 7923,8840
—

Ex. V. Add the following numbers together.

15836,071
20,09
347
583,27008
—
Sum 16474,13103
—

In the two last examples, where there are both integer and fractional numbers, it may be observed, that like integer places, and like fractional places, stand under each other; and the manner of adding them together, is the same as explained in the first example.

25. It frequently happens, that numbers are to be added together, the names of which do not increase in a tenfold manner, as in the last Examples; such as in adding different sums of money, weights or measures; in which, regard is to be had to the number of those of a lower name, contained in one of its next greater name, as shewn in the preceding tables: Examples of which follow.

Ex. VI. Add the following sums of money together.

£.	s.	d.
353	14	$8\frac{1}{2}$
276	10	4
89	17	$5\frac{1}{4}$
34	12	$10\frac{3}{4}$
754	15	$4\frac{1}{2}$

Ex. VII. Add the following sums of money together.

£.	s.	d.
7683	08	$2\frac{1}{2}$
954	19	$9\frac{1}{2}$
682	10	$7\frac{1}{4}$
63	15	$6\frac{1}{4}$
9384	14	$2\frac{1}{4}$

In these two examples the carriage is by 4 in the farthings; by 12 in the pence; by 20 in the shillings; and by 10 in the pounds.

Ex. VIII. Add the following Troy Weights together.

lb.	oz.	dwt.	gr.
218	10	13	18
176	9	19	23
85	11	17	11
24	8	15	21
506	5	07	01

Carry for 24, 20, 12, 10.

Ex. X. Add the following parts of Time together.

Weeks	Da.	Ho.	Min.	Sec.
21	4	18	37	59
11	6	13	25	47
19	3	23	59	28
38	4	08	22	39
91	5	16	25	53

Carry for 60, 60, 24, 7, 10.

Ex. IX. Add the following Avoirdupoise Weights together.

Tons.	Cwt.	qrs.	lb.	oz.
535	17	3	22	11
94	19	1	27	13
158	12	0	18	15
7	15	2	13	08
797	05	0	26	15

Carry for 16, 28, 4, 20, 10.

Ex. XI. Add the following parts of a Circle together.

Deg.	"	'	'''	iv
176	32	59	43	25
85	59	27	31	59
114	28	45	59	14
67	12	38	24	47
444	13	51	39	25

Carry for 60, 60, 60, 60, 10.

Explanation of Example VI.

Three farthings and 1 farthing is 4 farthings, and 2 farthings is 6 farthings; which is a penny halfpenny: set down $\frac{1}{2}$, and carry 1.

Then 1 and 10 is 11, and 5 is 16, and 4 is 20, and 8 is 28 pence; which is 2 shillings and 4 pence: set down 4, and carry 2.

Again, 2 and 12 is 14, and 17 is 31, and 10 is 41, and 14 is 55 shillings; which is 2 pounds 15 shillings: set down 15 shillings, and carry 2 pounds. The rest is easy.

26. SECTION III. SUBTRACTION.

SUBTRACTION is the method of taking one number from another, and shewing the remainder, or difference, or excess.

The *subducend* is the number to be subtracted, or taken away.

The *minuend* is the number from which the subducend is to be taken.

RULE 1st. Under the minuend write the subducend, so that like names stand under like names; and under them draw a line.

2^d. Beginning at the right-hand side, take each figure in the lower line from the figure standing over it, and write the remainder, or what is left, beneath the line, under that figure.

3^d. But if the figure below is greater than that above it, increase the upper figure by as many as are in an unit of the next greater name; from this sum take the figure in the lower line, and write the remainder under it.

4th. To the next name in the lower line, carry the unit borrowed, and thus proceed to the highest denomination or name.

EXAMPLES.

Ex. I. From 436565874 the minuend,
Take 249853642 the subducend,

Remains 186712232 the difference,

Here the five figures on the right of the subducend may be taken from those over them: but the 6th figure, viz. 8, cannot be taken from the 5 above it. Now as an unit in the 7th place makes 10 in the 6th place, therefore borrowing this unit makes the 5, 15: then say, 8 from 15 leaves 7, which set down; and say 1 carried and 9 is 10, 10 from 6 cannot be had, but 10 from 16 leaves 6, set it down; then 1 carried and 4 is 5, 5 from 13 leaves 8; set it down; then 1 carried and 2 is 3, 3 from 4 leaves 1.

Ex. II. From 7620908
Take 3875092
Remains 3745816

Ex. III. From 327,9563
Take 49,8697
Remains 278,0866

Ex. IV. From 30007,295
Take 2536,876
Leaves 27470,419

Ex. V. From 5000,0000
Take 479,6378
Leaves 4520,3622

Ex. VI. Borrowed £. s. d.
Paid 24 14 6½
18 12 4½
Remains 6 02 2½

Ex. VII. Lent £. s. d.
Received 294 15 9½
89 18 10½
Remains 204 16 10½

Ex. VII.

Ex. VIII. In Sexagesimals.

	o	i	ii	iii	iv
From	26	28	37	49	32
Take	65	29	16	53	45
Leaves	10	59	20	55	47

Ex. IX. In Sexagesimals.

	o	i	ii	iii	iv
From	218	46	32	50	18
Take	149	52	47	53	29
Leaves	68	53	44	56	49

QUESTIONS to exercise Addition and Subtraction.

QUEST. I. The share of Jack's prize money was 148 £. 17 s. 6½ d; and Tom received as much, beside 7 £. 18 s. smart money: How much money did Tom receive?

	£.	s.	d.
Tom's prize money	148	17	6½
Smart money	7	18	0
Tom received	156	15	6½

QUEST. III. What year was King George born in, he being 67 years old in the year 1749?

Current year	1749
Age	67 subtr.
Year born in	1682

QUEST. V. A seaman who had received 46 £. 17 s. 6 d. for wages, prize money, &c. meeting with bad company, was tricked out of 18 guineas: Now John had reckoned to pay his wife's debts of 13 £. 16 s. 6 d. and his landlady's bill of 16 £. 12 s. Required whether he can fulfil his intentions, and what the difference will be?

	£.	s.	d.
Money lost	18	18	0
Wife's debt	13	16	6
Landlady's bill	16	12	0
Total	49	6	6
Money received	46	17	6
He will want	2	9	0

QUEST. II The Spanish invasion was in the year 1588, and the French attempted an invasion in the year 1744: How many years were between these fruitless attempts?

French	1744
Spanish	1588
Years between	156

QUEST. IV. Two ships depart from the same port, one having sailed 835 miles, is got 48 miles a-head of the other: Required the aftermost ship's distance.

The first ship's distance	835
Their difference	48
Second ship's distance	787

QUEST. VI. Will and Frank, talking of their ages in the year 1749, Will said he was born in the year of the Rebellion, in 1715; and Frank said he remembered he was ten years old the year King George the second was crowned, in 1727: Required the age of each, and the difference of their age?

Current year	1749
Will was born	1715
Will's age	34
Current year	1749
King George crowned	1727
Years since	22
Frank's age then	10
Frank's age	32

So Will was oldest by two years.

28. SECTION IV. MULTIPLICATION.

MULTIPLICATION is the method of finding what a given number will amount to, when repeated as many times as is represented by another number.

The number to be multiplied, is called the *Multiplicand*.

The number multiplied by, is called the *Multiplier*.

And the number which the multiplication amounts to, is called the *Product*.

Both multiplicand and multiplier are called *Factors*.

Before any operation can be performed in Multiplication, it is necessary that the learner should commit to memory the following table.

29.

The MULTIPLICATION TABLE.

times	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3		9	12	15	18	21	24	27	30	33	36
4			16	20	24	28	32	36	40	44	48
5				25	30	35	40	45	50	55	60
6					36	42	48	54	60	66	72
7						49	56	63	70	77	84
8							64	72	80	88	96
9								81	90	99	108
10									100	110	120
11										121	132
12											144

Observe, that in multiplying any figure in the upper line by any figure in the left-hand column, the product will stand right against the figure used in the left-hand column, and under that used in the upper line. Thus were 6 to be multiplied by 9, seek the greater figure 9 in the upper line, and right under it, against 6 in the left-hand, stands 54 for the Product. And so of others.

The foregoing table being well known, the work of Multiplication will be performed as follows.

To multiply any number, as	37256
By any single figure, as by	7
Set them as in the margin, and proceed	<u>260792</u>

thus, 7 times 6 is 42, set down 2 and carry 4; 7 times 5 is 35 and 4 carried is 39, set down 9 and carry 3; 7 times 2 is 14 and 3 carried is 17, set down 7 and carry 1; 7 times 7 is 49 and 1 carried is 50, set down 0 and carry 5; 7 times 3 is 21 and 5 carried is 26, which set down, and the work is done. But for compound Multiplication take the following:

30. RULE 1st. Write the Factors so, that the right-hand place of the Multiplier may stand under the right-hand place of the Multiplicand.

2d. Multiply the Multiplicand severally by every figure of the Multiplier, setting the first figure of each line under the figure then multiplying by.

3d. Add the several lines together; and their sum is the Product.

4th. From the right hand of the Product point off, for fractions, as many places as there are fractional places in both Factors; and those to the left of the mark of distinction are integers; those to the right are fractions.

*

5th. If

5th. If the number of places in the Product are not so many as the number of fractional places in both Factors, make up that number by writing cyphers on the left hand, and to these prefix the mark of distinction.

EXAMPLE I. *Multiply 742 by 53.*

The lesser Factor being written under the greater Factor, as here shewn, and a line drawn under them; say 3 times 2 is 6, write 6 under the 3; then 3 times 4 is 12, write down 2 and carry 1; and 3 times 7 is 21, and 1 carried is 22, write the 22: Again, 5 times 2 is 10, write 0 under the 5, and carry 1; and 5 times 4 is 20 and 1 carried is 21, write down 1 and carry 2; then 5 times 7 is 35, and 2 carried is 37, write down the 37: Now add the two lines together found by multiplying by 3 and by 5, and their sum 39326 is the product required.

Multiplicand	742	}	Factors.
Multiplier	53		
		2226	
		3710	
Product		39326	

EXAMPLE II.

<i>Multiply</i>	28704
<i>by</i>	8631
	28704
	86112
	172224
	229632
	247744224

Product.

EXAMPLE III.

<i>Multiply</i>	3684,2795
<i>by</i>	7,594
	147371180
	331585155
	184213975
	257899565
	27978,4185230

Here, because there are 4 fractional places in the Multiplicand, and 3 in the Multiplier, which together make 7, therefore 7 places are pointed off on the right of the product for fractions.

EXAMPLE IV.

<i>Multiply</i>	936,287
<i>by</i>	607,02
	1872574
	65540090
	56177220
	568344,93474

The cyphers in the Multiplier of this examples are thus managed. Having multiplied by the 2 as before, say 0 times 7 is 0, write 0 under the 0, and proceed to the next figure 7, by which multiply as before, then coming to the second 0, say, 0 times 7 is 0, write 0 under the place of the second 0, and proceed to the next figure 6, by which multiply as before.

EXAMPLE V.

<i>Multiply</i>	0,34796
<i>by</i>	0,0258
	278368
	173980
	69592
	0,008977368

Here, because there are 5 fractional places in one Factor, and 4 in the other, there should be 9 fractional places in the Product; and there arising but 7, therefore two cyphers are set on the left hand to make 9 places.

31. SECTION V. DIVISION.

DIVISION is the method of finding how often one number is contained in another ; or may be taken from another.

The number to be divided, is called the *Dividend*.

The number dividing by, is called the *Divisor*.

The *Quotient* is the number arising from the division, and shews how many times the Divisor is contained in the Dividend.

The operations in Division are performed as follow :

32. RULE 1st. On the right and left of the Dividend draw a crooked line ; write the Divisor on the left side, and the Quotient, as it arises, on the right side of the Dividend.

2d. Seek how often the Divisor may be taken in as many figures on the left hand of the Dividend, as are just necessary ; write the number of times it may be taken, in the Quotient ; and there will be as many figures more in the Quotient, as there are figures remaining in the Dividend then not used.

3d. Multiply the Divisor by this Quotient-figure, set the Product under that part of the Dividend used ; subtract, and to the right hand of the remainder bring down the next figure of the Dividend : Divide as before ; and thus proceed until all the figures of the Dividend are used.

4th. If there is a remainder, to its right hand side annex a cypher or cyphers, as if brought down from the Dividend, and divide as before ; and thus it is that fractions arise, viz. from the remainders in division.

5th. When any figure of the Dividend is taken down, or annexed, as before shewn, and the Divisor cannot be taken in the number thus increased ; put 0 in the Quotient, and take down, or annex, another figure ; and proceed in this manner, until the Divisor can be taken from the number.

6th. When fractions are concerned : From the number of fractional places used in the Dividend, take those in the Divisor ; count the number of remaining places from the right of the Quotient, put the mark there ; and those to the left are integers, those to the right fractions.

7th. If there arise not so many places in the Quotient as the 6th article requires, supply the places wanting with cyphers on the left, and to those prefix the fractional mark.

Ex. I. Divide 3656 £. among 8 persons.

Set the given numbers as in Art. 1st. Now the two left hand figures contain 8 ; then say 8 is contained in 36, 4 times ; set 4 in the Quotient, and say 4 times 8 is 32, set 32 under 36, subtract, there remains 4, to which bring down the next figure of the Dividend 5, makes 45 ; then say 8 is contained in 45, 5 times ; set 5 in the Quotient, and say 5 times 8 is 40 ; write 40 under 45, subtract, and to the remainder 5 take down 6, the next figure of the Dividend, makes 56 ; then say 8 is contained in 56, 7 times ; write 7 in the Quotient, multiply 8 by 7 makes 56, which write under the other 56, and subtracting there remains 0 : So it may be concluded, that 3656 contains 8, 457 times : Or, if 3656 £. be divided among 8 persons, the share of each will be 457 £.

$$\begin{array}{r}
 8 \overline{) 3656(457} \\
 \underline{32} \\
 45 \\
 \underline{40} \\
 56 \\
 \underline{56} \\
 0
 \end{array}$$

Ex. II.

Ex. II. Divide 3125 by 25.

$$\begin{array}{r}
 25 \overline{) 3125} \\
 \underline{25} \\
 62 \\
 \underline{50} \\
 125 \\
 \underline{125} \\
 0
 \end{array}$$

Ex. IV. Divide 5859 by 124.

$$\begin{array}{r}
 124 \overline{) 5859} \quad (47, 25 \\
 \underline{496} \\
 899 \\
 \underline{868} \\
 310 \text{ for the Remaind.} \\
 248 \text{ See precept 4th.} \\
 \underline{620} \\
 620
 \end{array}$$

Ex. VI. Divide 2,3569 by 673,4.

$$\begin{array}{r}
 673,4 \overline{) 2,3569} \quad (35 \\
 \underline{20202} \text{ Quot. } 0,0035 \\
 33670 \text{ See precepts} \\
 33670 \text{ 4th, 6th, 7th.} \\
 \underline{}
 \end{array}$$

Ex. III. Divide 95269 by 47.

$$\begin{array}{r}
 47 \overline{) 95269} \quad (2027 \\
 \underline{94} \\
 \text{See precept 5th. } 126 \\
 \underline{94} \\
 329 \\
 \underline{329} \\
 0
 \end{array}$$

Ex. V. Divide 337,27368 by 6,28.

$$\begin{array}{r}
 6,28 \overline{) 337,27368} \quad (53,706 \text{ for the Quotient.} \\
 \underline{3140} \text{ See precept 6th.} \\
 2327 \\
 \underline{1884} \\
 4433 \\
 \underline{4396} \\
 \text{See precept 5th. } 3768 \\
 \underline{3768}
 \end{array}$$

In Ex. VI. the 4 fractional places given in the Dividend, and the 0 used with the Remainder, make 5 fractional places; from which 1 place in the Divisor being taken, leaves 4 fractional places for the Quotient; but in the Quotient are

only the two places 35, therefore 2 cyphers are prefixed, which make ,0035, before which, for form sake, an 0 is set for the place of units.

33. When the Divisor does not exceed the number 12, the Division may be performed in one line; by making the Multiplication and Subtraction mentally, or in the mind, and carrying the Remainder, as so many tens, to the next figure.

34. In all operations of Division, it must be observed, that the Product of the Divisor by the Quotient figure must not exceed that part of the Dividend then using; and the Remainder, by subtracting the Product, must ever be less than the Divisor.

As the Quotient multiplied by the Divisor makes the Dividend;

So the Product of two numbers being divided by one of them, will give the other; that is, Division is proved by Multiplication, and Multiplication is proved by Division.

35. SECTION VI. REDUCTION.

REDUCTION is the method of reducing numbers from one name, or denomination, to another; retaining the same value.

CASE I. To reduce a number consisting of several names, to their least name.

RULE 1st. Multiply the first, or greater name, by the parts which an unit of that name contains of the next less name; adding to the Product the parts of the second name in the given number.

2d. Multiply this sum by the number of times that an unit of the next less name is contained in one of the second name; adding to the Product the parts of the third name contained in the given number: And thus proceed, until the least name in the given number is arrived at.

Ex. I. In 23*l.* 14*s.* 6½*d.* how many farthings?

<i>l.</i>	<i>s.</i>	<i>d.</i>
23	14	6½
20		

474 Shillings.

12

5694 Pence.

4

Answer 22778 Farthings.

Ex. III. In a Cannon weighing 2 Tons, 14 C. 3 qrs. 19 lb. how many pounds?

T.	C.	Qrs.	lb.
2	14	3	19
20			

54 C. weight.

4

219 Qrs.

28

1771

438

6151 Pounds.

Ex. II. In 8 lb. 10 oz. of gold, how many grains?

lb.	oz.
8	10
12	

106 Ounces.

20

2120 Pennyweights.

24

8480

4240

50880 Grains.

Ex. IV. In 36 deg. 48'. 27". 56''' how many thirds?

deg.	'	"	'''
36	48	27	56
60			

2208 Minutes.

60

132507 Seconds.

60

7950476 Thirds.

An explanation of the first Ex. will make all the rest plain. Since pounds is the greatest name in the given number, and an unit thereof contains 20 of the next less name, or shillings; therefore multiply the pounds by 20, saying 0 times 3 is 0, to which adding the 4 in the 14 s. makes 4; then 2 times 3 is 6, and the 1, in the place of tens in the shillings, makes 7; then 2 times 2 is 4: Now multiply 474 s. by 12, saying 12 times 4 is 48, and the 6 in the pence makes 54; write 4 and carry 5; then 12 times 7 is 84, and 5 is 89, &c. Lastly, multiply the 5694 pence by 4, saying 4 times 4 is 16, and the two farthings in the given number is 18; write 8 and carry 1, &c.

36. CASE II. *A number of an inferior name being given; to find how many of each superior denomination are contained in it.*

RULE 1st. Divide the given number, by the number of times that one of its units is contained in an unit of the next superior name.

2^d. Divide this Quotient by the parts making one of the next name.

3^d. Divide this Quotient by the parts making one of the next name: And proceed in this manner, until the highest name is obtained.

4th. Then the last Quotient, and the several remainders, will be the parts of the different names contained in the given number.

Ex. I. In 22778 farthings, how many pounds, shillings, and pence?

$$\begin{array}{r} 4 \overline{)22778} \text{ (2 Farthings.} \\ \underline{} \\ \end{array}$$

$$\begin{array}{r} 12 \overline{)5694} \text{ (6 Pence.} \\ \underline{} \\ \end{array}$$

$$\begin{array}{r} 20 \overline{)474} \text{ (14 Shillings.} \\ \underline{} \\ \end{array}$$

23 Pounds.

Answer 23 £. 14 s. 6 $\frac{1}{2}$ d.

Ex. III. In 6151 pounds, how many Tons, Hundreds, Quarters, Pounds?

$$\begin{array}{r} 28 \overline{)6151} \text{ (219} \\ \underline{} \\ \end{array}$$

$$\begin{array}{r} 56 \overline{)4219} \text{ (3 Qrs.} \\ \underline{} \\ \end{array}$$

$$\begin{array}{r} 55 \overline{)28} \text{ (2,0)5,4 (14 C.} \\ \underline{} \\ \end{array}$$

$$\begin{array}{r} 271 \\ 252 \\ \hline \end{array}$$

19 Pounds.

Answer 2 T. 14 C. 3 Qrs. 19 lb.

Ex. II. In 7950476 thirds of a degree, how many $^{\circ}$ $'$ $''$ $'''$?

$$\begin{array}{r} 6,0 \overline{)7950476} \text{ (56 Thirds.} \\ \underline{} \\ \end{array}$$

$$\begin{array}{r} 6,0 \overline{)13250,7} \text{ (27 Seconds.} \\ \underline{} \\ \end{array}$$

$$\begin{array}{r} 6,0 \overline{)220,8} \text{ (48 Minutes.} \\ \underline{} \\ \end{array}$$

36 Degrees.

Answer 36 $^{\circ}$. 48'. 27''. 56'''.

Ex. IV. In 50880 grains, how many Pounds, Ounces, Pennyweights, Grs.

$$\begin{array}{r} 24 \overline{)50880} \text{ (2120} \\ \underline{} \\ \end{array}$$

$$\begin{array}{r} 48 \overline{)28} \text{ (2,0)212,0 (0 dwt.} \\ \underline{} \\ \end{array}$$

$$\begin{array}{r} 28 \overline{)24} \text{ (12) 106 (10 oz.} \\ \underline{} \\ \end{array}$$

$$\begin{array}{r} 48 \\ 48 \\ \hline \end{array}$$

0

Answer 8 lb. 10 oz.

Explanation of Ex. I. Since 4 of the given number make one of the next name, pence, then 22778 divided by 4, give 5694 pence, and a Remainder of 2 farthings; then 5694 pence divided by 12, the number of pence in one of the next name, shillings, the Quotient is 474 shillings, and a Remainder of 6 pence; then 474 shillings divided by 20, the number of shillings in one of the next name, pounds, the Quotient is 23 pounds, and a Remainder of 14 shillings. And by the 4th precept, the answer is collected.

A like operation will solve the other examples, having regard to the increase of the different names.

37. In any Division, if the Divisor has one or more cyphers on the right hand, those cyphers may be pointed off; but then as many places must be pointed off from the Dividend, which places are not to be divided, but annexed to the right hand of the Remainder. See the above examples.

38. CASE

38. CASE III. To reduce a vulgar fraction to its equivalent decimal fraction.

RULE. To the Numerator annex one or more cyphers; divide this by the Denominator, and the Quotient will be the fraction sought.

If the Division does not end when six figures are found in the Quotient, the work need not be carried any farther.

EXAM. I. To reduce $\frac{15}{423}$ to its equivalent decimal fraction.

Here 423 the Denominator is made the Divisor, and 15 the Numerator is set for the Dividend, to which annexing a cypher or two for fractional places, seek how often the Divisor can be had in 15, the integral part of the Dividend; and as it cannot be taken, put 0 in the Quotient for the place of units: Then taking in one fractional place, seek how oft the Divisor can be had in 150, say 0 times, and put another 0 in the Quotient for the place of primes: Now taking in two fractional places to the 15, the Divisor will be contained in it thrice, and thus proceed until the Division ends, or till 6 places arise in the Quotient: But in this example, as the 6th place would be 0, it is omitted, because cyphers on the right hand of decimal fractions are of no signification, as will evidently appear, Notation of Fractions being well understood.

$$\begin{array}{r}
 423 \overline{)15.00(0.03546} \\
 \underline{1269} \\
 2310 \\
 \underline{2115} \\
 1950 \\
 \underline{1692} \\
 2580 \\
 \underline{2538} \\
 420
 \end{array}$$

Ex. II. Reduce $\frac{1}{2}$ to a decimal fraction.

$$2 \overline{)1.0(0.5 \text{ Answer.}}$$

Ex. IV. Reduce $\frac{3}{4}$ to a decimal fraction.

$$4 \overline{)3.00(0.75 \text{ Answer.}}$$

Ex. VI. Reduce $\frac{1}{3}$ to a decimal fraction.

$$3 \overline{)1.00(0.33, \&c. \text{ Answer.}}$$

Ex. III. Reduce $\frac{1}{4}$ to a decimal fraction.

$$4 \overline{)1.00(0.25 \text{ Answer.}}$$

Ex. V. Reduce $\frac{5}{8}$ to a decimal fraction.

$$8 \overline{)5.000(0.625 \text{ Answer.}}$$

Ex. VII. Reduce $\frac{7}{12}$ to a decimal fraction.

$$12 \overline{)7.0000(0.5833, \&c. \text{ Answ.}}$$

39. In the two last Quotients, it may be observed, that 3 would continually arise; such decimal fractions are called circulating, or recurring fractions: These have a peculiar kind of operation belonging to them, which the inquisitive reader will find in a book intitled *A General Treatise of Mensuration**, the third edition, published in the year 1767; and also in other books.

* By the Author of these Elements,

40. CASE IV. *To reduce a number consisting of different names, to a decimal fraction of its greatest name.*

RULE 1st. Write the given names orderly under one another, the least name being uppermost; and on their left side draw a line: Let these be reckoned as Dividends.

2d. Against each name, on the left hand, write the number making one of its next superior name: And let these be the Divisors to the former Dividends.

3d. Begin with the upper one, and write the Quotient of each division as fractions, on the right of the Dividend next below it; then let this mixed number be divided by its Divisor, &c.

And the last Quotient will be the decimal fraction sought.

Ex. I. Reduce 15s. 9½d. to the fractional part of a pound sterling.

First set the three farthings, the 9 pence, the 15 shillings and 0 pounds under one another; and against the farthings set 4; against the pence set 12, and against the shillings, 20; then the three with cyphers supposed to be annexed, being divided by 4, the Quotient, 75 is written on the right hand of the 9 pence; and the mixed number 9,75 with cyphers annexed as they are wanted, being divided by 12, the Quotient, 8125 is written on the right hand of the 15s. then this mixed number 15,8125 being divided by 20, the Quotient 0,790625£. is the answer.

$$\begin{array}{r} 4 \overline{) 3} \\ 12 \overline{) 9,75} \\ 20 \overline{) 15,8125} \\ \hline 0,790625 \end{array}$$

Ex. II. Reduce 1s. 2¼d. to the fractional part of a pound sterling.

$$\begin{array}{r} 4 \overline{) 1} \\ 12 \overline{) 2,25} \\ 20 \overline{) 1,1875} \\ \hline 0,059375 \end{array}$$

Answer 1s. 2¼d. = 0,059375£.

Ex. III. Reduce 48'. 17". 53''' to the fractional part of a degree.

$$\begin{array}{r} 60 \overline{) 53} \\ 60 \overline{) 17,883333} \\ 60 \overline{) 48,298055} \\ \hline 0,804967 \end{array}$$

Answer 48'. 17". 53''' = 0,804967 Deg.

Ex. IV. Reduce 8 oz. 15 dwt. 18 gr. to the fractional part of a pound troy.

$$\begin{array}{r} 24 \left\{ \begin{array}{l} 4 \overline{) 18} \\ 6 \overline{) 4,5} \end{array} \right. \\ 20 \overline{) 15,75} \\ 12 \overline{) 8,7875} \\ \hline 0,732291 \end{array}$$

Ans. 8 oz. 15 dwt. 18 gr. = 0,732291lb.

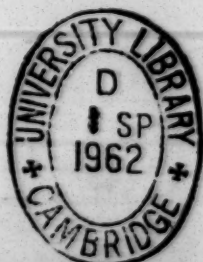
Ex. V. Reduce 3 qrs. 19 lb. 14 oz. to the fractional part of a C. weight.

$$\begin{array}{r} 16 \left\{ \begin{array}{l} 4 \overline{) 14} \\ 4 \overline{) 3,5} \end{array} \right. \\ 28 \left\{ \begin{array}{l} 4 \overline{) 19,875} \\ 7 \overline{) 4,96875} \end{array} \right. \\ 4 \overline{) 3,709821} \\ \hline 0,927455 \end{array}$$

Answer 3 qr. 19 lb. 14 oz. = 0,927455 C.

41. Here because 24 is a number too great to divide by in one line, therefore it is broken into the parts 4 and 6, which multiplied together make 24.

VOL. I.



C

42. CASE

42. CASE V. To reduce a decimal fraction of a superior name, to its value in inferior denominations.

RULE 1st. Multiply the given fraction by the number that an unit of its name contains units of the next lesser name; from the right hand of the Product point off as many places as there are in the given fraction.

2d. Multiply the places, so pointed off, by as many as an unit of this name contains of the next less name; point off as before.

And thus proceed until the multiplication is made by the least name.

3d. Then the integers, or the numbers on the left of the distinguishing marks in each Product, will be the parts in each name, which together are equal to the given fraction.

EXAMPLE I. What number of shillings, pence, and farthings, are equal in value to 0,790625 £. sterling.

Here an unit of the given name £. contains 20 of the next less name, shillings; then multiplying by 20, and pointing off 6 places on the right, because the given number 0,790625 contains 6 fractional places, the Product is 15,812500 shillings; then the fractions of this number, viz. 812500 multiplied by 12, the number that an unit of this name contains of the next less name, and the product pointed as before, there arises 9,750000 pence; the fractions of this number multiplied by 4, gives 3,000000 farthings; then the parts pointed off on the left, viz. 15 s. 9 $\frac{3}{4}$ d. are the value of the given fraction.

$$\begin{array}{r} \text{£. } 0,790625 \\ \quad 20 \\ \hline \text{s. } 15,812500 \\ \quad 12 \\ \hline \text{d. } 9,750000 \\ \quad 4 \\ \hline \text{far. } 3,000000 \end{array}$$

EXAMPLE II. What is the value of 0,056285 £. sterling?

$$\begin{array}{r} \text{£. } 0,056285 \\ \quad 20 \\ \hline \text{s. } 1,125700 \\ \quad 12 \\ \hline \text{d. } 1,5084 \\ \quad 4 \\ \hline \text{far. } 2,0336 \end{array}$$

EXAMPLE III. What is the value of 0,58695 degrees?

$$\begin{array}{r} \text{Deg. } 0,58695 \\ \quad 60 \\ \hline \text{Min. } 35,21700 \\ \quad 60 \\ \hline \text{Sec. } 13,020 \\ \quad 60 \\ \hline \text{Thirds } 0,120 \end{array}$$

EXAMPLE IV. What is the value of 0,732291 lb. troy?

This example worked as above, by multiplying by 12, 20, 24, the value will be found to be

8oz. 15dwts. 18gr. nearly.

EXAMPLE V. What is the value of 0,927455 part of a C. weight?

By operating as above, multiplying by 4, 28, 16, the answer will be

3qrs. 19lb. 14oz. nearly.

43. QUEST.

QUESTIONS to exercise the preceding rules.

43.

QUEST. I. *A sloop with the captain and 26 hands take a prize which sold for 1578 £. of which each seaman had 45 £. and the captain the rest: How much was his share?*

26 Men
45 £. to each
130
104
1170 £. the crew's share.
1578 £. the whole prize.
408 £. the captain's share.

subtract from 1170 £. the crew's share.
1578 £. the whole prize.
remains 408 £. the captain's share.

QUEST. III. *A seaman, whose wages was 35 s. 6 d. a month, returns home at the end of 29 months; he having taken up 12 £. 18 s.: How much has he to receive?*

s.	d.
35	6
mult. by 12	
426 pence a month,	
mult. by 29 months,	
3834	
852	
12) 12354 pence	
2,0) 102,9 6 d.	
from 51 £. 9 s. 6 d. = wages,	
take 12 £. 18 s. 0 d. received,	
remains 38 £. 11 s. 6 d. to receive.	

QUEST. V. *In 306 crowns, how many half crowns and pence?*

Answer { 612 half crowns.
18360 pence.

QUEST. VII. *A seaman's share of a prize was 14 guineas, 32 moidores, 12 thirty-six shillings pieces, and 52 pistoles at 17 s. each: How much sterling did the whole come to?*

Answer 123 £. 14 s.

QUEST. II. *A boat's crew of 15 men got by plunder 321 £. How much was the share of each?*

15) 321 (21 £.
30
21
15
6

remains 6 £. which
mult. by 20 s. in 1 £.

15) 120 s. (8 s.
120

Answer 21 £. 8 s. to each.

QUEST. IV. *Six mess-mates, who propose to live well during an East-India voyage of 22 months, agree to expend among them 5 s. a day, besides the ship's allowance: Now one of them having but 25 s. a month, how will matters stand with him at the end of the voyage?*

Now 28 days, at 5 s. a day, makes 140 s. or 7 £. a month; which for 22 months, is 154 £.

Then a sixth part of 154 £. is 25 £. 13 s. 4 d. for each man.

Also 25 s. a month for 22 months makes 27 £. 10 s. for wages; which will overpay his expences, by 1 £. 16 s. 8 d.

QUEST. VI. *In 30 chalders of coals, each of 36 bushels, how many pecks?*

Answer 4320 pecks.

QUEST. VIII. *Suppose a ship sails $5\frac{1}{2}$ miles an hour for 14 days: How many degrees and minutes has she sailed in the whole; 60 sea miles making one degree?*

Answer 30 deg. 48 min.

SECTION VII. OF PROPORTION:

Or, THE RULE OF THREE.

44. Four numbers are said to be proportional, when by comparing them together by two and two, they either give equal Products or equal Quotients.

Suppose these four numbers 3 8 12 32

In comparing them together by multiplication,

The Product of 3 and 8 is 24; of 12 and 32 is 384, unequal.
 of 3 and 12 is 36; of 8 and 32 is 256, unequal.
 of 3 and 32 is 96; of 8 and 12 is 96, equal.

Therefore 3 8 12 32, are called proportional numbers.

Now let them be compared together by division.

The Quotient of 8 by 3 is 2,6 &c. of 32 by 12 is 2,6 &c. equal.
 of 12 by 3 is 4 of 32 by 8 is 4, equal.
 of 32 by 3 is 10,6 &c. of 12 by 8 is 1,5, unequal.

Therefore by this comparison, the numbers are said to be proportional.

In this kind of comparing four numbers together, there is no need to try for more equal Products, or Quotients, than one set of either fort; for either case will determine the proportionality independent of the other.

But it must be observed, that among four proportional numbers, there will be but one set of equal Products, and two sets of equal Quotients, the smaller numbers being Divisors.

45. When four numbers are to be written as proportionals, they must be placed in such order, that the Product of the first and fourth be equal to the Product of the second and third.

A question is said to belong to the *Rule of Three*, when three numbers or terms are given to find a fourth proportional, which is the answer to the question.

And in order to resolve such questions, the three given terms must be first placed in a proper order, which is called stating the terms of the question.

46. Questions in the *Rule of Three* are stated, and resolved by the following precepts.

1st. Consider of what kind the fourth term, or number sought, will be, whether money, weight, measure, time, &c. and among the three numbers given in the question, let that which is of the same kind with what is required be placed for the third term.

2d. From the nature of the question, determine whether the number sought will be greater or less than the number which is placed for the third term.

3d. If

3d. If the fourth term will be greater than the third, set the greater of the remaining two terms for the second, and the less for the first.

But if the fourth term is to be less than the third, set the greater of the remaining two terms for the first, and the less for the second.

Then in either case, the given three terms are stated.

4th. Reduce those terms which consist of more names than one, to one name; and observe that the first and second terms are always to be of the same name.

5th. Multiply the second and third terms together, divide the product by the first term, and the Quotient will be the fourth term, of the same name the third term was reduced to.

47. QUEST. I. If 4 yards of cloth cost 18 s. what will 24 yards cost?

Here it is plain, that the term sought, or the worth of 24 yards, will be money; therefore the given money 18 s. is set for the third term; and as the worth of 24 yards must be greater than the worth of 4 yards, therefore the 24 is set for the 2d term, and the 4 for the 1st. Then the 2d term 24 being multiplied by the 3d, 18, the Product is 432, which divided by the 1st term 4, the Quotient or 4th term is 108, which are shillings, the same name of the 3d term; then 108 shillings divided by 20, gives 5 £. 8 s.

$$\begin{array}{r}
 \text{yds.} \quad \text{yds.} \quad \text{s.} \\
 4 \text{---} 24 \text{---} 18 \\
 \quad \quad 18 \\
 \hline
 \quad \quad 192 \\
 \quad \quad 24 \\
 \hline
 4 \overline{) 432} (108 \text{ shillings.} \\
 \quad \underline{20} \quad \quad \quad \\
 \quad \quad 20 \quad \quad \quad 10,8 \text{ shillings.} \\
 \quad \quad \quad \underline{\quad} \\
 \quad \quad \quad \quad 5 \text{ pounds.}
 \end{array}$$

Answer 5 £. 8 s.

QUEST. II. If I lend 200 £. for 12 months, how long ought I to have the use of 150 £. to recompence me?

Here the answer or 4th term is to be time; therefore let 12 months, the given time, be set for the 3d term: Now it is evident, that the 150 £. being less than the 200 £., must be kept a longer time, and so the 4th term will be greater than the 3d term: Therefore the 200 is put for the 2d term, and the 150 for the 1st. Then the 2d term multiplied by the 3d, the Product will be 2400; which being divided by the 1st term, the Quotient 16 is the 4th term; and because the 3d term was months, the 4th term will be months.

$$\begin{array}{r}
 \text{£.} \quad \text{£.} \quad \text{m.} \\
 150 \text{---} 200 \text{---} 12 \\
 \quad \quad 12 \\
 \hline
 15,0 \overline{) 240,0} (16 \text{ months.} \\
 \quad \underline{15} \quad \quad \quad \\
 \quad \quad 90 \quad \quad \quad \\
 \quad \quad \underline{90} \quad \quad \quad
 \end{array}$$

Answer 16 months.

QUEST. III. *What will 1836 lb. of raisins come to, at the rate of 6 s. 8 d. for 24 lb.?*

Here as money is the thing sought, money must be the 3d term: And as 6 s. 8 d. consists of two names, they must be reduced to one name, viz. pence.

lb.	lb.	s.	d.
24	1836	6	8
		6	8
	1836	12	
	80		
		80d = 3d term.	
24)	146880	(6120d = 4th term.	
	144		
	28	12)6120	
	24	2,0)51,0 (10s.	
	48	25 £.	
	48		
	0		

Here the 2d term being multiplied by the 3d, and the Product divided by the first, the quotient is 6120 pence; which being valued, gives 25 £. 10 s.

QUEST. IV. *If 20 yards of cloth, 5 quarters wide, will serve to hang a room: How many yards of 4 quarters wide will serve to hang the same room?*

Here yards of length are required; then 20 yards must be the 3d term.

qrs.	qrs.	yds.
4	5	20
		5
		4)100(

Answer 25 yards.

QUEST. V. *What will 420 yards of cloth come to, at 14 s. 10 $\frac{1}{4}$ d. for 1 ell English?*

The term sought being money, the 14 s. 10 $\frac{1}{4}$ d. must be the 3d term, and be reduced to farthings; also the 1st and 2d terms are to be reduced to quarters of a yard.

Ell Eng. yds.	s.	d.
1	420	14 10 $\frac{1}{4}$
1	420	14 10 $\frac{1}{4}$
5	4	12
5	1680	178
	715	4
	8400	715 far. = 3d term.
	1680	
	11760	
5)	1201200(	
4)	240240	farthings = 4th term.
12)	60060	pence.
2,0)	500,5	5 shillings.
	250	pounds.

Answer 250 £. 5 s.

The Divisor 5 being a single digit, the Quot. is written under the Divid.

QUEST. VI. *A owes to B 463 £. but compounds for 7 s. 6 d. in the pound: How much must B receive for his debt?*

Here composition money is the thing sought; then the 3d term must be the composition money, viz. 7 s. 6 d.

£.	£.	s.	d.
1	463	7	6
	90	12	
	12)41670	(6d 90 pence.	
	2,0)347,2	(12 s.	
	173		

Answer 173 £. 12 s. 6 d.

48. As it will be more convenient in most cases to reduce such numbers, or terms, which consist of several names, to the fractional parts of their greatest name, than to reduce them to their lowest name; therefore in the solution of some of the following questions, the inferior parts of the given terms are reduced by Case IV. of Reduction; and the answers are valued by Case V.

QUEST.

QUEST. VII. If 8 lb. of pepper cost 4 s. 8 d.: What will 7 C. 3 qrs. 14 lb. come to at that rate?

lb.	C.	qrs.	lb.	s.	d.
8	7	3	14	4	8
	4			12	
					56
			882	lb.	
			56		
			262		
			62		
			5292		
			4410		
			882		
			8)49392		
			12)6174	6 d.	
			2,0)51,4	14 s.	

25 £.

Answer 25 £. 14 s. 6 d.

QUEST. IX. What is the interest of 584 £. for a year, at 5 per cent. per annum: Or at the rate of 5 £. for the use of 100 £. for a year?

Here interest is the term required; therefore 5 £. the interest of 100 £. is to be the 3d term: And as the 4th term, or the interest of 584 £. is greater than the 3d term; then the 2d term is to be greater than the 1st.

£.	£.	£.
100	584	5
	5	

1,00)29,20(See Case V. of Reduction.

20

4,00

Answer 29 £. 4 s.

QUEST. XI. What is the interest of 542 £. 10 s. for 219 days, at 5 £. per cent. per annum?

To solve this question, find the interest for 1 year; multiply this interest by 219, and divide the Product by 365, the Quotient will be the answer; and is 16 £. 5 s. 6 d.

QUEST. VIII. One bought 4 Hhds. of sugar, each containing 6 C. 2 qrs. 14 lb. at 2 £. 8 s. 6 d. for each C. weight: What did the whole come to?

C.	C.	qrs.	lb.	£.	s.	d.
1	6	2	14	2	8	6

Now 1 C. weight is 112 lb.

And 4 Hh. at 6 C. 2 q. 14 lb. = 2968 lb.

Also 2 £. 8 s. 6 d. is 582 d.

Then the Product of the 2d and 3d terms is 1727376.

Which divided by the 1st term 112, the Quotient is 15423 pence, whose value is 64 £. 5 s. 3 d.

QUEST. X. What is the interest of 387 £. 12 s. for three years and 4 months, at 3½ per cent. per annum?

Find the interest for 1 year; then thrice that, together with ⅓ of one year, will be the interest sought.

£.	£.	£.
100	387,6	3,5
	3,5	

19380

11628

100) 1356,60(

13,566 for 1 year.

3

40,698 for 3 years.

⅓ of 1 year = 4,522 for 4 months.

The sum 45,220 is the interest.

Answer 45 £. 4 s. 5 d.

QUEST. XII. For how long must 487 £. 10 s. be at simple interest, at 4½ £. per cent. per annum, to gain 95 £. 1 s. 3 d.?

Find what will be the interest of 487 £. 10 s. for 1 year; divide 95 £. 1 s. 3 d. by this interest, and the Quotient will be 4½ years.

QUEST. XIII. One bought 14 pipes of wine, and is allowed 6 months credit: But for ready money gets it 6 d. in a gallon cheaper: How much did he save by paying ready money?

Answer 44 £. 2 s.

QUEST. XV. One bought 3 tons of oil for 153 £. 9 s. which having leaked 74 gallons, he would make the prime cost of the remainder: How must it be sold per gallon?

Now 1 T. = 252 Gall. And 3 T. = 756
Subtract the gallons leaked = 74

Remains 682

G. G. £.
Then 682 — 1 — 153.45

Answer 4 s. 6 d. a gallon.

QUEST. XVII. At 13 £. for 100 lb. of goods: What will 895 lb. come to, allowing 4 lb. upon every 100 lb. for tret, or waste?

Since 4 lb. is to be allowed on the 100 lb. therefore 104 lb. is given for 100.

lb. lb. £.
Then 104 — 895 — 13

Answer 111 £. 17 s. 6 d.

QUEST. XIX. If 100 pounds of sugar be worth 36 s. 8 d. What will be the worth of 875 lb. rebating 4 lb. upon every 100 lb. for tare?

Here the buyer has 100 lb. on paying for 96 lb.

lb. lb. £.
Then 100 — 96 — 875
And the 4th term will be 840 lb.
Also 100 — 840 — 1,833333

Then the 4th term will be 15 £. 8 s. and so much will the sugar come to.

QUEST. XIV. A clothier sold 50 pieces of kersey, each piece containing 34 ells Flemish, at the rate of 8 s. 4 d. per ell English: What did the whole come to?

Answer 425 £.

QUEST. XVI. A broker sold $\frac{2}{3}$ of $\frac{3}{4}$ of a ship for 147 £. 11 s. 3 d.: How much was the whole ship valued at?

Now $\frac{2}{3}$ of $\frac{3}{4} = \frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$ by art. 38.
For $2 \times 3 = 6$, a new numerator.

And $5 \times 4 = 20$, a new denominator.

Also 147 £. 11 s. 3 d. = 147.5625. art.

40.

share share £.

Then 0.3 — 1 — 147.5625 art. 46.

Answer 491 £. 17 s. 6 d.

QUEST. XVIII. One has cloth which cost 2 s. 8 d. a yard: For how much must it be sold a yard on 3 months credit, to gain 25 £. per cent. per annum?

mon. mon. £. £.
First 12 — 3 — 25 — 6.25

£. £. £.
Secondly 100 — 6.25 — 0.133333

By multiplying and dividing, the 4th term will be found 2 d.

Then 2 s. 8 d. + 2 d. = 2 s. 10 d. a yard, the selling price.

QUEST. XX. A chapman bought 81 kerseys for 135 £.: How must he sell them per piece to gain 15 £. per cent?

Find how much 135 £. will be advanced to, at 15 £. per cent.

Then this sum divided by 81 will be the selling price of each piece.

£. £. £. £.
Now 100 — 115 — 135 — 155.25
Then 81) 155.25 (1.916666 £.

Answer 1 £. 18 s. 4 d. a-piece.

QUEST.

QUEST. XXI. *A merchant who is to receive a sum of money, is offered ducats at 6 s. 4 d. which are worth but 6 s. 2½ d. or chequins at 8 s. 2 d. each, that are worth but 8 s. : By which specie will he sustain the least loss?*

6 s. 2½ d. = 74, 5 d. } the real value.
 8 s. 0 d. = 96
 6 s. 4 d. = 76 } the advan. val.
 8 s. 2 d. = 98

r. val. r. val. ad. val. ad. val.
 Then 74,5 — 96 — 76 — 97,93

But the chequins are valued at 98 d. Therefore the ducats are most advantageous.

QUEST. XXIII. *A person wants 750 pieces of foreign coin, each worth 11 s. 4 d. : How much will they come to, allowing the broker the worth of 2 pieces upon every 100?*

Now 100 — 102 — 750 — 765.
 He must pay for 765 pieces, which will come to 433 £. 10 s.

QUEST. XXV. *A grocer bought 4¼ C. of pepper for 15 £. 17 s. 4 d. which proving to be damaged, he is willing to lose 12½ £. per cent. : For how much must he sell it a lb.?*

Since he is to lose 12½ per cent. he must take 87 £. 10 s. for 100 £. Now diminish the 15 £. 17 s. 4 d. in this proportion, and this sum divided by the pounds in 4¼ C. will give 7 d. for what each pound is to be sold at.

QUEST. XXII. *One who had sold a parcel of cloths at 2 s. 10 d. a yard on 3 months credit, found he had gained 25 £. per cent. per annum : What did the cloth cost per yard?*

mo. £. mo. £.
 Now 12 — 25 — 3 — 6,25
 And 100 + 6,25 = 106,25 £.
 Then 106,25 — 100 — 0,14166

The fourth term to which will be a fraction, the value of which will be 2 s. 8 d. which is the prime cost per yard of the cloth.

QUEST. XXIV. *A gentleman would exchange 729 pieces of 4 s. 2 d. each into sterling money : How much will he receive for them, allowing the broker 1¼ £. per cent.?*

P. P. £. £.
 Now 1 — 729 — 0,208333 — 151,875
 the worth of the pieces.
 Then 101,25 — 100 — 151,875 — 150 £.
 He will receive 150 £. for them.

QUEST. XXVI. *Suppose 42 gallons of honey be valued at 2 £. and the duty is 15 £. per cent. on this value, and a drawback of 5 £. per cent. on the duty for prompt payment : What will the ready money duty of 672 gallons come to?*

Now 42 G. : 672 G. :: 2 £. : 32 £.
 And 100 £. : 15 £. :: 32 £. : 4,8 £.
 Also 100 £. : 95 £. :: 4,8 £. : 4,56 £.

Answer 4 £. 11 s. 2½ d.

The Rule of Proportion is of almost universal use in all business where computation is required ; as in buying and selling, values of stocks and their dividends ; the interest and discount of money ; the customs and duties on goods, &c. But the designed brevity of this book will not permit farther illustrations.

SECTION VIII. OF THE POWERS OF NUMBERS, AND OF THEIR ROOTS.

49. The POWER of a number, is a product arising by multiplying that number by itself, the product by the same number, this product by the same number again, &c. to any number of multiplications.

50. The given number is called the first power or root.

The Product of the 1st power by itself, is the second power, or square.

The Product of the 2d power by the 1st, is the 3d power, or cube.

The Product of the 3d power by the 1st, is the 4th power, &c.

51. Here follow the 1st, 2d, and 3d powers of the nine digits.

Roots, or 1st power	1	2	3	4	5	6	7	8	9
Squares, or 2d power	1	4	9	16	25	36	49	64	81
Cubes, or 3d power	1	8	27	64	125	216	343	512	729

Ex. I. What is the 2d power, or square of the number 24?

$24 \times 24 = 576$ is the 2d power.

Ex. II. What is the 3d power, or cube of 38?

Now $38 \times 38 = 1444$ the 2d power.

Then $1444 \times 38 = 54872$ the 3d power.

The figure, or number, shewing the name of any power, is called the index of that power.

Thus 1 is the index of the first power : 2 is the index of the 2d power ; 3 of the third power, &c. Also $\frac{1}{2}$ is the index of the square root ; $\frac{1}{3}$, the index of the cube root, &c.

52. Any number may be considered as a power of some other number.

Thus 64 may be taken as the 2d power of 8, and the third power of 4, &c.

53. The root of a given number, considered as a power, is a number which being raised to the index of that power, will either be equal to the given number, or approach very near to it.

54. To extract the Square Root of a given number.

RULE 1st. Begin at the unit's place, put a point over it, and also over every next figure but one, reckoning to the left for integers, and to the right for fractions ; and there will be as many integer places in the root, as there are points over the integers in the given number.

The figure under a point, with its left-hand place, is called a period.

2d. Under the left-hand period write the greatest square contained in it, and set the root thereof in the Quotient ; subtract the square, and to the remainder bring down the next period, as in Division.

3d. On the left of this Remainder write the double of the Root or Quotient for a Divisor ; seek how often this may be had in the Remainder, except the right-hand place ; write what ariseth both in the Root, and on the right of the Divisor.

4th. Multiply this increased Divisor by the last Quotient-figure ; subtract, and to the Remainder bring down the next period ; double the Root for a Divisor, and proceed as before.

55. Fractional places will arise in the Root, by annexing to the Remainders, periods of two cyphers each, and renewing the operation.

Ex.

Ex. I. *What is the Square Root of 1444?*

Put a point over the units place 4, and also over the place of 100s. Now the number consists of 2 periods, and will have 2 integer places in the Root: Then the greatest Square in 14, the left-hand period, is 9, and its Root is 3; write 9 under the period, and three in the Root; now 9 from 14 leaves 5, to which annex the next period 44; the Root 3 doubled makes 6, which in 54 is contained 8 times, annex 8 to the 3 in the Quotient, and to the Divisor 6, makes the Root 38, and the Divisor 68; then 8 times 68 is 544; and there remaining 0, on subtraction, it may be concluded, that 38 is the true Root.

$$\begin{array}{r}
 \cdot \cdot \\
 1444 \text{ (38 Root.} \\
 \underline{9} \\
 68) \ 544 \\
 \underline{544} \\
 0
 \end{array}$$

Ex. II. *What is the Square Root of 36372961?*

$$\begin{array}{r}
 \cdot \cdot \cdot \cdot \\
 36372961 \text{ (6031 Root.} \\
 \underline{36} \\
 1203) \ 3729 \\
 \underline{3609} \\
 12061) \ 12061 \\
 \underline{12061} \\
 0
 \end{array}$$

Ex. IV. *What is the Square Root of 24681024?*
Answer 4968Ex. VI. *What is the Square Root of 76395820?*

$$\begin{array}{r}
 \cdot \cdot \cdot \cdot \cdot \cdot \\
 76395820 \text{ (8740.4702} \\
 \underline{64} \\
 167) \ 1239 \\
 \underline{1169} \\
 1744) \ 7058 \\
 \underline{6976} \\
 174804) \ 822000 \\
 \underline{699216} \\
 1748087) \ 12278400 \\
 \underline{12236609} \\
 174809402) \ 417910000 \\
 \underline{349618804} \\
 68291196
 \end{array}$$

Ex. III. *What is the Square Root of 1,0609?*

$$\begin{array}{r}
 \cdot \cdot \\
 1,0609 \text{ (1,03 Root.} \\
 \underline{1} \\
 203) \ 0609 \\
 \underline{609} \\
 0
 \end{array}$$

Ex. V. *What is the Square Root of 911236798,794365?*
Answer 30186,699, &c.

$$\begin{array}{r}
 \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\
 911236798,794366 \text{ (30186,6} \\
 601) \ 1123 \\
 6028) \ 52267 \\
 60366) \ 404398 \\
 603726) \ 4220279 \\
 \underline{597923} \\
 \text{\&c.}
 \end{array}$$

Here the products are omitted, the multiplication and subtraction being made in the mind.

In the VIth Example, after all the periods given were brought down, there remained 8220, to which a period of two cyphers was annexed, and the operation renewed, and continued until 4 decimal places were obtained in the Root; every period brought down giving one place.

56. *To extract the Cube Root of a given Number.*

RULE 1st. Over the unit place of the given number put a point, and also over every third figure from the unit place, to the left for integers, and to the right for fractions; and the root will have as many integer places, as there are points, or periods, in the integral part of the given number.

2d. Under the left hand period, write the greatest Cube it contains, the root of which set in the Quotient: Subtract the Cube from the period, and to the Remainder annex the remaining periods; call this the Resolvend.

3d. To the Quotient annex as many cyphers as there were periods remaining; call this the Root.

4th. Divide the Resolvend by the Root, add the Quotient to thrice the Square of the Root, let the Sum be a Divisor to the Resolvend, and the Quotient-figures annexed to the right of the first Root, without the cyphers, will be the Cube Root sought.

5th. If the second figure of the Root be 1, or 0; then generally 3 or 4 figures of the Root will be obtained at the first operation: But if the second figure exceeds 2, it will be best to find only two places at first.

6th. To renew the operation; subtract the Cube of the figures found in the Root from the given number; then form a Divisor, and divide as directed in the fourth precept; and this will give the Root true to 5 or six places: For each operation commonly triples the figures found in the last Root.

Ex. I. *What is the Cube Root of 9800344?*

Put a point over the unit place 4, another over the place of thousands, and another over that of millions; and because there are 3 points, there will be 3 places in the Root. Under the left hand period 9, write 8, the greatest Cube in it, and its Root 2 write in the Quotient, then subtracting, the Resolvend is 1800344: Now because there are two periods remaining, therefore two cyphers annexed to the Root 2, make it 200, by which dividing the Resolvend, the Quotient is 9001; also the square of 200 is 40000, the triple thereof 120000 being added to 9001, makes 129001 for a Divisor, by which dividing 1800344, the Quotient is 14 nearly, and is taken as 14, because it is much nearer to it than to 13; now 14 being annexed to the former Root 2, makes 214, the Root sought.

For $214 \times 214 \times 214 = 9800344$.

$$\begin{array}{r}
 \cdot \quad \cdot \quad \cdot \\
 9800344(2 \\
 \underline{8} \\
 2,00) 18003,44 \text{ Resolvend.} \\
 \underline{9001} = \text{Quotient.} \\
 120000 = \text{thrice the Sq. of the R.} \\
 \underline{129001} 1800344(14 \\
 \underline{129001} \\
 510344 \\
 \underline{516004}
 \end{array}$$

The Root is 214.

Ex. II.

SECTION IX. OF NUMERAL SERIES.

58. A rank of three or more numbers that increase or decrease by an uniform progression, is called a *Numerical Series*.

59. If the Progression is made by equal differences, that is by the constant addition or subtraction of the same number; the series is called an *Arithmetic Progression*.

Thus $\left\{ \begin{array}{l} 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ \&c. \text{ increasing by adding } 1, \\ 3 \ 6 \ 9 \ 12 \ 15 \ 18 \ 21 \ 24 \ 27 \ \&c. \text{ increasing by adding } 3, \\ 49 \ 43 \ 37 \ 31 \ 25 \ 19 \ 13 \ 7 \ 1 \ \&c. \text{ decreasing by subtracting } 6, \end{array} \right.$
are ranks of numbers in Arithmetic Progression: And of such ranks there may be an infinite variety.

60. If the Progression is made by a constant multiplication or division with the same number, the series is called a *Geometric Progression*.

Thus $\left\{ \begin{array}{l} 1 \ 2 \ 4 \ 8 \ 16 \ 32 \ 64 \ \&c. \text{ increasing by } 2, \\ 1 \ 5 \ 25 \ 125 \ 625 \ 3125 \ 15625 \ \&c. \text{ increasing by } 5, \\ 6561 \ 2187 \ 729 \ 243 \ 81 \ 27 \ 9 \ 3 \ \&c. \text{ decreasing by } 3, \\ 16384 \ 4096 \ 1024 \ 256 \ 64 \ 16 \ 4 \ 1 \ \&c. \text{ decreasing by } 4, \end{array} \right.$
are ranks of numbers in Geometric Progression: And of such ranks there may be an infinite variety.

61. The common Multiplier or Divisor is called the *ratio*.

Thus 2 is the ratio in the 1st rank, 5 in the 2d rank, 3 is the ratio in the 3d rank, and 4 in the 4th rank.

62. In any series of terms in Arithmetic Progression, the sum of any two terms, considered as extremes, is equal to the sum of any two terms taken as means equally distant from the extremes.

Thus in 3 terms (where the 1st and 3d are extremes, and the other the mean)
viz. 6 . 9 . 12, then $6 + 12 = 9 + 9 = 18$.

And in 4 terms, viz. 13 . 19 . 25 . 31.

Then $13 + 31 = 19 + 25 = 44$.

Also in the terms 49 . 43 . 37 . 31 . 25 . 19 . 13 . 7 . 1.

Then $49 + 1 = 43 + 7 = 37 + 13 = 31 + 19 = 25 + 25 = 50$.

63. In a series of terms in Geometric Progression, the Product of any two terms considered as extremes, is equal to the Product of any two intermediate equidistant terms considered as means.

Thus in 3 terms, viz. 5 . 25 . 125. Or 3 . 9 . 27.

Then $5 \times 125 = 25 \times 25 = 625$. Also $3 \times 27 = 9 \times 9 = 81$.

And in 4 terms 4 . 8 . 16 . 32.

Then $32 \times 4 = 16 \times 8 = 128$.

Also in the terms 1 . 4 . 16 . 64 . 256 . 1024 . 4096 . 16384.

Then $16384 \times 1 = 4096 \times 4 = 1024 \times 16 = 256 \times 64 = 16384$.

64. In any Arithmetic Progression, the sum of any two terms lessened by the first term; or their difference increased by the first term, will be a term also in that progression.

Thus in the Progression 1 . 3 . 5 . 7 . 9 . 11 . 13 . 15 . 17 . 19 . 21 &c.
Then $7 + 11 = 18$, and $18 - 1 = 17$ is a term of the Progression.
Also $11 - 7 = 4$, and $4 + 1 = 5$ is a term of the Progression.

65. In any Geometric Progression, the product of any two terms divided by the first term; or the Quotient of any two terms multiplied by the first term, will give a term also in that series.

Thus in the Progression 3 . 6 . 12 . 24 . 48 . 96 . 192 . 384 . 768 &c.
Then $\frac{12 \times 96}{3} = 384$; and $\frac{192}{12} \times 3 = 48$, are terms in the Progression.

66. If over a series of terms in Geometric Progression, be written a series of terms in Arithmetic Progression, the first term of which is 0, and common difference is 1, term for term; then any term in the Arithmetic Series will shew how far its corresponding term in the Geometric Series is distant from the first term.

Thus

0	1	2	3	4	5	6
1	3	9	27	81	243	729

&c. Arithmetic Series.
&c. Geometric Series.
Here 729 is distant from the 1st term, 6 terms; 243 is distant 5 terms, 81 is distant 4 terms.

67. The terms of the Arithmetical Series are called indices to the terms of the Geometric Series.

Thus 5 is the index to 243; 3 is the index to 27; 1 is the index to 3; &c.

68. PROBLEM I. In an Arithmetic Progression: Given the first term, the common difference, and the number of terms.

Required the last term.

RULE. Subtract 1 from the number of terms, multiply the remainder by the common difference; to the product add the first term, and the sum will be the last term.

Ex. I. Suppose 1 and 9 to be the first and second terms, of an Arithmetic Progression of 1074 terms: What is the last term?

Here $9 - 1 = 8$ is the com. diff. Now $1074 - 1 = 1073$.

And $1073 \times 8 = 8584$. Then $8584 + 1 = 8585 = \text{last term}$.

Ex. II. A person agrees to discharge a certain debt in a year, by weekly payments, viz. the first week 5s. the 2d week 8s. &c. constantly increasing each week by 3s.: How much was the last payment?

1st term.

Now $52 - 1 = 51$.

com. diff.

And $51 \times 3 = 153$.

N^o of terms.

Then $153 + 5 = 158s. = 7\text{ } \text{£}. 18s. = \text{last Payment}$.

69. PRO-

69. PROBLEM II. *In an Arithmetic Progression: Given the first term, last term, and the number of terms.*

Required the sum of all the terms.

RULE. Add the first and last terms together, the sum multiplied by half the number of terms, gives the sum of all the terms.

Ex. I. *Required the sum of the first 1000 numbers in their natural order.*

Here 1 = 1st term, 1 = com. diff.

1000 = N^o of terms, its $\frac{1}{2}$ is 500.

Now $1000 + 1 = 1001$.

Then $1001 \times 500 = 500500$ is the sum required.

Ex. II. *A debt is to be discharged in a year by weekly payments equally increasing; the 1st to be 5s. and the last 7 £. 18s. How much was the debt?*

Here 7 £. 18s. = 158s. = last term.

52 = N^o of terms, its $\frac{1}{2}$ is 26.

Now $158 + 5 = 163$.

Then $163 \times 26 = 4238s. = 211 £. 18s.$ is the sum of the terms, or debt.

Ex. III. *Suppose a basket and 500 stones were placed in a straight line, a yard distant from one another: Required in what time a man could bring them one by one to the basket, allowing him to walk at the rate of 3 miles an hour?*

Between the basket and stones are 500 spaces, which is the number of terms.

Now $500 + 1 = 501$. Then $501 \times 250 = 125250 =$ sum of the terms.

But as he goes backwards and forwards, he walks 250500 yards.

Which divided by 1760 (the yards in 1 mile) gives 142,329 miles.

Which at 3 miles an hour, will take 47 h. 26 min. 35 seconds nearly.

70. PROBLEM III. *In a Geometric Progression: Given the first term, the ratio and the last term.*

Required the sum of all the terms.

RULE. Multiply the last term by the common ratio, from the Product subtract the first term for a Dividend.

Subtract 1 from the ratio for a Divisor; then divide, and the Quotient will be the sum of all the terms.

Ex. I. *Suppose the first term of a series to be 3, the ratio 3, and the last term 6561: Required the sum of all the terms.*

Now 6561 = last term.

And 3 = ratio.

Mult. by 3 = ratio.

Sub. 1

19683 = Product

Rem. 2 = Divisor:

Subtr. 3 = first term.

19680 = Dividend.

Then $2) 19680 (9840$ is the sum of all the terms.

Ex. II. *Let the first term be 2, the second term 10, and the last term 156250: Required the sum of all the terms.*

Here 2) 10 (5 is the common ratio.

Now $156250 \times 5 = 781250$. And $781250 - 2 = 781248 =$ Dividend.

Also $5 - 1 = 4$ the Divisor.

Then $4) 781248 (195312$ is the sum of all the terms.

71. PROBLEM IV. In a Geometric Progression: Given the first term, the ratio, and the number of terms.

Required the last term.

RULE 1st. Write down 6 or 7 of the leading terms in the Geometric Series, and over them their Indices.

2d. Add together the most convenient Indices to make an index less by unity than the number expressing the place of the term sought.

3d. Multiply together the terms of the Geometric Series, belonging to those Indices which were added; make the product a dividend.

4th. Raise the first term to a power whose index is one less than the number of terms multiplied; make the result a Divisor to the former Dividend, and the Quotient will be the term sought.

Ex. I. What is the 12th term of a Geometric Series, the first term of which is 3, and second term is 6?

Now $\frac{6}{3} = 2$ is the common ratio.

And $\left\{ \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \text{ \&c. Indices.} \\ 3 & 6 & 12 & 24 & 48 & 96 & 192 \text{ \&c. Geometric terms.} \end{array} \right.$

Then $6 + 5 = 11$, is the index to the 12th term.

And $192 \times 96 = 18432$, is the Dividend.

The number of terms multiplied together is 2; and $2 - 1 = 1$, the power to which the first term 3 is to be raised; but the first power of 3 is 3.

Then $\frac{18432}{3} = 6144$ is the 12th term of the given series.

Ex. II. A person being asked to dispose of a fine horse, said he would sell him on condition of having one farthing for the 1st nail in his shoes, two farthings for the 2d nail; one penny for the 3d nail; two pence for the 4th; four pence for the 5th; 8 pence for the 6th, &c.; doubling the price of every nail to 32, the number of nails in the four shoes: How much would that horse be sold for at that rate?

Here the first term is 1, the ratio 2, and the number of terms 32.

First. To find the last term.

Now $\left\{ \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \text{ \&c. Indices.} \\ 1 & 2 & 4 & 8 & 16 & 32 & 64 & 128 & 256 \text{ \&c. Geometric terms.} \end{array} \right.$

And 31 is the index to the 32d term.

Then $8 + 8 = 16$; $16 + 8 = 24$; $24 + 7 = 31$.

The 1st term being 1, any power thereof is 1; so the 4th article of the rule is useless in this question.

Now $256 \times 256 = 65536$ is the 17th term.

$65536 \times 256 = 16777216$ is the 25th term.

$16777216 \times 128 = 2147483648$ is the 32d term.

Then 2147483648

$\frac{2147483648}{2} = 1073741824$

$\frac{1073741824}{2} = 536870912$

$2 - 1 = 1$ the 1st term. 4294967295 the sum of the terms: or the price, in farthings, of the horse.

VOL. I.

D

SECTION

41294967295
12 1073741823 3f.
20 89478485 - 3d.
4473924 - 5s.
Answer 4473924L. 5s. 3 $\frac{3}{4}$ d.

SECTION X. OF LOGARITHMS.

72. LOGARITHMS are a series of numbers so contrived, that by them the work of multiplication may be performed by addition; and the operation of division may be done by subtraction.

73. Or, Logarithms are the Indices to a series of numbers in Geometrical Progression.

Thus	{	0	1	2	3	4	5	6	Indices or Logarithms.
		1	2	4	8	16	32	64	Geometric Progression.
		0	1	2	3	4	5	6	Indices or Logarithms.
		1	3	9	27	81	243	729	Geometric Series.
or	{	0	1	2	3	4	5	6	Indices or Logarithms.
		1	10	100	1000	10000	100000	1000000	Geometric Series.

Where the same Indices serve equally for any Geometric Series.

74. Hence it is evident, there may be as many kinds of Indices or Logarithms, as there can be taken kinds of Geometric Series.

But the Logarithms most convenient for common uses, are those adapted to a Geometric Series increasing in a tenfold Progression, as in the last of the examples above.

75. In the Geometric Series 1 . 10 . . 100 . 1000 . &c. if between the terms 1 and 10, the numbers 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 were interposed, Indices might also be adapted to them in an Arithmetic Progression, suited to the terms interposed between 1 and 10, considered as a Geometric Progression. Moreover proper Indices may be found to all the numbers that can be interposed between any two terms of the Geometric Series.

But it is evident that all the Indices to the numbers under 10 must be less than 1; that is, are fractions: Those to the numbers between 10 and 100 must fall between 1 and 2; that is, are mixed numbers consisting of 1 and some fraction: And so the Indices to the numbers between 100 and 1000 will fall between 2 and 3; that is, are mixed numbers consisting of 2 and some fraction: And so of the other Indices.

76. Hereafter, the integral part only of these Indices will be called the Index; and the fractional part will be called the Logarithm: And the computing of those fractional parts is called the making of Logarithms; the most troublesome part of this work is to make the Logarithms of the prime numbers; that is, of such numbers which cannot be divided by any other number than by itself and unity.

77. To find the Logarithms of prime numbers.

RULE 1st. Let the sum of the proposed number and its next less number be called A.

2d. Divide 0,868588963 * by A, reserve the Quotient.

* The number 0,868588963 is the Quotient of 2 divided by 2,302585093, which is the Logarithm of 10, according to the first form of the Lord Napier, who was the inventor of Logarithms. The manner by which Napier's Log. of 10 is found, may be seen in many books of Algebra; but is here omitted, because this treatise does not contain the elements of that science: However, those who have not opportunity to enter thoroughly into this subject, had better grant the truth of one number, and thereby be enabled to try the accuracy of any Logarithm in the tables, than to receive those tables as truly computed, without any means of examining the certainty thereof.

3d. Divide

3d. Divide the reserved Quotient by the Square of A, reserve this Quotient.

4th. Divide the last reserved Quotient by the Square of A, reserving the Quotient; and thus proceed as long as division can be made.

5th. Write the reserved Quotients orderly under one another, the first being uppermost.

6th. Divide these Quotients respectively by the odd numbers 1 . 3 . 5 . 7 . 9 . 11, &c. that is, divide the first reserved Quotient by 1, the 2d by 3, the 3d by 5, the 4th by 7, &c. let these Quotients be written orderly under one another, add them together, and their sum will be a Logarithm.

7th. To this Logarithm, add the Logarithm of the next less number, and the sum will be the Logarithm of the number proposed.

Ex. I. *Required the Logarithm of the number 2.*

Here the next less number is 1, and $2 + 1 = 3 = A$.

And the Square of A is 9. Then;

$\begin{array}{r} 0,868588963 \\ \hline 3 \\ 0,289529654 \\ \hline 9 \\ 0,032169962 \\ \hline 9 \\ 0,003574440 \\ \hline 9 \\ 0,000397160 \\ \hline 9 \\ 0,000044129 \\ \hline 9 \\ 0,000004903 \\ \hline 9 \\ 0,000000545 \\ \hline 9 \end{array}$	$\begin{array}{r} \text{And } 0,289529654 \\ \hline 1 \\ 0,032169962 \\ \hline 3 \\ 0,003574440 \\ \hline 5 \\ 0,000397160 \\ \hline 7 \\ 0,000044129 \\ \hline 9 \\ 0,000004903 \\ \hline 11 \\ 0,000000545 \\ \hline 13 \\ 0,000000060 \\ \hline 15 \end{array}$
$\begin{array}{r} =,289529654 \\ =,032169962 \\ =,003574440 \\ =,000397160 \\ =,000044129 \\ =,000004903 \\ =,000000545 \\ =,000000060 \end{array}$	$\begin{array}{r} =,289529654 \\ =,010723321 \\ =,000714888 \\ =,000056737 \\ =,000004903 \\ =,000000445 \\ =,000000042 \\ =,000000004 \end{array}$

To this Log.

Add the Log. of

$0,301029994$

$1 = 0,000000000$

Their sum is the Log. of $2 = 0,301029994$

This process needs no other explanation than comparing it with the rule.

That the manner of computing these Logarithms may be familiar to the Reader, the operations of making several of them are here subjoined.

Ex. II. Required the Logarithm of the number 3.

Here the next less number is 2; and $3+2=5=A$, whose Square is 25.

$\begin{array}{r} 0,868588963 \\ \hline 5 \\ 0,173717792 \\ \hline 25 \\ 0,006948712 \\ \hline 25 \\ 0,000277948 \\ \hline 25 \\ 0,000011118 \\ \hline 25 \\ 0,000000445 \\ \hline 25 \end{array}$	$\begin{array}{r} 173717792 \\ \hline 1 \\ 6948712 \\ \hline 3 \\ 277948 \\ \hline 5 \\ 11118 \\ \hline 7 \\ 445 \\ \hline 9 \\ 18 \\ \hline 11 \end{array}$
--	--

To this Logarithm
Add the Log. of 2

$$\begin{array}{r} 0,176091258 \\ 0,301029994 \\ \hline \end{array}$$

The sum is the Logarithm of 3

$$\begin{array}{r} 0,477121252 \\ \hline \end{array}$$

78. Since the Logarithms are the Indices of numbers considered in a Geometric Progression; therefore the sums, or differences of these Indices, will be Indices or Logarithms belonging to the Products, or Quotients, of such terms in the Geometric Progression as correspond to those Logarithms which were added or subtracted (71).

Ex. III. Required the Log. of 4.

Now $4=2 \times 2$.

Then to the Log. of 2 $0,301029994$
Add the Log. of 2 $0,301029994$

Sum is the Log. of 4 $0,602059988$

Ex. IV. Required the Log. of 6.

Now $3 \times 2=6$.

Then to the Log. of 3 $0,477121252$
Add the Log. of 2 $0,301029994$

Sum is the Log of 6 $0,778151246$

Ex. V. Required the Log of 10.

In the original Series, 1 is assumed for the Logarithm of 10.

Ex. VI. Required the Log. of 5.

Now 10 divided by 2 gives 5.

Then from Log. of 10 $1,000000000$
Take Log. of 2 $0,301029994$

Leaves the Log. of 5 $0,698970006$

Ex. VII. Required the Log. of 8.

Now $8=2 \times 2 \times 2$.

Therefore the Log. of 2 taken thrice gives the Log. of the number 8.

The Log. of 2 is $0,301029994$
Which multiply by 3

Gives the Log. of 8 $0,903089982$

Ex. VIII. Required the Log. of 9.

Now $9=3 \times 3$

Therefore the Log. of 3 doubled gives the Log of 9.

The Log. of 3 is $0,477121252$
Which multiplied by 2

Gives the Log. of 9 $0,954242504$

Ex. IX.

Ex. IX. Required the Logarithm of 7.

Here 6 is the next less. Then $7 + 6 = 13 = .1$; and $169 = \text{Square of } A$.

$$\frac{0,868588963}{13} = ,066814536. \quad \text{And} \quad \frac{,066814536}{1} = ,066814536$$

$$\frac{0,066814536}{169} = 395352. \quad \& \quad \frac{395352}{3} = 131784$$

$$\frac{0,000395352}{169} = 2339. \quad \& \quad \frac{2339}{5} = 468$$

$$\frac{0,000002339}{169} = 14. \quad \& \quad \frac{14}{7} = 2$$

To this Logarithm	0,066946790
Add the Logarithm of 6	0,778151246

The sum is the Logarithm of 7	0,845098036
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The Log. of 12 is equal to the sum of the Logs. of 3 & 4, or of 2 & 6,

The Log. of 14 is equal to the sum of the Logs. of 7 & 2.

of 15

of 3 & 5.

of 16

of 4 & 4, or of 8 & 2.

of 18

of 3 & 6, or of 9 & 2.

of 20

of 4 & 5, or of 10 & 2.

79. The Logs. of the prime numbers 11, 13, 17, 19, are to be found as in the examples I. II. IX. and in like manner is the Log. of any other prime number to be found; but it may be observed, that the operation is shorter in the larger prime numbers; for any number exceeding 400, the first Quotient added to the Logarithm of its next lesser number, will give the Logarithm sought, true to 8 or 9 places; and therefore it will be very easy to examine any suspected Logarithm in the tables.

80. The manner of disposing the Logarithms, when made, into tables, is various: But in this treatise they are ordered as follows:

Any number under 100, or not exceeding two places, and its Logarithm, are found in the first page of the table, where they are placed in adjoining columns; and distinguished by the title Num. for the common numbers; and by Log. for the Logarithms.

These tables are at the end of Book IX.

A number of three or four places being given, its Logarithm is thus found.

Seek for a page in which the given number shall be contained between the two numbers marked at the top, annexed to the letter N: Then right against the three first figures of the given number, found in the column signed Num. and in the column signed by the fourth, stands the Logarithm belonging to that number of four places.

If the number consisted of 3 places only; then these places found as before directed, the Logarithm stands against them in the column signed 0.

Thus, to find the *Logarithm* of 5738. Seek for a page in which stands at top N^o 5200 to 5800; then in the column signed N^o find 573, right against which in the column signed 8 at top or bottom stands, 75876, which is the *Logarithm* to 5738, exclusive of its Index.

81. *A Logarithm being given, its number is thus found.*

Seek for a page in which the three first figures of the given *Logarithm* are found at top annexed to the letter *L*; then in one of the columns signed with the figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, find a number the nearest to the given *Logarithm*; against this number in the column signed N^o, stand three figures; to the right of these annex the figure with which the column was signed at top or bottom, and this will be the number corresponding to the given *Logarithm*, not regarding the Index.

82. All numbers consisting of the same figures, whether they be integral, fractional, or mixed, have the fractional parts of their *Logarithms* the same.

If the following examples be well attended to, there will be no difficulty in finding the *Logarithm* to a proposed number, or the number to a proposed *Logarithm*, within the limits of the table of *Logarithms* here used.

Num.	Logarithms.	Logarithms.	Numbers.
5874	3,76893	0,37295	2,360
587,4	2,76893	1,28631	19,33
58,74	1,76893	2,51947	330,7
5,874	0,76893	3,75062	5632,
0,5874	1,76893	3,18397	0,001527
0,05874	2,76893	2,43020	0,02693
0,005874	3,76893	1,85962	0,7238

83. A general rule to find the Index to the Log. of a given number.

To the left of the *Logarithm*, write that figure (or figures) which expresses the distance from unity, of the highest-place digit in the given number, reckoning the unit's place 0, the next place 1, the next place 2, the next place 3, &c.

When there are integers in the given number, the Index is always affirmative; but when there are no integers, the Index is negative, and is to be marked by a little line drawn above it: thus $\bar{2}$.

Thus a number having 1 . 2 . 3 . 4 . 5 &c. integer places.

The Index of its *Logarithm* is 0 . 1 . 2 . 3 . 4 . &c.

And a fraction having a digit in the place of Primes, Seconds, Thirds, Fourths, &c.

Then the Index of its *Logarithms* will be $\bar{1}$. $\bar{2}$. $\bar{3}$. $\bar{4}$ &c.

By the above rule, the place of the fractional comma, or mark of distinction, in the number answering to a given *Logarithm*, will be always known.

84. The more places the *Logarithms* consist of, the more accurate, in general, will be the result of any operation performed with them: But for the purposes of Navigation, as five places, exclusive of the Index, are sufficient, therefore the logarithmic tables in this treatise are not extended any farther,

85. MUL-

85. MULTIPLICATION BY LOGARITHMS;

Or, Two or more numbers being given, to find their Product by Logarithms.

RULE. Add together the Logarithms of the Factors, and the sum is a Logarithm, the corresponding number of which is the Product required.

Observing to add what is carried from the Logarithm to the sum of the affirmative Indices.

And that the difference between the affirmative and negative Indices are to be taken for the Index to the Logarithm of the Product.

Ex. I. Multiply 86,25 by 6,48.

86,25 its Log. is	1,93576
6,48 its Log. is	0,81157

Product 558,9	2,74733
---------------	---------

Ex. III. Multiply 3,768 by 2,053 and by 0,007693.

3,768 its Log. is	0,57611
2,053	0,31239
0,007693	3,88610

Product 0,05951	2,77460
-----------------	---------

The 1 carried from the left hand column of the Logs. being affirmative, reduces $\bar{3}$ to 2.

Ex. II. Multiply 46,75 by 0,3275.

46,75 its Log. is	1,66978
0,3275 its Log. is	1,51521

Product 15,31	1,18499
---------------	---------

Ex. IV. Multiply 27,63 by 1,859 and by 0,7258 and by 0,03591.

27,63 its Log. is	1,44138
1,859	0,26928
0,7258	1,86082

0,03591	2,55521
Product 1,339	0,12669

Here 2 being carried to the Index 1, makes 3; which takes off the $\bar{1}$ and $\bar{2}$.

86. DIVISION BY LOGARITHMS.

Or, Two numbers being given, to find how often the one will contain the other, by Logarithms.

RULE. From the Log. of the Dividend, subtract the Log. of the Divisor; then the number agreeing to the Remainder, will be the Quotient required.

But observe to change the Index of the Divisor from negative to affirmative, or from affirmative to negative: And then let the difference of the affirm. and neg. Indices be taken for the Index to the Log. of the Quotient.

When an unit is borrowed in the left-hand place of the Logarithm, add it to the Index of the Divisor, if affirmative; but subtract it if negative; and let the Index arising be changed and worked with as before.

Ex. I. Divide 558,9 by 6,48.

Log. of Divid. 558,9 is	2,74733
Log. of Divisor 6,48 is	0,81157

The Quotient is 86,25	1,93576
-----------------------	---------

Ex. II. Divide 15,31 by 46,75.

Log. of Divid. 15,31 is	1,18497
Log. of Divisor 46,75 is	1,66978

The Quotient is 0,3275	1,51519
------------------------	---------

Ex. III. Divide 0,05951 by 0,007693.

Log. of Divid. 0,05951 is	2,77459
Log. of Divisor 0,007693 is	3,88610

The Quotient is 7,735	0,88849
-----------------------	---------

Ex. IV. Divide 0,6651 by 22,5.

Log. of Divid. 0,6651 is	1,82289
Log. of Divisor 22,5 is	1,35218

The Quotient is 0,02956	2,47071
-------------------------	---------

87.

OF PROPORTION.

1st. State the terms of the question (by 46) and let them be written orderly under one another, prefixing to the first term the word *As*, to the second *To*, to the third *So*, and under them set the word *To*.

2d. Against the first term, write the arithmetical complement of its Logarithm. See Art. 88.

3d. Against the second and third terms, write their Logarithms.

4th. The sum of those three Logarithms, abating 10 in the Index, will be the Logarithm of the 4th term; which sought in the tables, the number answering to it is the answer or term sought.

88. The arithmetical complement of a Logarithm is thus found. Beginning at the Index, write down what each figure wants of 9, except the last, or right-hand figure, which take from 10.

But if the Index is negative, add it to 9; and proceed with the rest as before.

Ex. I. Find a fourth proportional number to 98,45 and 1,969 and 347,2

As 98,45 its * Ar. Co. Log.	8,00678
To 1,969	0,29425
So 347,2	2,54058
To 6,944	0,84161

Ex. II. Find a third proportional number to 9,642 and 4,821.

As 9,642 its Ar. Co. Log.	9,01583
To 4,821	0,68314
So 4,821	0,68314
To 2,411	0,38211

Ex. III. What will a gunner's pay amount to in a year at 2*£*. 12*s*. 6*d*. a month of 28 days?

As 28 days its Ar. Co. Log.	8,55284
To 365 days	2,56229
So 2 <i>£</i> . 12 <i>s</i> . 6 <i>d</i> . = 2,625 <i>£</i> .	0,41913
To 34 <i>£</i> . 4 <i>s</i> . 5 <i>d</i> . = 34,22 <i>£</i> .	1,53426

Ex. IV. If $\frac{3}{4}$ of a yard of cloth cost $\frac{3}{4}$ of a guinea: How many ells English for 3*£*. 10*s*.?

As $\frac{3}{4}$ Guin. = 14 <i>s</i> .	Ar. Co.	8,85387
To 3 <i>£</i> . 10 <i>s</i> = 70 <i>s</i> .		1,84510
So $\frac{3}{4}$ ell = 0,6		1,77815
To 3 ells.		0,47712

Ex. V. What number will have the same proportion to 0,8538 as 0,3275 has to 0,0131?

As 0,0131 its Ar. Co. Log.	11,88273
To 0,3275	1,51521
So 0,8538	1,93136
To 21,35	1,32930

Ex. VI. How many yards of shalloon of $\frac{3}{4}$ ell wide will be enough to line a coat containing $3\frac{1}{2}$ ells of $1\frac{1}{4}$ yards wide?

As $\frac{3}{4} \times \frac{5}{4}$ yd. w. = 0,9375		10,02803
To $1\frac{1}{4}$ yd. w. = 1,75		0,24304
So $3\frac{1}{2} \times \frac{5}{4}$ yd. l. = 4,375		0,64098
To 8 $\frac{1}{2}$ yd. long = 8,167		0,91295

where w. stands for wide, l. for long.

* Ar. Co. Log. stands for the Arithmetical Complement of the Logarithm.

OF POWERS AND THEIR ROOTS.

89. *A number being given, to find any proposed power of that Number.*

RULE 1st. Seek the Logarithm of the given number.

2^d. Multiply this Logarithm by the Index of the proposed power.

3^d. Find the number corresponding to the Product, and it will be the power required.

90. In multiplying a Logarithm having a negative Index, the Product of that Index is negative.

But the carriage from the Logarithm is affirmative.

Therefore the difference will be the Index of the Product.

And is to be of the same kind with the greater, or that which was made the minuend.

Ex. I. *What is the second power of the number 3,874?*

To 3,874 its Log. is 0,58816
The Index is 2

The power sought is 15,01 1,17632

Ex. II. *What is the 3^d power of the number 2,768?*

The N^o 2,768 its Log. is 0,44217
The Index is 3

The power sought is 21,21 1,32651

Ex. III. *What is the 12th power of the number 1,539?*

1,539 its Log. is 0,18724
The Index is 12

The power sought is 176,6 2,24688

Ex. IV. *What is the 365th power of the number 2?*

2 Its Log. is 0,30103
The Index is 365

150515
180618
90309
109,87595

In the IVth Ex. the Index of the Product being 109, shews that the required power will consist of 110 integer places; of which no more than 4 places are found in these tables; therefore the number sought may be thus expressed, 7515 [106]: That is, 7515 with 106 cyphers annexed.

Ex. V. *What is the 2^d power of the number 0,2857?*

To 0,2857, its Log. is 1,45591
The Index of 2^d power 2

The power 0,08162 2,91182

Ex. VI. *What is the 3^d power of the number 0,7916?*

To 0,7916, its Log. is 1,89851
The Index of 3^d power 3

The power is 0,4961 1,69553

Here, there being no carriage from the Product of the Log. the whole Product of the negative Index is negative, viz. 2.

Here the carriage from the Product of the Log. is 2; then the Product of the negative Index viz. 3, being lessened by 2, leaves 1, the Index of the Product.

91. *A number being given, to find any proposed Root of it.*

RULE 1st. Seek the Logarithm of the given number.

2^d. Divide this Logarithm by the denominator of the Index of the proposed root.

3^d. The number corresponding to the Quotient will be the root.

When the Index to the Logarithm to be divided is negative, and less than the Divisor, or Denominator of the root. Then

Increase the negative Index by as many units, borrowed, as shall be equal to the Divisor, and the Quotient will give $\bar{1}$ for the Index.

Carry the units borrowed as tens to the left-hand place of the Logarithm, and then divide that Logarithm as in whole numbers.

Ex. I. *What is the Square Root of the number 1501?*

$$\begin{array}{r} 2)3,17638 \\ \underline{1,58819} \\ \text{Root sought is } 38,74 \end{array}$$

Ex. II. *What is the Cube Root of the number 2121?*

$$\begin{array}{r} 3)3,32654 \\ \underline{1,10885} \end{array}$$

Ex. III. *What is the Root, of which 176,6 is the 12th power?*

$$\begin{array}{r} 12)2,24699 \\ \underline{0,18725} \\ \text{Root sought is } 1,539 \end{array}$$

Ex. IV. *What is the Root, of which 2 is the 365th power?*

$$\begin{array}{r} 365)0,30103 \\ \underline{0,00082} \end{array}$$

Ex. V. *What is the Square Root of the number 0,08162?*

$$\begin{array}{r} \text{Log. of } 0,08162, \text{ div. by } 2)2,91180 \\ \underline{1,45590} \\ \text{Gives Root } 0,2857 \text{ to} \end{array}$$

Ex. VI. *What is the Cube Root of the number 0,496?*

$$\begin{array}{r} \text{Log. of } 0,496 \text{ div. by } 3)1,69548 \\ \underline{1,89849} \\ \text{Gives Root } 0,7916 \text{ to} \end{array}$$

Here the Divisor 2 can be taken in the Index $\bar{2}$, and gives for the Quotient $\bar{1}$.

Here the Divisor 3 cannot be taken in the Index $\bar{1}$; then $\bar{2}$ borrowed makes with $\bar{1}$, $\bar{3}$; in which the Divisor 3 will go $\bar{1}$: the 2 borrowed carried to the 6, &c. makes 269548, which divided by 3, gives 89849.



THE
ELEMENTS
OF
NAVIGATION.

BOOK II.
OF GEOMETRY.

SECTION I.

Definitions and Principles.

1. **G**EOMETRY is a science which treats of the descriptions, properties, and relations of magnitudes in general: That is of things in which length, or length and breadth, or length, breadth, and thickness, are considered.

2. A POINT is that which has neither parts nor dimensions.

3. A LINE is length without breadth: It is called a **RIGHT LINE** when it is the shortest distance between two points, as AB: Or a **CURVED LINE** when it is not the shortest distance, as CD.

A line is usually denoted by two letters; viz. one at each end, as AB or CD.

4. A **SUPERFICIES** or **SURFACE** is that magnitude which has only length and breadth, and is bounded by lines: as FG.

5. A **SOLID** is that magnitude which has length, breadth, and thickness.

6. A **FIGURE** is a bounded space, the limits or bounds of which may be either lines or surfaces.

7. A **PLANE**, or a **PLANE FIGURE**, is a superficies which lies evenly, or perfectly flat, between its limits, and may be bounded by one curve line; but not with less than three right lines, as A, B, C, D, or E, &c.

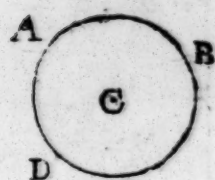
A ————— B

C ————— D

F
G

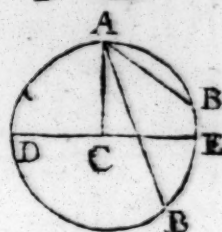
A B
C
D E

8. A **CIRCLE** is a plain figure, bounded by an uniformly curved line, called the **Circumference**, as ABD , which is every where equally distant from one point, as C within the figure called the **CENTER**.



9. A **RADIUS** is a right line drawn from the center to the circumference as CA , CD , or CE .

All the radii of the same circle are equal.



10. An **ARC** is any part of the circumference.

As the arc AB , or the arc AD .

11. A **CHORD** is a right line joining the ends of an arc, as AB , and is said to subtend that arc; it divides the circle into two parts, called **SEGMENTS**.

12. A **DIAMETER** is a chord passing through the center, as DE , and divides the circle into two equal parts, called **SEMICIRCLES**.

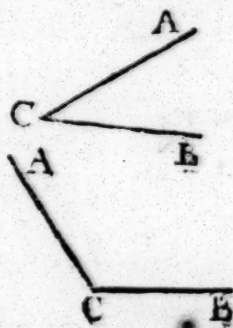
13. The **CIRCUMFERENCE** of every circle is supposed to be divided into 360 equal parts, called **DEGREES**; each degree into 60 equal parts, called **MINUTES**; each minute into 60 equal parts, called **SECONDS**, &c.

14. A **PLANE ANGLE**, is the inclination of two lines on the same plane meeting in a point, as ACB .

A right lined angle is formed by two right lines. The point where the lines meet is called the *angular point*, as C .

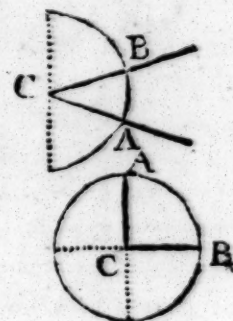
The lines which form the angle are called *legs*.

Thus CA and CB are the legs of the angle ACB .



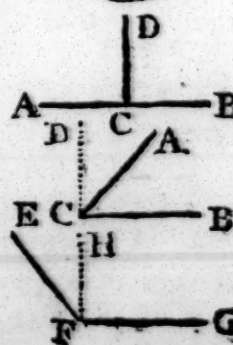
An Angle is usually marked by three letters, viz. one at the angular point, and one at the other end of each leg; but that at the angular point is always to be read the middle letter, as ACB , or BCA .

15. The measure of a right lined angle is an arc, as BA contained between the legs CB , CA , including the angle, the angular point C being the center of that arc.



16. A **RIGHT ANGLE** is that, the measure of which is a fourth part of the circumference of a circle, or ninety degrees. *Thus the angle ACB is a right angle.*

17. A **PERPENDICULAR** is that right line which cuts another at right angles; or which makes equal angles on both sides. *Thus DC is perpendicular to AB , when the angles DCA and DCB are equal, or are right angles.*



18. An **ACUTE ANGLE**, as ACB , is that which is less than a right angle DCB .

19. An **OBTUSE ANGLE**, as EFG , is that which is greater than a right angle HFG .

Acute and obtuse angles are called **OBLIQUE ANGLES**.

20. PARALLEL LINES, are right lines in the same plane, which do not incline to one another, as AB, CD.

21. A TRIANGLE is a plane figure bounded by three lines.

22. An EQUILATERAL TRIANGLE is that in which the three lines, or sides, are equal, as A.

23. An ISOSCELES TRIANGLE is that which has only two equal sides, as B or C.

24. A RIGHT ANGLED TRIANGLE, as ABC, is that which has one right angle B.

25. An OBTUSE ANGLED TRIANGLE, as DEF, has one obtuse angle E.

26. An ACUTE ANGLED TRIANGLE, as G, has all its angles acute.

27. A QUADRANGLE, or QUADRILATERAL, is a plane figure bounded by four right lines, or sides.

A Quadrangle is usually expressed by letters at the opposite angles.

28. A PARALLELOGRAM is a quadrangle, the opposite sides of which are parallel and equal, as P.

29. A RECTANGLE is a parallelogram with right angles: and in which the length is greater than its breadth, as R.

30. A SQUARE is a parallelogram having four equal sides and right angles, as S.

31. A TRAPEZIUM is a quadrangle the opposite sides of which are not parallel, as T.

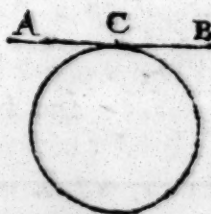
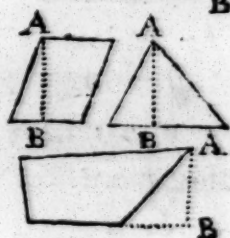
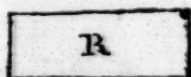
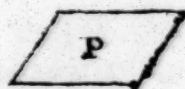
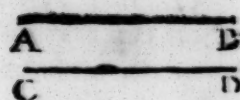
32. The DIAGONAL of a quadrangle, is a line, as AB, drawn from one angle, to its opposite angle.

33. The BASE of a figure, is the line it is supposed to stand on.

34. The ALTITUDE or HEIGHT of a figure, is the perpendicular distance AB, between the base and the vertex, or part most remote from the base.

35. CONGRUOUS FIGURES, are those which agree, or correspond, with one another, in every respect.

36. A TANGENT to a circle is a right line, as AB, touching its circumference, but not cutting; and the point C, where it touches, is called the point of contact.



37. An **ANGLE**, BAC , in a *Segment*, $CADB$, is, when the angular point is in the circumference of the segment, and the legs including the angle pass through the ends B and C , of the chord of the segment.

Such an angle is said to be in a circumference; and to stand on the arc BC , included between the legs, AB and AC , of the angle.



38. Right lined figures, having more than four sides, are called *Polygons*; and have their names from the number of their angles, or sides; as those of five sides are called *Pentagons*; of six sides, *Hexagons*; of seven sides, *Heptagons*; of eight sides, *Octagons*, &c.

39. A regular Polygon is a figure with equal sides and equal angles.

40. A figure is said to be inscribed in a circle, when all the angles of that figure are in the circumference of the circle.

41. A figure is said to circumscribe a circle, when every side of the figure is touched by the circumference of the circle.

42. A Proposition is something proposed to be considered; and requires either a solution or answer, or that something be made out, or proved.

A Problem is a practical proposition, in which something is proposed to be done, or effected.

A Theorem is a speculative proposition, or rule, in which something is affirmed to be true.

A Corollary is some conclusion gained from a preceding proposition.

A Scholium is a remark on some proposition; or an exemplification of the matter which it contains.

An Axiom is a self-evident truth, or principle, that every one assents to upon hearing it proposed.

A Postulate is a principle, or condition, requested; the simplicity or reasonableness of which cannot be denied.

In Mathematics, the following Postulates and Axioms, are some of the principal ones that are generally taken for granted.

When a proposition, from supposed premises, asserts such and such consequences; and subjoins, *And the contrary*: it is to be understood, that if the consequences be assumed as premises; then what were first taken as premises, would become consequences.

Thus, in Article 95, it is premised, *that if two parallel right lines are cut by another right line, there results this consequence; The alternate angles are equal.* And the contrary means; that *where equal alternate angles are made by a right line cutting two other right lines; the right lines so cut, are parallel lines.*

Postulates,

Postulates.

43. I. That a right line may be drawn from any given point to another given point.

44. II. That a given right line may be continued, or lengthened at pleasure.

45. III. That from a given point, and with any radius, a circle may be described.

Axioms.

46. I. Things equal to the same thing, are equal to one another.

47. II. If equal things are added to equal things, the sums or wholes will be equal. But if unequals be added, the sums are unequal.

48. III. If equal things are taken from equal things, the remainders, or differences, are equal : but are unequal, when unequals are taken.

49. IV. Things are equal which are double, triple, quadruple, &c. or half, third part, &c. of one and the same thing, or of equal things.

50. V. Things which have equal measures, are equal. And the contrary.

51. VI. Equal circles have equal radii.

52. VII. Equal arcs in equal circles have equal chords, and are the measures of equal angles. And the contrary.

53. VIII. Parallel right lines have each the same inclination to a right line cutting them.

In what follows, it is to be understood, that right lines (*viz.* straight lines) are drawn by the edge of a straight ruler: circles or arcs, are described with one foot of a pair of compasses, the other foot resting on the point which is taken for the center; and the distance of the feet, or points, of the compasses is taken as the radius: also, that the point marked out by a letter is to be understood, when the reference is made to that letter.

54. It is also taken for granted, that a line or distance can be taken between the compasses, and may be transferred or applied from one place to another. Also, that one figure can be applied to, or laid upon another, or conceived to be so applied.

In any problem, when a line, angle, or figure is said to be given; that line, angle, or figure must be made, before any part of the operation is performed.

SECTION II.

Geometrical Problems.

55-

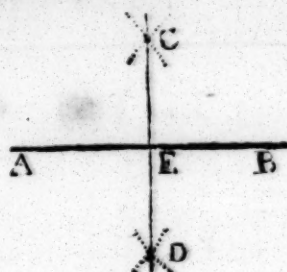
PROBLEM I.

To bisect, or divide into two equal parts, a given line AB.

OPERATION. 1st. From the ends A and B *, with one and the same radius, greater than half AB, describe arcs cutting in c and D. (45)

2d. A line drawn from c to D, gives E, the middle of AB, as required.

The proof of this operation depends on articles 101, 99.



56.

PROBLEM II.

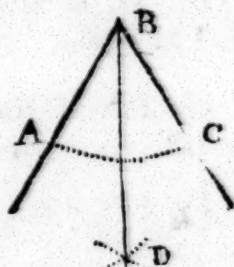
To bisect a given right lined angle ABC.

OPERATION. 1st. From B, describe an arc AC.

2d. From A and c *, with one and the same radius, describe arcs cutting in D. (45)

3d. A right line drawn from B to D will divide the angle into two equal parts, as required.

The proof depends on article 101.



57.

PROBLEM III.

From a given point B, in a given right line AF, to draw a right line perpendicular to the given line.

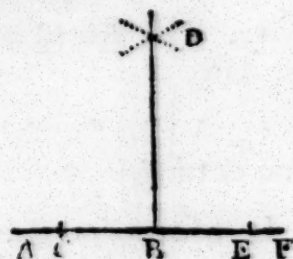
CASE I. *When B is near the middle of the line.*

OPERATION. 1st. On each side of B, take the equal distances BC, and BE. (54)

2d. On c and E * describe, with the same radius, arcs cutting in D. (45)

3d. A right line drawn through B and D will be the perpendicular required. (43)

The proof depends on article 103.



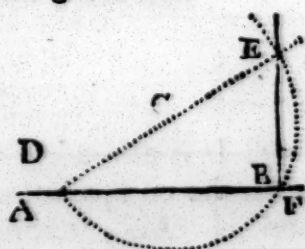
58. CASE II. *When B is at, or near the end of the given line.*

OPERATION. 1st. On any convenient point c, taken at pleasure, with the distance, or radius CB, describe an arc DBE, cutting AF in D and B. (45)

2d. A line drawn through D and c will cut this arc in E.

3d. A right line drawn through B and E will be the perpendicular required.

This depends on article 130.



* That is, first describe an arc from one point; then describe an arc from the other point with the same opening of the compasses.

59.

PROBLEM IV.

To draw a line perpendicular to a given right line AB, from a point C without that line.

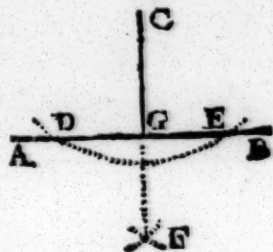
CASE I. When the point C is nearly opposite to the middle of the given line.

OPERATION. 1st. On C, with one radius, cut AB in D and E. (45)

2d. On D and E, with one radius, describes arcs cutting in F. (45)

A line drawn from C to F will be the perpendicular required.

This depends on articles 101, 99.



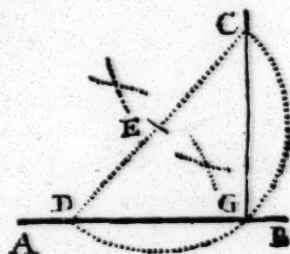
60. CASE II. When C is nearly opposite to one end of the given line AB.

OPERATION. 1st. To any point D in AB, draw the line CD. (43)

2d. Bisect the line CD in E. (55)

3d. On E, with the radius EC, cut AB in G. Then CG being drawn, will be the perpendicular required.

This depends on article 130.



61.

PROBLEM V.

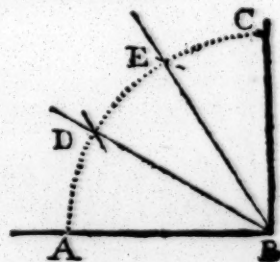
To trisect, or divide into three equal parts, a right angle ABC.

OPERATION. 1st. From B, with any radius BA, describe the arc AC, cutting the legs BA, BC, in A, C.

2d. From A, with the radius AB, cut the arc AC in E, and from C, with the same radius, cut AC in D.

3d. Draw BE, BD, and the angle ABC will be divided into three equal parts.

This depends on article 193.



62.

PROBLEM VI.

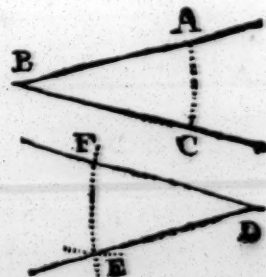
At a given point D, to make a right lined angle equal to a given right lined angle ABC.

OPERATION. 1st. From D and B, with the same radius describe the arcs EF, and AC, cutting the legs of the given angle in the points A, C.

2d. Transfer the distance AC to the arc EF, from F to E. (54)

3d. Lines drawn from D, through E and F, will form the angle EDF equal to the angle ABC.

This depends on article 101.



63.

PROBLEM VII.

To draw a line parallel to a given right line AB.

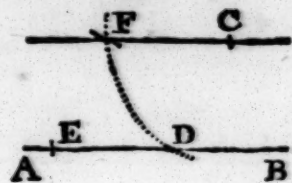
CASE I. When the parallel line is to pass through a given point, c.

OPERATION. 1st. From c, with any convenient radius, describe an arc DF, cutting AB in D.

2d. Apply the radius CD from D to E; and from E, with the same radius, cut the arc DF in F.

3d. A line drawn through F and c will be parallel to AB.

This depends on article 101, 95.



64. CASE II. When the parallel line is to be at the given distance c from AB.

OPERATION. 1st. From the points A and B, with the radius c, describes arcs D and E.

2d. Lay a ruler to touch the arcs D and E, and a line drawn in that position is the parallel required.

This operation is mechanical.



65.

PROBLEM VIII.

Upon a given line AB, to make an equilateral triangle.

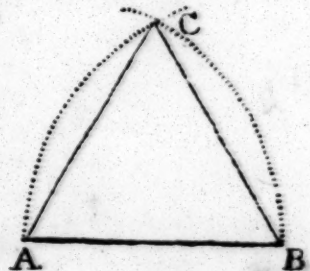
OPERATION. 1st. From the points A and B, with the radius AB, describe arcs cutting in c.

(45)

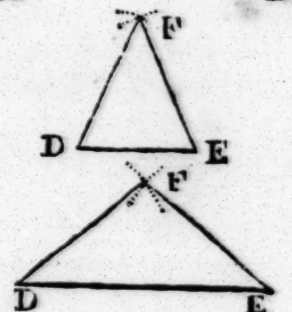
2d. Draw CA, CB, and the figure ABC is the triangle required.

(43.)

The truth of this operation is evident; for the sides are radii of equal circles.



66. By a like operation, an Isosceles triangle DEF may be constructed on a given base DE, with the given equal legs DF, EF, either greater, or less, than the base DE.



67.

PROBLEM IX.

To make a right lined triangle, the sides of which shall be respectively equal, either to those of a given triangle ABC, or to three given lines, provided any two of them taken together are greater than the third.

OPERATION. 1st. Draw a line DE equal to the line AB.

(54)

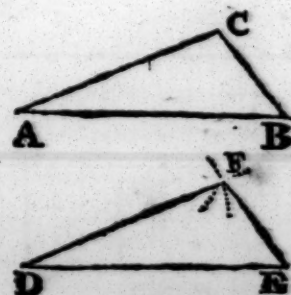
2d. On D, with a radius equal to AC, describe an arc F.

(45)

3d. On E, with a radius equal to BC, describe another arc, cutting the former arc in F.

(45)

4th. Draw FD, FE, and the triangle DFE will be that required.



68. PROB.

68.

PROBLEM X.

Upon a given line AB, to describe a square.

OPERATION. 1st. Draw BC perpendicular, and equal to AB. (58)

2d. On A and C, with the radius AB, describe arcs cutting in D. (45)

3d. Draw DC, DA; and the figure ABCD is the square required.

This depends on articles 101, 104, 95, 30.

69.

PROBLEM XI.

To describe a rectangle whose length shall be equal to a given line EF, and breadth equal to another given line G.

OPERATION. 1st. At E and F erect two perpendiculars, EH and FI, each equal to the given line G.

2d. Draw HI, and the figure EFIH will be the rectangle required.

This depends on articles 29, 101, 104, 95.

70.

PROBLEM XII.

To find the center of a circle.

OPERATION. 1st. Draw any chord AB.

2d. Bisect AB with the chord CD. (55)

3d. Bisect CD with the chord EF, and their intersection G will be the center required.

This depends on article 125.

71.

PROBLEM XIII.

To divide the circumference of a circle into two, four, eight, sixteen, thirty-two, &c. equal parts.

OPERATION. 1st. A diameter AB divides the circle into two equal parts. (12)

2d. A diameter DE, perpendicular to AB, divides the circumference into four equal parts.

3d. On A, D, B, describe arcs cutting in a, b; then by the intersections a, b, and the center, the diameters FG, HI, being drawn, divide the circumference into eight equal parts; and so on by continual bisection.

For at each operation, the intercepted arcs are bisected, and the parts doubled.

72.

PROBLEM XIV.

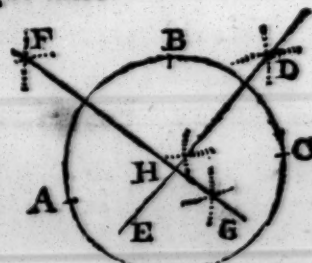
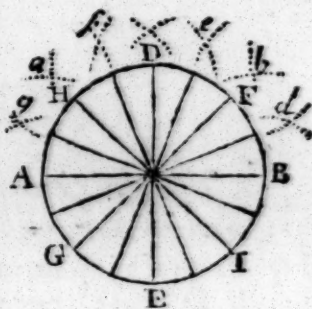
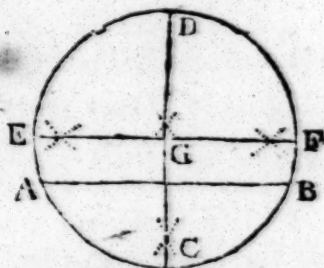
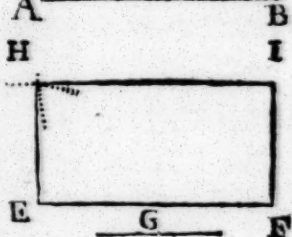
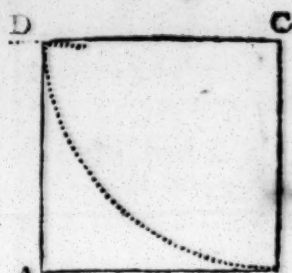
To describe a circle, the circumference of which shall pass through three given points A, B, C, provided they do not lie in one right line.

OPERATION. 1st. Bisect the distance CB with the line DE. (55)

2d. Bisect the distance AB with the line FG.

3d. On H, the intersection of these lines, with the distance to either of the given points, describe the circle required.

This depends on article 125.



73.

PROBLEM XV.

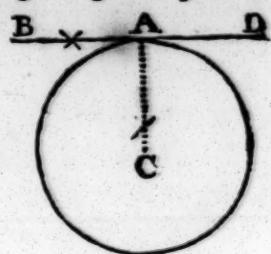
To draw a tangent to a given circle, that shall pass through a given point A.

CASE I. When A is in the circumference of the circle.

OPERATION. 1st. From the center c, draw the radius CA. (43)

2d. Through A draw BD perpendicular to CA (58), and BD is the tangent required.

This depends on article 126.



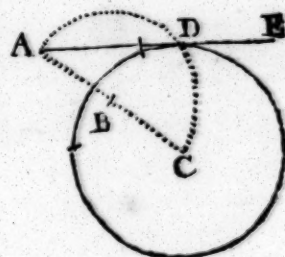
74. CASE II. When the given point A is without the given circle.

OPERATION. 1st. From the center c, draw CA, which bisect in B. (55)

2d. On B, with the radius BA, cut the given circumference in D.

3d. Through D, the line AE being drawn, will be the tangent required.

This depends on articles 130, 126.



75.

PROBLEM XVI.

To two given right lines A, B, to find a third proportional.

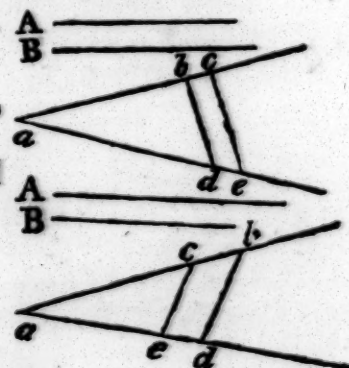
OPERATION. 1st. Draw two right lines making any angle, and meeting in a.

2d. In these lines, take ab = first term, and ac , ad , each equal to the second term.

3d. Draw bd , and through c , draw ce parallel to bd ; then ae is the third proportional sought.

And $ab : ac :: ad : ae$ *.

This depends on article 165.



76.

PROBLEM XVII.

To three given right lines A, B, C, to find a fourth proportional.

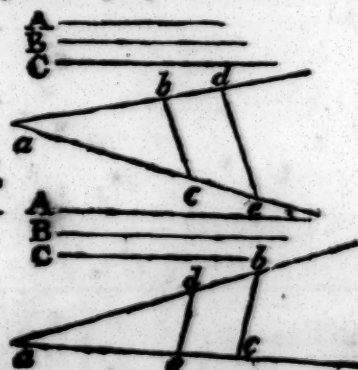
OPERATION. 1st. Draw two right lines making any angle, and meeting in a.

2d. In these lines take ab = first term, ac = second term, and ad = third term.

3d. Draw bc , and parallel to it, through d , draw de ; then ae is the fourth proportional required.

And $ab : ac :: ad : ae$.

This depends on article 165.



* And is thus read: ab is to ac , as ad is to ae .

The character (:) standing for is to, and the character (::) for as.

77. PROB

77.

PROBLEM XVIII.

Between two given right lines A, B, to find a mean proportional.

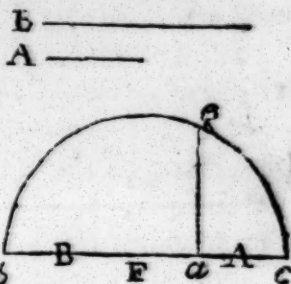
OPERATION. 1st. Draw a right line, in which take $ac=A$, $ab=B$.

2. Bisect bc in F (55); and on F, with the radius Fb , describe a semicircle bec .

3d. From a draw ae perpendicular to bc (57); then ae is the mean proportional required.

And $ac : ae :: ae : ab$.

This depends on article 171.



78.

PROBLEM XIX.

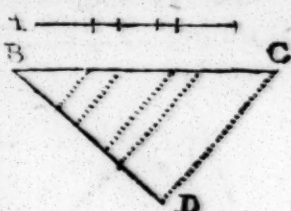
To divide a given line BC in the same proportion as a given line A is divided.

OPERATION. 1st. From one end B of BC draw BD, making any angle with BC.

2d. In BD apply from B the several divisions of A; so BD will be equal to A, and alike divided.

3d. Draw CD; then lines drawn parallel to CD through the several divisions of BD, will divide the line BC in the manner required.

This depends on article 165.



79.

PROBLEM XX.

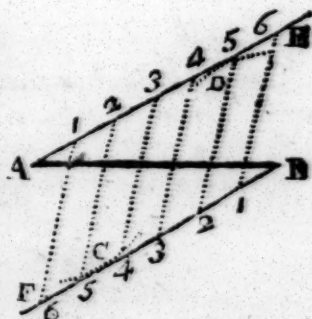
To divide a given right line AB into a proposed number of equal parts (suppose 7).

OPERATION. 1st. From A, one end of AB, draw AE to make any angle with AB; and from B, the other end, draw BF, making the angle ABF equal to the angle BAE. (62)

2d. Begin at A and B, and set off, on these lines, as many equal parts, except one, as AB is to be divided into, viz. 1, 2, 3, 4, 5, 6.

3d. Lines drawn from 1 to 6, 2 to 5, 3 to 4, &c. will divide AB as was required.

This depends on article 165.



80.

Another Method.

OPERATION. 1st. Through one end A, draw a line CC nearly perpendicular to AB.

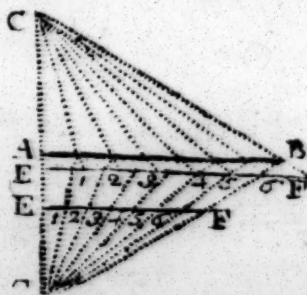
2d. Draw EF parallel to AB, at any convenient distance.

3d. In EF take, of any length, as many equal parts as AB is to be divided into; as 1, 2, 3, 4, 5, 6, to F.

4th. Through B and F, where the divisions terminate, draw BC, meeting CC in c.

5th. Lines drawn from c through the several divisions of EF, will cut AB into the equal parts required.

Note, If the sum of the divisions from E to F chance to be less than AB, the point c, and the line EF, will be on the same side of AB; but if greater, c falls on the contrary side.



P R O B L E M. XXI.

To make Scales of equal parts.

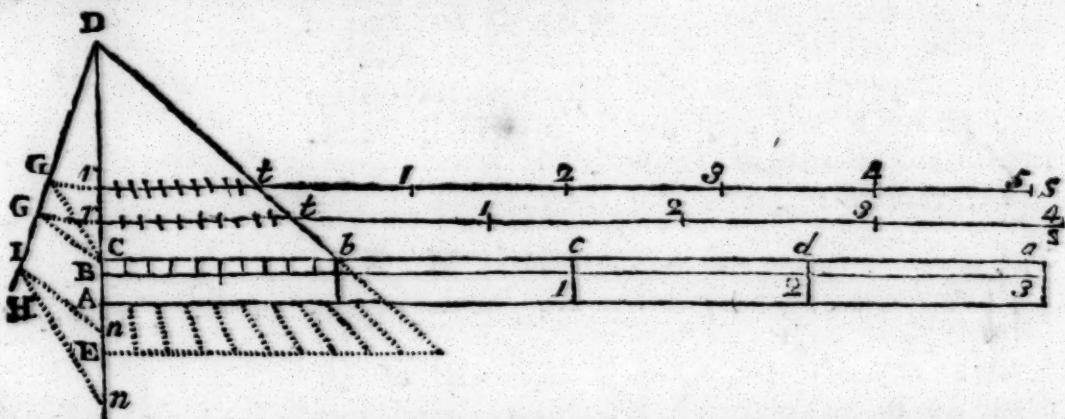
OPERATION. 1st. Draw three lines, A, B, C , parallel to one another, and at convenient distances, such as are here expressed.

2d. In the line c , take the equal parts, cb, bc, cd, da , &c. each equal to some proposed length.

3d. Through c , draw the line DE perpendicular to ca ; and parallel to DE , through the several points b, c, d, a , &c. draw lines across the three parallels A, B, C ; then are the spaces or distances cb, bc, cd , &c. called the primary divisions.

4th. Divide the left-hand space into 10 equal parts (80), and through these points let lines, parallel to DE , be drawn across the parallels BC ; and the primary division cb will be parted into 10 equal spaces, called subdivisions.

5th. Number the primary divisions from the left toward the right beginning at c , and the scale is constructed.



Such Scales are useful for constructing figures, the sides of which are expressed in measures of length; as feet, yards, miles, &c. Or to find the measures of the length of lines in a given figure.

Thus a line of 36, or 360, feet, yards, &c. is taken by setting one point of the compasses on the third primary division, and extending the other point to the sixth subdivision: and so of others.

In these Scales it is usually supposed, that an inch is divided into some number of such parts, as are expressed by the subdivisions.

82. *To find the divisions of a Scale of equal parts, when any given number of them are to make an inch.*

OPERATION. 1st. Make a Scale ca , where the primary divisions shall be each one inch; let DC be at right angles to ca , and the left-hand space cb be divided into 10 equal parts, and draw Db from any point D .

2d. Draw DH , making with DC any angle; and make $Dr = cb$.

3d. Take the number of parts proposed in an inch, from the Scale ca , and apply them from D to n , in the line DC , continued if necessary.

4th. Draw nI , and CG parallel to nI ; and make $Dr = DG$.

5th. Through r draw rs parallel to ca , cutting Db in t ; then rt will be one of the primary divisions, containing 10 of the parts the inch was to be divided into.

6th. Lines drawn from D to the divisions of cb , divide rt into 10 equal parts.

83.

P R O B L E M XXII.

To divide the circumference of a circle into degrees; and thence to make a Scale of chords.

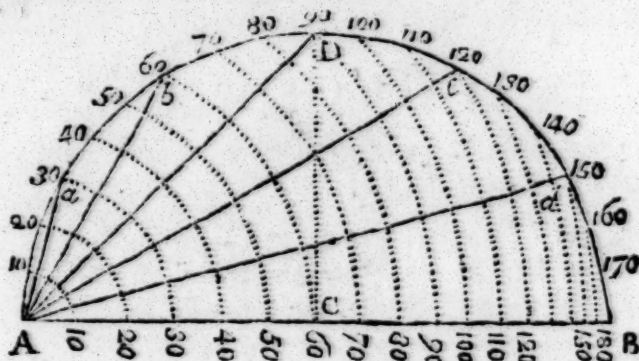
OPERATION. 1st. Describe the semicircle ABD, the center of which is C, and draw CD at right angles to AB.

2d. Trisect the angles ACD, DCB, in the points $a, b; c, d$ (61): then by trials, divide the arcs Aa, ab, bD, Dc, cd, dB , each into three equal parts, and the semicircumference will be divided into 18 equal parts, of 10 degrees each.

3d. These arcs being bisected, will give arcs of 5 degrees:

4th. And these arcs being divided into 5 equal parts, by trials, will give arcs of 1 degree each.

5th. The chords of the arcs Ad, Ac, AD, Ab, Aa , and of every other arc, being applied from A on the diameter AB, will divide AB into a Scale of chords; which are to be numbered from A towards B.



A Scale of chords is useful for constructing an angle of a given number of degrees: And for finding the measure of a given angle.

84.

P R O B L E M XXIII.

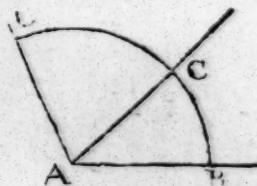
To make an angle of a proposed number of degrees.

OPERATION. 1st. Take the first 60 deg. from the Scale of chords, and with this radius describe an arc BC, the center of which is A.

2d. Take the chord of the proposed number of degrees from the Scale, reckoning from its beginning, and apply this distance to the arc BC, from B to c.

3d. Lines drawn from A, through the points B and c, will form an angle BAC, the measure of which is the degrees proposed.

If the angle exceed 90° , take half the number of degrees, and set it off twice, from B to c, and from c to c; or set off 90° first, and afterward the number of degrees which the angle exceeds 90° .



P R O B L E M XXIV.

To measure a given angle BAC.

From the angular point A, with the chord of 60 degrees, describe the arc BC, cutting the legs in the points B and c.

Then the distance BC being applied to the chords, from the beginning, will shew the degrees which measure the angle BAC.

85.

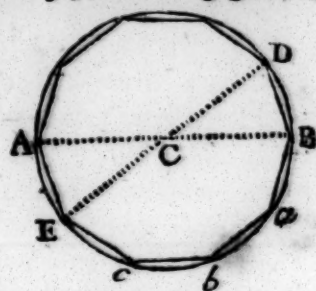
PROBLEM XXIV.

In a given circle to inscribe a regular polygon, the number of sides being given.

OPERATION. 1st. Divide 360 degrees by the number of sides, and the Quotient will be the degrees which measure the angle at the center of the circle, subtended by a side of the polygon *.

2d. Draw the radius CB, make an angle BCD equal to those degrees (84), and draw the chord DB; then will DB be the side of the polygon required; which applied to the circumference from B to a, a to b, b to c, &c. will give the points to which the sides of the polygon are to be drawn.

If the polygon has an even number of sides, draw the diameter AB; and divide half the circumference, as before; then lines drawn from these points through the center, as ED, will give the remaining points, in the other semicircumference.



86.

PROBLEM XXV.

On a given right line AB, to construct a regular polygon of any assigned number of sides.

OPERATION. 1st. Divide 360 degrees by the number of sides; subtract the Quotient from 180 degrees, the remainder will be the degrees which measure the angle made by any two adjoining sides of that polygon, and is called the angle of the polygon †.

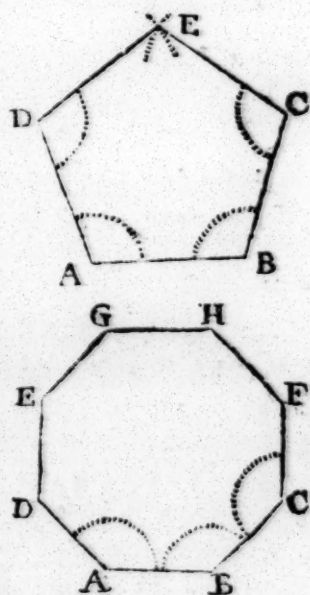
2d. At the ends A, B, of the line AB, make angles ABC, BAD, equal to the angle of the polygon.

3d. Make AD, BC, each equal to AB.

4th. At the points C, D, make angles equal to that of the polygon as before; and let the sides including those angles be each equal to AB; and thus proceed until the polygon is constructed.

In figures of any number of sides, the two last sides DE, CE; or EG, HG; are readiest found by describing arcs from C and D, or from E and H, with the radius AB, intersected in E, or in G.

In figures of an even number of sides, having drawn half the number, AD, AB, BC, CF, by means of the angles; the remaining sides may be found, by drawing through the points D and F the lines DE, FH, parallel and equal to their opposite sides CF, AD; and so of the rest.



* For there will be as many equal angles at the center as there are equal sides. And the whole circumference measures the sum of all the angles at the center.

† For each side of the polygon with radii drawn to its ends form an Isosceles triangle.

Then the angle of the polygon is derived from articles 85, 96, 104.

87. PROB.

87.

PROBLEM XXVI.

About a given regular polygon, to circumscribe a circle: or within that polygon to inscribe a circle.

OPERATION. 1st. Bisect any two angles FAB, CBA, with the lines AG, BG, (56) and the point G, where they intersect one another, will be the center of the polygon.

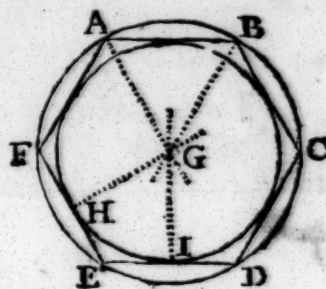
2d. A circle described from G, with the radius GA, will circumscribe the given polygon.

This depends on articles 85, 96, 104.

88. Again, 1st. Bisect any two sides FE, ED, in the points H and I (55); and draw HG, IG, at right angles to FE, ED (57); then the point G, where they intersect each other, will be the center of the polygon.

2d. A circle described from G, with the radius GH, will be inscribed in the given polygon.

This depends on article 126.



89.

PROBLEM XXVII.

On a given right line AB, to describe a segment of a circle, that shall contain an angle equal to a given right lined angle c.

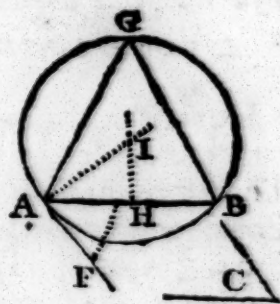
OPERATION. 1st. Make an angle BAF equal to the given angle c. (62)

2d. From H, the middle of AB, draw HI at right angles to AB (57), and from A draw AI at right angles to AF (58), cutting HI in I.

3d. From I, with the radius IA, describe a circle.

Then will the segment AGB contain an angle AGB equal to the given angle c.

This depends on articles 125, 126, 132.



90.

PROBLEM XXVIII.

To divide a right line in continued proportion, in the ratio of two given right lines AB, AC.

OPERATION. 1st. From B, with the radius AB (the antecedent) describe an arc Ac.

2d. In that arc apply (the consequent) AC from A to c; draw Ac, and apply cA from c to D.

3d. Apply the following lines in the order directed, viz. AD from A to d, and from d to E; AE from A to e, and from e to F; AF from A to f, and from f to G; AG from A to g, and from g to H, &c. Then will the proportional lines be

AB, AC, AD, AE, AF, AG, &c. And

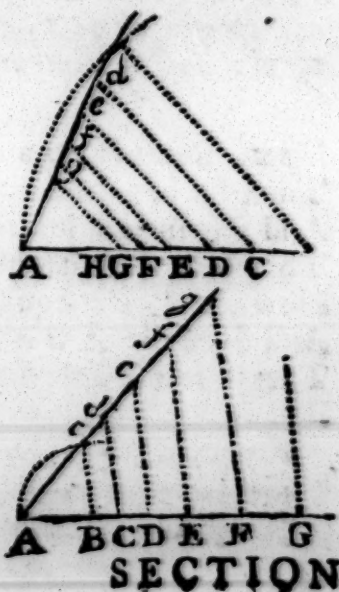
$AB : AC :: (Ac =) AC : (Ad =) AD.$

$AB : AC :: (Ad =) AD : (Ae =) AE.$

$AB : AC :: (Ae =) AE : (Af =) AF.$

&c.

This depends on articles 104, 95, 165.



SECTION III.

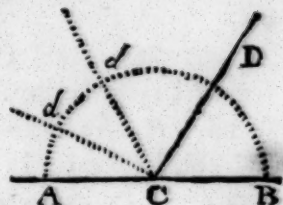
Geometrical Theorems.

OF RIGHT LINES AND PLANES.

91.

THEOREM I.

When one right line CD stands upon another right line AB , they make two angles BCD , ACD , which together are equal to two right angles.



DEMONSTRATION. For describing a semicircle ADB , on C . (45)

Then the arc DB measures the angle BCD . (15)

And the arc DA measures the angle ACD . (15)

But the arcs DB and AD together measure two right angles. (13, 16)

Therefore BCD and ACD together, are equal to two right angles (50)

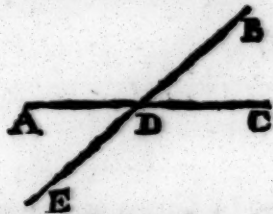
92. COROLLARY. Hence if any number of right lines cd stands on one point C , on the same side of another right line AB ; the sum of all the angles are equal to two right angles; or are measured by 180 degrees.

93.

THEOREM II.

If two right lines AC , EB intersect each other in D , the opposite angles are equal, viz.

* $\angle CDB = \angle ADE$, and $\angle CDE = \angle ADB$.



† DEM. For the angles ADE and ADB together make two right angles. (91)

And the angles CDB and ADB together make two right angles. (91)

Therefore the sum of ADE and $ADB =$ sum of CDB and ADB . (46)

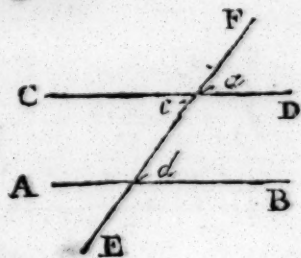
Consequently the $\angle CDB$ is equal to the $\angle ADE$. (48)

94. † COROL. Hence if any number of right lines cross each other in one point, the sum of all the angles which they make about that point, is equal to four right angles; or is measured by 360 degrees.

95.

THEOREM III.

If a right line FE cut two parallel right lines AB , CD ; then is the outward angle a || equal to the inward and opposite angle d ; and the alternate angles c , d , are equal: and the contrary.



DEM. Because CD and AB are parallel by supposition:

Then FE has the same inclination to CD and AB . (53)

And this inclination is expressed by the $\angle a$ or $\angle d$: (14)

Therefore the outward $\angle a$ is equal to the inward and opposite $\angle d$.

Now the $\angle a$ is equal to the $\angle c$; (93)

And since the $\angle a$ is equal to the $\angle d$;

Therefore the alternate angles c and d are equal. (46)

* The mark $\angle$ stands for the word angle.

† DEM. stands for Demonstration.

‡ COROL. stands for Corollary.

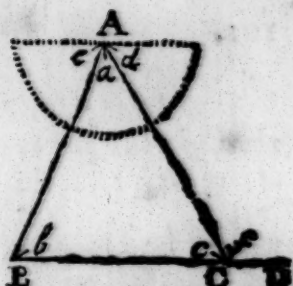
[|| Angles are sometimes marked by a single letter. Thus angle a is used for angle FAD .

96. THEO.

96.

THEOREM IV.

In any right lined triangle ABC, the sum of the three angles a, b, c , is equal to two right angles: And if one side BC be continued, the outward angle f is equal to the sum of the two inward and opposite angles a, b .



DEM. Through A, draw a right line parallel to BC, (63) making with AB the $\angle e$, and with AC the $\angle d$.

Now $\angle e = \angle b$; and $\angle d = \angle c$. being alternate. (95)

And two right angles measure the $\angle e + \angle a + \angle d$. (92)

Therefore $\angle b + \angle a + \angle c = \angle e + \angle a + \angle d$. (47)

Consequently $\angle b + \angle a + \angle c = \text{two right angles}$. (46)

Moreover $\angle c + \angle f = \text{two right angles}$. (91)

Therefore $\angle b + \angle a = \angle f$. (46)

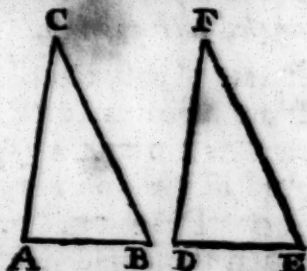
97. Hence, if one angle is right or obtuse, each of the other is acute.

98. If two angles of one triangle are equal to two angles of another triangle, the remaining angles are equal. And if one angle in one triangle is equal to one angle in another triangle, then is the sum of the remaining angles in one, equal to the sum of the remaining angles in the other.

99.

THEOREM V.

If two sides AB, AC, and the included angle A, in one triangle ABC, are respectively equal to two sides DE, DF, and the included angle D of another DEF, each to each; then are those triangles congruous.



DEM. Apply the point D to the point A, and the line DE to AB. (54)

Now as $DE = AB$ (by supposition); therefore the point E falls on B.

But $\angle D = \angle A$ (by sup.); therefore DF will fall on AC.

And since $DF = AC$ (by sup.); therefore the point F falls on C.

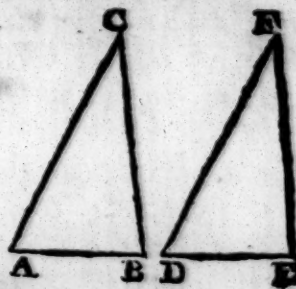
Consequently FE will fall on CB.

Therefore the triangles ACB, DFE, are congruous, since every part agrees.

100.

THEOREM VI.

If two triangles ABC, DEF, have two angles A, B, and the included side AB in one respectively equal to two angles D, E, and the included side DE in the other, each to each; then are those triangles congruous.



DEM. Apply the point D to A, and the line DE to AB.

Now as $DE = AB$ (by sup.); therefore the point E falls on B.

And as $\angle D = \angle A$ (by sup.); therefore the line DF falls on AC.

Now if the line AC is less or greater than the line DF;

Then the line FE not falling on CB, makes the $\angle B$ less or greater than $\angle E$.

But $\angle B = \angle E$ (by sup.); therefore AC is neither less nor greater than DF.

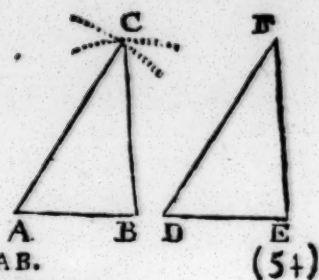
Or the line $AC = DF$; consequently $FE = CB$.

Therefore the triangles are congruous.

101. THEO-

101. THEOREM VII.

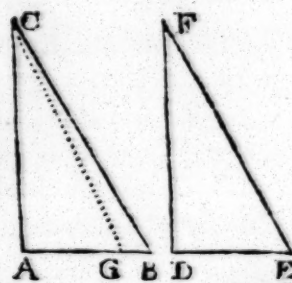
Two triangles ABC, DEF, are congruous, when the three sides in the one are equal to the three sides in the other, each to each.



DEM. Apply the point D to A, and the line DE to AB.
 Now as $DE = AB$ (by sup.); therefore the point E falls on B.
 On A, with the radius AC, describe an arc.
 Then as $DF = AC$, the point F will fall in that arc.
 Also on B, with the radius BC, describe another arc, cutting the former in c.
 And since $EF = BC$, the point F will fall in this arc also.
 But if the point F can fall in both these arcs, it can be only where they intersect, as in c.
 Consequently the triangles are congruous.

102. THEOREM VIII.

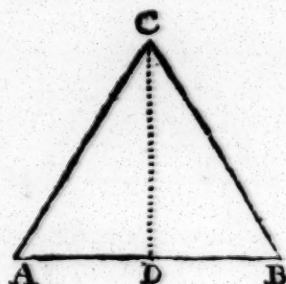
Two triangles ABC, DEF are congruous, when two angles A, B and a side AC opposite to one of them, in one triangle, are respectively equal to two angles D, E, and a side DF opposite to a like angle in the other triangle, each to each.



DEM. Apply the point D to A, and the line DF to AC.
 Now as $DF = AC$ (by sup.); therefore the point F falls on C.
 And as $\angle D = \angle A$ (by sup.); the line DE will fall on the line AB.
 And if the point E does not fall on B, it must fall on some other point G. Draw CG.
 Then the angle AGC is equal to the angle DEF. (99)
 And the angle $ABC = (\angle DEF =) \angle AGC$, which is not possible. (96)
 Therefore the point E can fall no where but on the point B.
 Consequently the triangles are congruous.

103. THEOREM IX.

In the Isosceles, or equilateral triangle ACB; a line drawn from the vertex C to the middle of the base AB is perpendicular to the base, and bisects the vertical angle: and the contrary.*



DEM. The triangles ADC, BDC, are congruous.
 Since $CA = CB$ (23); $CD = CD$, and $AD = DB$ by supposition:
 Therefore $\angle A = \angle B$, $\angle ACD = \angle BCD$, $\angle ADC = \angle BDC$. (101)
 Consequently CD is at right angles to AB. (17)

104. COROL. Hence in any right lined triangle where there are equal sides, or angles;

The angles A, B, opposite to equal sides BC, AC, are equal.
 And the sides BC, AC, opposite to equal angles A, B, are equal.

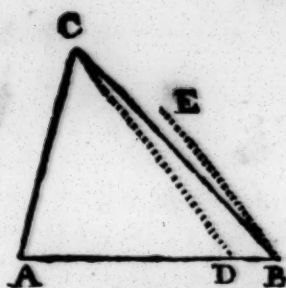
* That is, if a line drawn perpendicular to the base of a triangle bisects the vertical angle; then that triangle must be Isosceles, and the perpendicular is drawn from the middle of the base.

105. THEO-

105.

THEOREM X.

In every right lined triangle ABC, the greater angle C is opposite to the greater side AB.



DEM. In the greater side AB, take $AD = AC$; draw CD, and through B draw BE parallel to CD. (63)

Then the angles ADC, ACD, are equal. (104)

And $\angle ADC = \angle ABE$. (95)

Therefore $\angle ACD = \angle ABE$. (46)

That is, a part of the angle ACB is greater than the angle ABC.

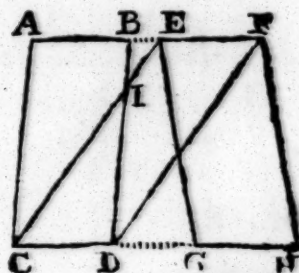
Consequently the $\angle C$ is greater than the $\angle B$; and in the same manner it may be proved to be greater than the $\angle A$, if the side AB be greater than CB.

106. COROL. Hence in every right lined triangle, the greater side is opposite to the greater angle.

107.

THEOREM XI.

Parallelograms ACDB, ECDF, EGHF, standing on the same base CD, or on equal bases CD, GH, and between the same parallels, CH, AF, are equal.



DEM. For $AB = EF$, being each equal to CD . (28, 46)

To each add BE , and $AE = BF$. (47)

Now $AC = BD$, and $CE = DF$. (28)

The triangles ACE, BDF, are therefore congruous. (101)

Now if from each of the triangles ACE and BDF, be taken the triangle BIE, the remaining trapeziums ABIC and FEID are equal. (48)

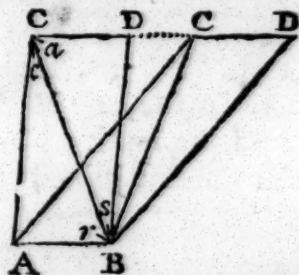
Then if to each of the trapeziums ABIC, FEID, be added the triangle CID, their sum will be the parallelograms AD and DE, which are equal. (47)

And in like manner it may be shewn, that the parallelogram EH is equal to the parallelogram $ED = AD$.

108.

THEOREM XII.

A triangle ABC is the half of a parallelogram AD, when they stand on the same base AB, and are between the same parallels AB, CD.



DEM. AC is equal to DB, and AB to DC. (28)

Also BC is a side common to both the triangles ABC and DCB.

These triangles are therefore congruous. (101)

Consequently the triangle ABC is half the parallelogram AD.

109. COROL. I. Hence every parallelogram is bisected by its diagonal.

110. COROL. II. Also, triangles standing on the same base, or on equal bases, and between the same parallels, are equal.

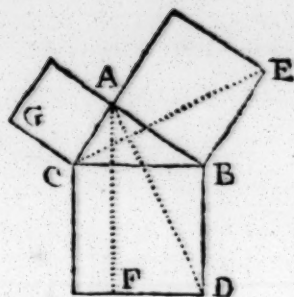
They being the halves of equal parallelograms under like circumstances.

III. THEO.

III.

THEOREM XIII.

In every right angled triangle BAC the square on the side BC opposite to the right angle A is equal to the sum of the squares of the two sides AB, AC containing the right angle,



DEM. On the sides AB, AC, BC, construct the squares AG, AE, CD (68): draw AD, CE; and draw AF parallel to BD. (63)
 Then the triangles ABD, EBC, are congruous. (99)
 For the $\angle ABE = \angle CBD$, being right angles. (30)
 To each add the angle ABC, then $\angle EBC$ and $\angle ABD$ are equal. (47)
 Therefore EB, BC, $\angle EBC$ are respectively equal to AB, BD, $\angle ABD$.
 Also the triangle EBC is half the parallelogram AE. (108)
 For they stand upon the same base EB, and are between the same parallels EB and AC; BA making right angles with BE and CA continued.
 Likewise the triangle ABD is half the parallelogram BF. (108)
 For they stand upon the same base BD, and are between the same parallels BD, AF.
 Therefore, as the halves of the parallelograms EA and BF are equal, consequently the parallelogram BF is equal to the square AE. (49)
 In the same manner may it be shewn, that the parallelogram CF is equal to the square AG.
 But the parallelograms BF and CF together, make the square CD.
 Therefore the square CD is equal to the squares EA and AG.

112. COROL. I. Hence if any two sides of a right angled triangle are known, the other side is also known.

For $BC = \text{square root of the sum of the squares of AC and AB.}$
 $AC = \text{square root of the difference of the squares of BC and AB.}$
 $AB = \text{square root of the difference of the squares of BC and AC.}$

113. Or thus, making the quantities $\overline{BC^2}$, $\overline{AB^2}$, $\overline{AC^2}$, to stand for the squares made on those lines.

And the mark $\sqrt{}$ to stand for the square root of such quantities as stand under the line joined to the top of this mark.

$$\begin{aligned} \text{Then } BC &= \sqrt{\overline{AC^2} + \overline{AB^2}}; \\ AC &= \sqrt{\overline{BC^2} - \overline{AB^2}}; \\ AB &= \sqrt{\overline{BC^2} - \overline{AC^2}}. \end{aligned}$$

Scholium. The lines of the lengths 5, 4, 3, (or their doubles, triples, &c.) will form a right angled triangle.

$$\text{For } 5^2 = 4^2 + 3^2. \text{ Or } 25 = 16 + 9.$$

114. COROL.

114. COROL. II. Of all the lines drawn from a given point to a given line, the perpendicular is the shortest.

115. COROL. III. The shortest distance between two parallel right lines, is a right line drawn from one to the other perpendicular to both.

116. COROL. IV. Parallel right lines are equidistant; and the contrary. For two opposite sides of a rectangular parallelogram are equal (28); and each is the shortest distance between the other sides.

THEOREM XIV.

117. If a right line AB be divided into any two parts AC, CB; then will the square on the whole line be equal to the sum of the squares on the parts, together with two rectangles under the two parts.

That is $\overline{AB}^2 = \overline{AC}^2 + \overline{CB}^2 + 2 \times AC \times CB$.*

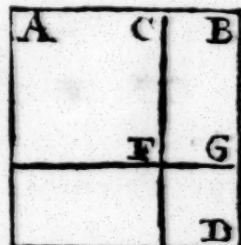
DEM. Let AD, AF, be squares on AB, AC. (68)

Then will FG, and GD, be each equal to CB. (48)

Hence FD is a square on a line equal to CB. (30)

Also FB and FE are rectangles on lines equal to AC, CB.

But the squares AF, FD, and the rectangles FB, FE, fill up the square AD, or are equal to AD.



118. COROL. I. Hence the square of AC the difference between two lines AB, CB, is equal to the square of the greater AB, lessened by the square of the less CB, and by two rectangles under the lesser line CB, and the said difference.

That is $\overline{AB} - \overline{CB}^2 = \overline{AB}^2 - \overline{CB}^2 - 2BC \times AC$.

For $\overline{AB} - \overline{CB} = \overline{AC}$.

Then $\overline{AC}^2 = \overline{AB}^2 - \overline{CB}^2 - 2BC \times AC$. (48)

119. COROL. II. The difference between the squares on two lines AB, AC, is equal to the rectangle under the sum $\overline{AB} + \overline{BC}$ and difference $\overline{AB} - \overline{BC}$ of those lines.

That is $\overline{AB}^2 - \overline{AC}^2 = \overline{AB} + \overline{BC} \times \overline{AB} - \overline{AC}$.

For $\overline{AB}^2 - \overline{AC}^2 = \overline{CB}^2 + 2\overline{AC} \times \overline{CB}$.

$= \overline{CB} + \overline{AC} + \overline{AC} \times \overline{CB}$.

$= \overline{AB} + \overline{AC} \times (\overline{CB} =) \overline{AB} - \overline{AC}$.

(117, 48)

* The rectangle under two lines is generally expressed by three letters; the first two letters stand for one line, and the last two for the other line.

Thus for $AC \times CB$, is written ACB.

for $AB \times BC$, is written ABC.

And for $2 \times AC \times CB$, is written 2ACB.

120.

THEOREM XV.

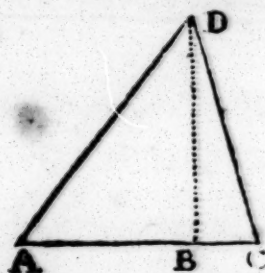
In every triangle, ADC the square of a side CD subtending an acute angle A, is equal to the squares of the sides AD, AC, about that angle, lessened by two rectangles under one of those sides AC, and that part AB contained between the acute angle and the perpendicular DB drawn to that side AC from its opposite angle D.

That is, $\overline{DC}^2 = \overline{AD}^2 + \overline{AC}^2 - 2CAB$

DEM. $\overline{DC}^2 - \overline{BC}^2 = (\overline{DB}^2 =) \overline{AD}^2 - \overline{AB}^2.$ (111)

And $\overline{AC}^2 = 2ABC + \overline{BC}^2 + \overline{AB}^2.$ (117)

Then $\overline{DC}^2 - \overline{BC}^2 - \overline{AC}^2 = \overline{AD}^2 - \overline{BC}^2 - 2\overline{AB}^2 - 2ABC.$ (48)



And $\overline{DC}^2 = \overline{AD}^2 + \overline{AC}^2 - 2\overline{AB}^2 - 2ABC,$ by adding $\overline{BC}^2 + \overline{AC}^2.$ (47)

$$\left(\begin{array}{l} -\overline{AB} \times 2\overline{AB} - \overline{BC} \times 2\overline{AB} \\ -\overline{AB} + \overline{BC} \times 2\overline{AB} \end{array} \right)$$

Then $\overline{DC}^2 = \overline{AD}^2 + \overline{AC}^2 - (\overline{AC} \times 2\overline{AB} =) 2CAB.$

121. COROL. Hence $AB = \frac{\overline{AD}^2 + \overline{AC}^2 - \overline{DC}^2}{2CA}.$

122.

THEOREM XVI.

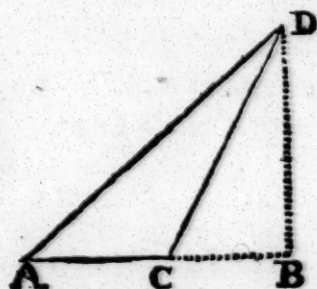
In an obtuse angled triangle ACD, the square of the side AD opposite to the obtuse angle C is equal to the sum of the squares of the sides AC, CD, about the obtuse angle; together with two rectangles under one side AC, and the continuation CB of that side to meet a perpendicular DB drawn to it from the opposite angle D.

That is, $\overline{AD}^2 = \overline{AC}^2 + \overline{CD}^2 + 2AC \times CB.$

DEM. For $\overline{AD}^2 - \overline{AB}^2 = (\overline{DB}^2 =) \overline{CD}^2 - \overline{CB}^2.$ (111)

And $\overline{AB}^2 = 2ACB + \overline{AC}^2 + \overline{CB}^2.$ (117)

Then $\overline{AD}^2 = \overline{AC}^2 + \overline{CD}^2 + 2ACB.$ (47)



123. COROL. Hence $CB = \frac{\overline{AD}^2 - \overline{AC}^2 - \overline{CD}^2}{2AC}.$

124. THEO.

124.

T H E O R E M XVII.

In any circle, a diameter, AB, drawn perpendicular to a chord, DE, bisects that chord and its subtended arc DBE.

DEM. From the center C, draw the radii CD, CE, to the extremities of the chord DE.

Then the triangles CFE, CFD, are congruous. (102)

For CF being at right angles to DE, the $\angle CFD = \angle CFE$.

And the triangle CDE being isosceles (23), the $\angle D = \angle E$ (104.) Also CF is common.

Therefore $DF = FE$: And the arc $DB = BE$:

For those arcs measure the equal angles FCE, FCD. (15)

125. COROL. Hence, in a circle, a right line drawn through the middle of a chord at right angles to it, passes through the center of that circle; and the contrary.

126.

T H E O R E M XVIII.

A tangent, AB, to a circle is perpendicular to a diameter, DC, drawn to the point of contact, C.

DEM. If it be denied that DC is perpendicular to AB, Then from the center D, let some other line DB, cutting the circle in E, be drawn perpendicular to AB.

Now the angle DEB being right, the angle DCB is acute.

Consequently DC is greater than DB.

But $DC = DE$ (9). Therefore DE is greater than DB, which is absurd.

Therefore no other line passing through the center can be perpendicular to the tangent, but that which meets it at the point of contact.

127.

T H E O R E M XIX.

An angle, BCD, at the center of a circle, is double of the angle, BAD, at the circumference, when those angles stand on the same arc, BD.

DEM. Through the point A draw the diameter AE.

Then the angle $ECD = \angle CAD + \angle CDA$. (96)

But the $\angle CAD = \angle CDA$. (104)

Therefore the $\angle ECD$ is equal to twice the angle CAD.

In the same manner it may be shewn, that the angle BCE is equal to twice the angle BAE.

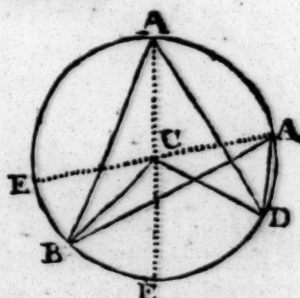
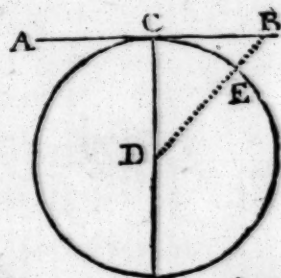
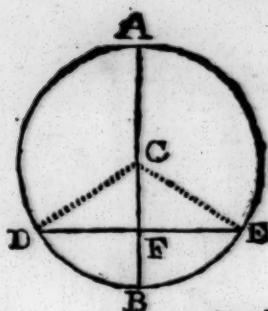
Consequently the angle BCD ($= \angle ECD + \angle BCE$) is equal to twice the angle BAD ($= \angle EAD + \angle BAE$). (47, 48)

128. COROL. I. Hence an angle, BAD, at the circumference, is measured by half the arc, BD, on which it stands.

For the angle at the center BCD is measured by the arc BD. (15)

Consequently the angle BAD = half the angle BCD, is measured by half the arc DB.

129. COROL. II. All angles in the circumference, and standing on the same arc, are equal.



* The mark $\pm$, signifies the sum or difference.

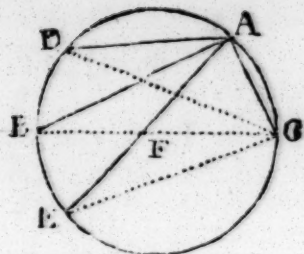
130.

THEOREM XX.

An angle, BAC, in a semicircle, is a right one.

An angle, DAC, in a segment less than a semicircle, is obtuse.

An angle, EAC, in a segment greater than a semicircle, is acute.



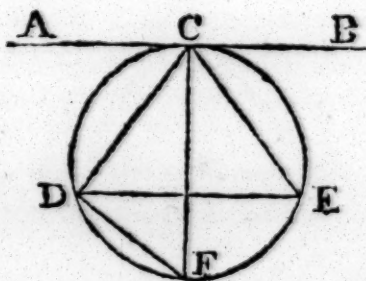
DEM. For the angle BAC is measured by half the semicircular arc BEC or is measured by half of 180 degrees; that is, by 90 degrees. (128, 16) And $\angle DAC$ is measured by half the arc DEC, greater than 180°*. Also $\angle EAC$ is measured by half the arc EC, less than 180°. Therefore these angles are respectively equal to, greater, or less than 90 degrees.

131. COR. Hence in a right angled triangle BAC; the angular point A of the right angle, and the ends, B, C, of the opposite side, are equally distant from F, the middle of that side; that is, a circle will always pass through the right angle, and the ends of its opposite side taken as a diameter.

132.

THEOREM XXI.

The angle ACD, formed by a tangent AB, to a circle, CDE, and a chord, CD, drawn from the point of contact, C; is equal to an angle, CED, in the alternate segment; and is measured by half the arc DC of the included segment.



DEM. Draw the diameter FC, and join DF.

The $\angle^s$ ACF and CDF are both right.

(126, 130)

Therefore the $\angle^s$ DCF and DFC are together = a right $\angle$.

(96)

But the $\angle^s$ ACD and DCF are together = a right $\angle$.

Consequently the $\angle^s$ DCF and DFC = the $\angle^s$ ACD and DCF.

(46)

Take the $\angle$ DCF from each, and the $\angle$ DFC = the $\angle$ ACD.

(48)

But the $\angle^s$ DFC and DEC are equal.

(129)

Consequently the $\angle^s$ DEC and ACD are equal also.

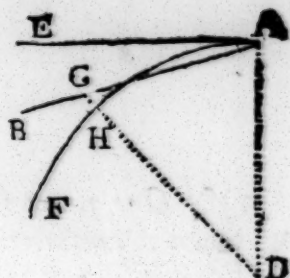
(46)

* A small ° put above any figure, signifies degrees.

133.

THEOREM XXII.

Between a circular arc AHF, and its tangent AE, no right line can be drawn from the point of contact A.



DEM. For if any other right line can be drawn, let it be the right line AB.

From D, the center of AHF, draw DG perpendicular to AB, cutting AB in G, and the arc in H.

Now as $\angle DGA$ is right; therefore DA is greater than DG. (106)

But $DA = DH$ (9). Therefore DH is greater than DG, which is absurd.

Consequently no right line can be drawn between the tangent AE and the arc AHF.

134. COROL. I. Hence the angle DAH, contained between the radius DA, and an arc AH, is greater than any right lined acute angle.

For a right line AB must be drawn from A, between the tangent AE and radius AD, to make an acute angle.

But no such right line can be drawn between AE and the arc AH. (133)

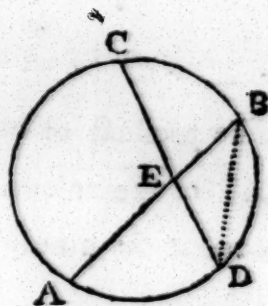
135. COROL. II. Hence the angle EAH, between the tangent EA and arc AH, is less than any right lined acute angle.

136. COROL. III. Hence it follows, that at the point of contact the arc has the same direction as the tangent, and is at right angles to the radius drawn to that point.

137.

THEOREM XXIII.

If two right lines, AB, CD, intersect any how (in E) within a circle, their inclination, AED, or CEB, is measured by half the sum of the intercepted arcs, AD, CB.



DEM. For drawing DB;

The $\angle AED = \angle EDB + \angle EBD$. (43)

But the $\angle EDB$ is measured by $\frac{1}{2}$ arc CB. (90)

And the $\angle EBD$ is measured by $\frac{1}{2}$ arc AD. (128)

Consequently the $\angle AED$ is measured by half the arc CB, together with half the arc AD. (128)

(50)

SECTION IV.

Of Proportion.

DEFINITIONS and PRINCIPLES.

138. One quantity A, is said to be measured or divided by another quantity B, when A contains B some number of times, exactly.

Thus if $A=20$, and $B=5$; then A contains B four times.

A is called a multiple of B; and B is said to be part of A.

139. If a quantity A ($=20$) contain another B ($=5$) as many times as a quantity C ($=24$) contains another D ($=6$); then

A and C are called like multiples of B and D.

B and D are called like parts of A and C: And

A is said to have the same relation to B, as C has to D.

Or, like multiples of quantities are produced, by taking their Rectangle, or Product, by the same quantity, or by equal quantities.

The Rectangle or Product of quantities, A and B, is expressed by writing this mark $\times$ between them. Thus, $A \times B$, or $B \times A$, expresses the rectangle contained by A and B.

140. When two quantities of a like kind are compared together, the relation which one of them has to the other, in respect to quantity, is called Ratio.

The first term of a ratio, or the quantity compared, is called the Antecedent; and the second term, or the quantity compared to, is called the Consequent.

A ratio is usually denoted by setting the antecedent above the consequent, with a line drawn between them.

Thus $\frac{A}{B}$ signifies, and is thus to be read, the ratio of A to B.

The multiple of a ratio $\frac{A}{B}$, is the product of each of its terms by the same quantity, or by equal quantities. Thus $\frac{A \times C}{B \times C}$ is the ratio $\frac{A}{B}$ taken c times.

The product of two or more ratios, $\frac{A}{B}, \frac{C}{D}$, is expressed by taking the product of the antecedents for a new antecedent, and the product of the consequents for a new consequent. Thus $\frac{A \times C}{B \times D} = \frac{A}{B} \times \frac{C}{D}$.

141. Equal ratios are those where the antecedents are like multiples or parts of their respective consequents.

Thus in the quantities A, B, C, D: Or 20, 5, 24, 6.

In the ratio of A to B, or of 20 to 5, the antecedent is a multiple of its consequent four times.

And in the ratio of C to D, or 24 to 6, the antecedent is a multiple of its consequent four times.

That is, the ratio of A to B is the same as the ratio of C to D.

And this equality of ratios is thus expressed, $\frac{A}{B} = \frac{C}{D}$.

142. Ratio

142. *Ratio of equality is, when the antecedent is equal to the consequent.*

Thus when $A=B$, then $\frac{A}{B}$, or $\frac{A}{A}$, or $\frac{B}{B}$, is a ratio of equality.

143. *Four quantities are said to be proportional, which, when compared together by two and two, are found to have equal ratios.*

Thus, let the quantities to be compared be A, B, C, D : Or 20, 5, 24, 6.
Now in the ratio of A to B , or of 20 to 5; A contains B four times.
And in the ratio of C to D , or of 24 to 6; C contains D four times.

Then the ratios of A to B , and of C to D , are equal: Or $\frac{A}{B} = \frac{C}{D}$. * (141)

And their proportionality is thus expressed, $A : B :: C : D$. (75)

Also in the ratio of A to C , or of 20 to 24; C contains A , once and $\frac{1}{3}$.

And in the ratio of B to D , or of 5 to 6; D contains B , once and $\frac{1}{3}$.

Where the ratios are likewise equal, viz. $\frac{A}{C} = \frac{B}{D}$.

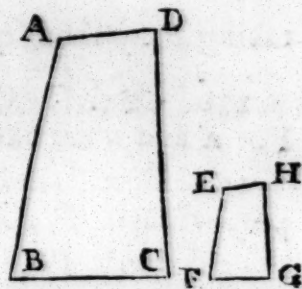
And these are also proportional, $A : C :: B : D$.

144. *So that when four quantities of the same kind are proportional, the ratio between the first and second is equal to the ratio between the third and fourth; and this proportionality is called Direct.*

145. *Also the ratio between the first and third is equal to the ratio between the second and fourth; and this proportionality is called Alternate.*

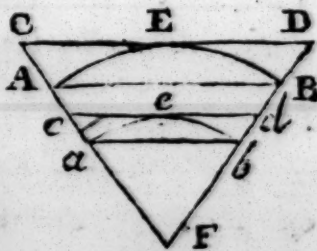
146. *Similar, or like, right lined figures, are those which are equiangular; (that is, the several angles of which are equal one to the other;) and also, the sides about the equal angles proportional.*

Thus if the figures AC and EG are equiangular,
And $AB : BC :: EF : FG$; Or $BC : CD :: FG : GH$;
Then are those figures called similar, or like figures.
And the like in triangles, or other figures.



147. *Like arcs, chords, or tangents, in different circles, are those which subtend, or are opposite to, equal angles at the center.*

Let F be the center of two concentric arcs AEB , aeb , terminated by the radii FAA , FbB , produced; AB , ab , their chords, and CD , cd , their tangents. Then as the angle CFD is measured either by the arc AEB , or aeb , those arcs are said to be alike, or similar; that is, the arc aeb is the same part of its whole circumference, as the arc AEB is of its whole circumference.



148.

THEOREM XXIV.

Quantities, and their like multiples, have the same ratio.

That is, the ratio of A to B is equal to the ratio of twice A to twice B, or thrice A to thrice B, &c. Or thus $\frac{A}{B} = \frac{2A}{2B} = \frac{3A}{3B}$, &c. $= \frac{C \times A}{C \times B}$; that is, equal to the ratio of c times A to c times B.

DEM. For the ratio of A to B must either be equal to the ratio of like multiples of A and B, or to the ratio of unlike multiples of them.

Now suppose the ratio of A to B is equal to the ratio of their unlike multiples, c times A, d times B; that is, $\frac{A}{B} = \frac{C \times A}{D \times B}$.

Then $A : B :: C \times A : D \times B$ (143). And $A : C \times A :: B : D \times B$. (145)

Therefore $\frac{A}{C \times A} = \frac{B}{D \times B}$ (144). Where the consequents are unequal multiples of their antecedents, by supposition.

But $\frac{A}{C \times A}$ is not equal to $\frac{B}{D \times B}$, (141)

Then $A : C \times A :: B : D \times B$ is not true. Also $A : B :: C \times A : D \times B$ is not true.

Consequently $\frac{A}{B}$ is unequal to $\frac{C \times A}{D \times B}$.

Therefore the ratio of unlike multiples of two quantities, is not equal to the ratio of those quantities.

Consequently the ratio of two quantities, and the ratio of their multiples, are the same. Or $\frac{A}{B} = \frac{C \times A}{C \times B}$.

149. COR. I. In any ratio, if both terms contain the same quantity or quantities; the value of the ratio will not be altered by omitting, or taking away those quantities. For $\frac{C \times A}{C \times B} = \frac{A}{B}$, by taking away c.

150. COR. II. Quantities, and their like parts, have equal ratios. For A and B are like parts of $C \times A$ and $C \times B$.

151. COR. III. Quantities, and their like multiples, or like parts, are proportional. For $A : B :: C \times A : C \times B$. And $C \times A : C \times B :: A : B$ (148)

152. COR. IV. If quantities are equal, their like multiples, or like parts, are also equal. For if $A = B$; and $\frac{A}{B} = \frac{C \times A}{C \times B}$;

Then are the antecedents and consequents in a ratio of equality. (141)

153. COR. V. If the parts of one quantity are proportional to the parts of another quantity, they are like parts of their respective quantities. For only like parts are proportional to their wholes. (151)

154. COR. VI. Ratios, which are equal to the same ratio, are equal to one another. For the ratio of $\frac{A}{B} = \frac{C \times A}{C \times B} = \frac{D \times A}{D \times B}$, &c. (148)

155. COR.

155. COR. VII. Proportions, which are the same to the same proportion, are the same to one another.

If $A : B :: C : D$; and $A : B :: E : F$; Then $C : D :: E : F$.

For $\frac{A}{B} = \frac{D}{C}$; and $\frac{A}{B} = \frac{F}{E}$ (144). Then $\frac{C}{D} = \frac{E}{F}$. (46)

156. COR. VIII. If two ratios or products are equal, their like multiples, either by the same or by equal quantities, or by equal ratios, are also equal.

That is, if $\frac{A}{B} = \frac{C}{D}$: Then $\frac{A \times E}{B \times E} = \frac{C \times E}{D \times E}$.

And if $E = F$: Then $\frac{A \times E}{B \times E} = \frac{C \times F}{D \times F}$.

And if $\frac{E}{F} = \frac{G}{H}$: Then $\frac{A \times E}{B \times F} = \frac{C \times G}{D \times H}$.

For in either case, the ratios may be considered as quantities.

157.

T H E O R E M XXV.

Equal quantities, A and B, have the same ratio or proportion to another quantity C. And any quantity has the same ratio to equal quantities.

That is, if $A = B$: Then $A : C :: B : C$. And $C : A :: C : B$.

DEM. Since $A = B$; then C is the like multiple, or part of B, as it is of A.

And $A : B :: C : C$ (151). Therefore $A : C :: B : C$. (145)

Also $C : C :: A : B$ (151). Therefore $C : A :: C : B$. (145)

158. COR. I. Hence, when the antecedents are equal, the consequents are equal; and the contrary.

159. COR. II. Quantities are equal, which have the same ratio to another quantity: or to like multiples or parts of another quantity.

Thus, if $A : C :: B : C$. Then $A = B$.

160. COR. III. Since $A : C :: B : C$; and $C : A :: C : B$. Therefore, when four quantities are in proportion, As antecedent is to consequent, so is antecedent to consequent: Then shall the first consequent be to its antecedent, as the second consequent to its antecedent: and this is called *the inversion of ratios*.

161.

T H E O R E M XXVI.

In two, or more, sets of proportional quantities, the rectangles under the like terms are proportional.

That is, if $A : B :: C : D$; and $E : F :: G : H$.

Then $A \times E : B \times F :: C \times G : D \times H$.

DEM. Since $\frac{A}{B} = \frac{C}{D}$; and $\frac{E}{F} = \frac{G}{H}$. (144)

Therefore $\frac{A \times E}{B \times F} = \frac{C \times G}{D \times H}$. (156)

Consequently $A \times E : B \times F :: C \times G : D \times H$. (143)

162.

THEOREM XXVII.

In four proportional quantities $A : B :: C : D$. Then the Rectangle or Product of the two extremes is equal to the Rectangle or Product of the two means. That is, $A \times D = B \times C$.

DEM. Since $A : B :: C : D$ by supposition. Therefore $\frac{A}{B} = \frac{C}{D}$. (144)

And $\frac{A}{B} = \frac{A \times D}{B \times D}$ (156). Also $\frac{C}{D} = \frac{C \times B}{D \times B}$. (156)

Therefore $\frac{A \times D}{B \times D} = \frac{C \times B}{D \times B}$ (46), where the consequents are equal.

Consequently $A \times D = C \times B$. (158)

163. Hence, if the Rectangle or Product of two quantities is equal to the Rectangle or Product of other two quantities; those four quantities are proportional.

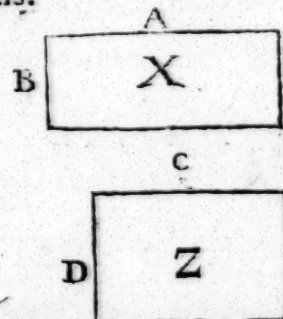
Thus, suppose the two Rectangles, X, Z, are equal;
Where A, C, are their lengths, and B, D, their breadths.

Then $A \times B = C \times D$ by supposition.

Therefore $A : C :: D : B$.

That is, As the length of X is to the length of Z.

So the breadth of Z is to the breadth of X.



In such cases, the lengths are said to be to one another reciprocally, as their breadths.

Or that proportion $A : C :: D : B$ is reciprocal, when $A \times B = C \times D$.

164.

THEOREM XXVIII.

If four quantities are proportional, then will either of the extremes, and the ratio of the product of the means to the other extreme, be in the ratio of equality: And either mean, and the ratio of the product of the extremes to the other mean, will be also in a ratio of equality.

That is, if $A : B :: C : D$. Then $A = \frac{B \times C}{D}$. And $B = \frac{A \times D}{C}$.

DEM. Since $A : B :: C : D$ by supposition.

Therefore $A \times D = B \times C$. (162)

And $\frac{A \times D}{C} = \frac{B \times C}{C}$ (157). Also $\frac{B \times C}{D} = \frac{A \times D}{D}$. (157)

But $B = \frac{B \times C}{C}$ (149). And $A = \frac{A \times D}{D}$. (149)

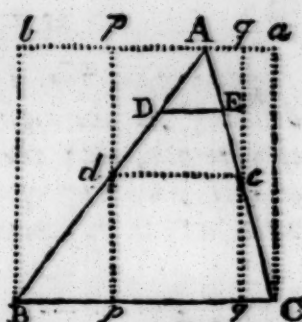
Therefore $\frac{A \times D}{C} = B$. And $\frac{B \times C}{D} = A$. (46)

165. THEO-

165. THEOREM XXIX.

In any plane triangle, ABC , any two adjoining sides, AB , AC , are cut proportionally by a line DE , drawn parallel to the other side BC , viz.

$$AD : DB :: AE : EC.$$



DEM. Through B and C draw Bb , Cc , at right angles to BC , meeting ba , drawn through A , parallel to BC : Through p , q , the middles of Ab , Aa , draw pp' , qq' , parallel to Bb or Cc , meeting AB , AC , in the points d , c ; and join d .

Now the triangles Adp , Bdp , and Acq , Ccq , are congruous. (95, 100)

Therefore $Ad = Bd$, $Ac = Cc$, $pd = pd$, $qc = qc$.

But $pp' = qq'$ (116): Therefore $pd = qc$. (49)

And dc is parallel to BC . (116)

In the same manner it may be shewn, that lines parallel to Bb , drawn through the middles of Ap , $p'b$; Aq , qa ; will also bisect Ad , Bd ; Ac , Cc ; and that lines joining these points of bisection will also be parallel to BC : And the same may be proved at any other bisections of the segments of the lines AB , AC : Also the like may be readily inferred at any other divisions of the lines Ab , Aa .

Therefore lines parallel to BC , cut off like parts from the lines AB , AC .

Then $AB : AC :: AD : AE$. And $AB : AC :: BD : CE$. (151)

Therefore $AD : AE :: BD : CE$. (155)

And by Alternation $AD : BD :: AE : CE$. (145)

166. COR. Hence, when the sides AB , AC , of a triangle are cut proportionally, in D , E , the segments AD , AE ; DB , EC ; of those sides are proportional to the sides: And the line DE , drawn to those sections, is parallel to the other side BC .

167. THEOREM XXX.

In equiangular triangles, ABC , abc , the sides about the equal angles are proportional; and the sides opposite to equal angles are also proportional.

DEM. In CA , CB , take $CD = ca$, $CE = cb$; and draw DE .

Then the triangles CDE , cab , being congruous. (99)
The $\angle CDE = (\angle a =) \angle A$. Therefore DE is parallel to AB . (95)

In the same manner, taking $AF = ac$, $AG = ab$; also $BH = bc$, $BI = ba$; and drawing FG , HI , the triangles AGF , IBH , abc , are congruous; therefore FG is parallel to CB , and HI is parallel to CA .

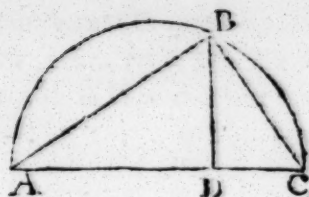
Then $(CD =) ca : CA :: (CE =) cb : CB$.
 $(AF =) ca : CA :: (AG =) ab : AB$.
 $(BH =) bc : BC :: (BI =) ab : AB$. } (165)

168. COR. Hence, Triangles have one angle in each equal, and the sides about those equal angles proportional, those triangles are equiangular and similar.

169. THEO-

169. THEOREM XXXI.

In a right angled triangle, ABC , if a line, BD , be drawn from the right angle B , perpendicular to the opposite side, AC ; then will the triangles ABD , BCD , on each side the perpendicular, be similar to the whole ABC , and to one another.



DEM. For in the triangles ABC , ADB , the $\angle A$ is common; And the right angle $ABC = \text{right angle } ADB$.

Therefore the remaining $\angle C = \angle ABD$. (98)

In the same manner it will appear, that the triangles ABC , BDC , are like.

Therefore the triangles ABD , BCD , are also similar.

170. COR. I. Hence, $AC : AB :: AB : AD$.

$AC : BC :: BC : DC$.

$AD : DB :: DB : DC$.



(167)

171. COR. II. Hence a right line BD , drawn from a circumference of a circle perpendicular to the diameter AC , is a mean proportional between the segments AD , DC , of the diameter.

And $AD \times DC = DB^2$. (162)

For a circle, the diameter of which is AC , will pass through A , B , C . (131)

Scholium. This corollary includes what is usually called one of the chief properties of the circle, namely;

The square of the Ordinate is equal to the rectangle under the two Abscissas.

Here, the ordinate is the perpendicular BD ; and the two Abscissas are the two segments AD , DC , of the diameter AC .

172. THEOREM XXXII.

In a circle, if two chords, AB , CD , intersect each other in E , either within the circle, or without, by prolonging them; then the rectangle under the segments, terminated by the circumference and their intersection, will be equal.

That is, $AE \times EB = CE \times ED$.

DEM. Draw the lines BC , DA .

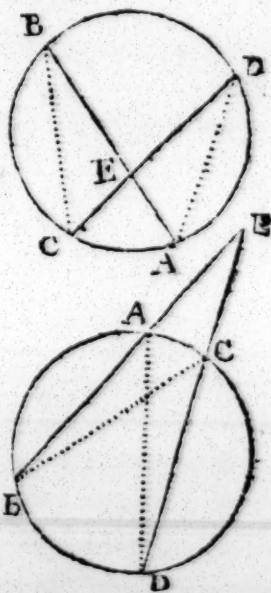
Then the triangles DEA , BEC , are similar.

For the angle at E is equal (93), or common.

And the $\angle D = \angle B$, as standing on the same arc AC (129). Then the other angles are equal. (98)

Therefore $AE : CE :: ED : EB$. (167)

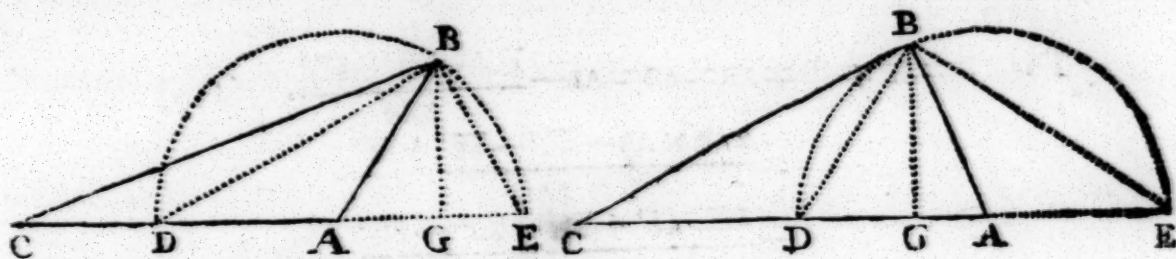
Consequently $AE \times EB = CE \times ED$. (162)



173.

THEOREM XXXIII.

If with the least side AB of a given triangle ABC, a semicircle be described from the angular point A; meeting the side AC, produced in the points D, E; and from B, the lines BE, BD, be drawn, and also BG perpendicular to DE: Then the values of the several lines AG, CG, GE, GD, BE, BD, BG, may be expressed in terms of the sides of the triangle ABC, as follow.



$$174. \quad AG = \frac{\overline{BC}^2 - \overline{AC}^2 - \overline{AB}^2}{2AC}. \quad (123)$$

$$\text{Or } AG = \frac{\overline{AC}^2 + \overline{AB}^2 - \overline{BC}^2}{2AC}. \quad (121)$$

$$175. \quad CG = \frac{\overline{AC}^2 - \overline{AB}^2 + \overline{BC}^2}{2AC}. \quad \text{For } CG = AC \pm AG, \text{ or to } AC - AG.$$

$$\text{And } AC + AG = AC \pm \frac{\overline{BC}^2 - \overline{AC}^2 - \overline{AB}^2}{2AC}. \quad (174)$$

$$= \frac{2\overline{AC}^2 + \overline{BC}^2 - \overline{AC}^2 - \overline{AB}^2}{2AC}. \quad (149)$$

$$176. \quad GE = \frac{2H \times H - CB}{2AC}; \quad \text{Here } 2H = AC + AB - BC$$

$$\text{For } GE = AB(AE) \pm AG = AB + \frac{\overline{AC}^2 + \overline{AB}^2 - \overline{CB}^2}{2AC}. \quad (174)$$

$$= \frac{2AC \times AB + \overline{AC}^2 + \overline{AB}^2 - \overline{BC}^2}{2AC}. \quad (149)$$

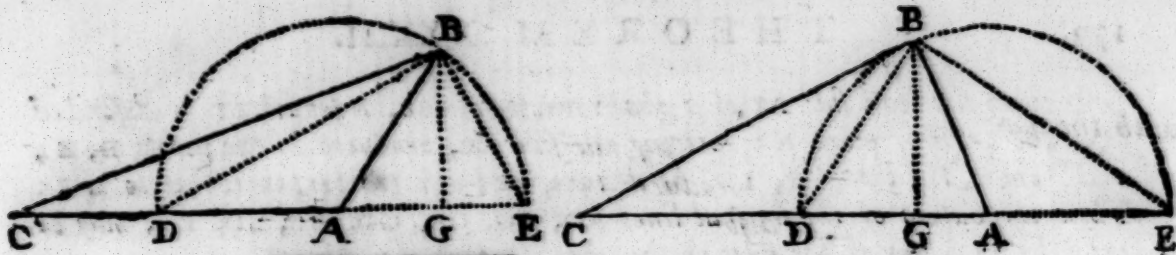
$$\text{But } 2AC \times AB + \overline{AC}^2 + \overline{AB}^2 = (\overline{AC} + \overline{AB})^2 = \overline{CE}^2. \quad (117)$$

$$\text{Then } GE \left(= \frac{\overline{CE}^2 - \overline{BC}^2}{2AC} \right) = \frac{\overline{CE} + \overline{BC} \times \overline{CE} - \overline{BC}}{2AC}. \quad (119)$$

$$= \frac{\overline{CA} + \overline{AB} + \overline{BC} \times \overline{CA} + \overline{AB} - \overline{BC}}{2AC}.$$

$$= \frac{2H \times 2H - 2BC}{2AC} = \frac{2H \times H - BC}{AC}.$$

$$177. \quad GD =$$



$$177. \quad GD = \frac{H - AC \times 2H - 2AB}{AC}. \quad \text{Here } 2H = AC + AB + BC.$$

$$\text{For } GD = AD - AG = AB - AG = AB - \frac{AC^2 - AB^2 + BC^2}{2AC}. \quad (174)$$

$$= \frac{2AC \times AB - AC^2 - AB^2 + BC^2}{2AC} \\ = \frac{AC - AB^2 + BC^2}{2AC} = \frac{BC^2 - CD^2}{2AC}. \quad (118)$$

$$= \frac{BC - CD \times BC + CD}{2AC}. \quad (119)$$

$$= \frac{BC + AB - AC \times BC + AC - AB}{2AC} \\ = \frac{2H - 2AC \times 2H - 2AB}{2AC}.$$

$$178. \quad BE = \sqrt{\frac{AB}{AC} \times 2H \times H - CB}.$$

$$\text{For } BE^2 = DE \times GE. \quad (170)$$

$$= 2AB \times \frac{CE^2 - BC^2}{2AC}. \quad (176)$$

$$\text{Therefore } BE = \sqrt{\frac{2AB}{2AC} \times CE^2 - BC^2} = \sqrt{\frac{AB}{AC} \times 2H \times H - CB}. \quad (176)$$

$$179. \quad BD = \sqrt{\frac{AB}{AC} \times H - AC \times 2H - 2AC}.$$

$$\text{For } BD^2 = DE \times GD. \quad (170)$$

$$= 2AB \times \frac{BC^2 - CD^2}{2AC}. \quad (177)$$

$$\text{Therefore } BD = \sqrt{\frac{2AB}{2AC} \times BC^2 - CD^2}. \quad (177)$$

$$180. \quad BG = \frac{2}{AC} \times \sqrt{H \times H - CB \times H - AC \times H - AB}.$$

$$\text{For } BG^2 = GE \times GD. \quad \text{Therefore } BG = \sqrt{GE} \times \sqrt{GD}. \quad (170)$$

$$\text{And } GE = \frac{2}{AC} \times H \times H - CB. \quad (176) \quad GD = \frac{2}{AC} \times H - AC \times H - AB. \quad (177)$$

181. COROL. Hence is derived the Rule usually given for finding the area, or superficial content, of a Triangle, the three sides being known.

RULE. I. From half the sum of the three sides, subtract each side severally, noting the three remainders.

2d. Multiply the said half sum, and the three noted remainders continually.

3d. The square root of the product is the area of the Triangle.

182. THEO.

182.

THEOREM XXXIV.

If a regular polygon, $ABCDEF$, be inscribed in a circle; and parallel to these sides if tangents to the circle be drawn, meeting one another in the points a, b, c, d, e, f ; then shall the figure formed by these tangents circumscribe the circle, and be similar to the inscribed figure.



DEM. Since the circle touches every side of the figure $abcdef$, by construction; therefore the circle is circumscribed by that figure. (41)

Through A and B , draw the radii SA, SB , prolonged till they meet the tangent ab , in a, b .

Then the triangles ASB, asb , are equiangular.

For the $\angle$ at s is common; and the other angles are equal, because AB and ab are parallel, by supposition.

Also $sa = sb$: For the triangles ASB, asb , are isosceles. (104)

And the same may be proved of the other triangles; and also, that they are equal to one another.

Therefore the figure $abcdef$ has equal sides, and is equiangular to the figure $ABCDEF$.

Now $SA : sa :: AB : ab$; and $SA : sa :: AF : af$. (167)

Therefore $AB : ab :: AF : af$. And the like of the other sides. (155)

Consequently the figures $ABCDEF, abcdef$, are similar. (145)

183. COR. I. If two figures are composed of like sets of similar triangles, those figures are similar.

184. COR. II. Hence, if from the angles a, b , of a regular polygon circumscribed a circle, lines as, bs , be drawn to the center s ; the chords AB of the intercepted arcs will be the sides of a similar polygon, inscribed in the circle: and the sides AB, ab , of the inscribed and circumscribing polygons will be parallel.

185. COR. III. The chords or tangents of like arcs in different circles, are in the same proportion as the radii of those circles.

For if a circle circumscribe the polygon $abcdef$; then the sides of the polygons $abcdef, ABCDEF$, are chords of like arcs in their respective circumscribing circles.

And if a circle be inscribed in the polygon $ABCDEF$, the sides $AB, ab, \&c.$ are tangents of like arcs also: And these have been shewn to be proportional to their radii SA, sa .

186. COR. IV. The Perimeters of like polygons (or the sum of their sides) are to one another as the radii of their inscribed or circumscribed circles.

For $SA : sa :: AB : ab$. (182)

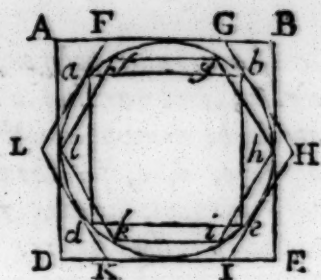
And AB, ab , are like parts of the perimeters of their polygons.

Therefore $SA : sa :: \text{perimeter } ABCDEF : \text{perimeter } abcdef$. (151)

187. THEO-

187. THEOREM XXXV.

If there be two regular and like polygons applied to the same circle, the one inscribed and the other circumscribed: Then will the circumference of that circle, and half the sum of the perimeters of those polygons, approach nearer to equality, as the number of sides in the polygons increase.



DEM. It is evident at sight, that the circumscribing hexagon $FGHIKL$ is less than the circumscribing square $ABED$.

And also that the inscribed hexagon $fgbikl$ is greater than the inscribed square $abed$.

And in both cases, the difference between the hexagon and the circle is less than the difference between the circle and the square.

Therefore the polygon, whether inscribed or circumscribed, differs less from the circle, as the number of its sides is increased.

And when the number of sides in both is very great, the perimeters of the polygons will nearly coincide with the circumference of the circle; for then the difference of the polygonal perimeters becomes so very small, that they may be esteemed as equal.

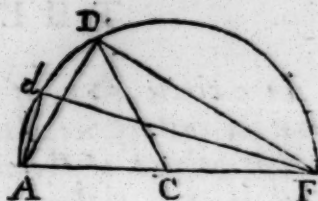
And yet so long as there is any difference between these polygons, though ever so small, the circle is greater than the inscribed, and less than the circumscribed polygons: Therefore half their sums may be taken for the circumference of the circle, when the number of those sides is very great.

Hence, the circumferences of circles are in proportion to one another, as the radii of those circles, or as their diameters.

For the perimeters of the inscribed and circumscribing polygons are to one another, as the radii of the circles. (186)

And these perimeters and circumferences continually approach to equality.

192. EXAMPLE. Required the chord of the $\frac{1}{3072}$ part of the circumference of a circle, the radius of which is 1. Or, required the side of a regular polygon of 3072 sides, inscribed in a circle, the diameter of which is 2.



Let ADF be a semicircle, the diameter $AF=2$, and center C . Take the arc $AD=\frac{1}{2}$ of the semicircumference, or equal to 60 degrees; and draw DC , DA , DF .

Let d represent the point where the arc is bisected; Fd the supplemental chord to that bisection: and let the marks d , d' , d'' , d''' , &c. express the bisected points agreeable to the number of bisections.

193. Now since $\angle ACD=60^\circ$.
Therefore $\angle CAD + \angle ADC = (180^\circ - 60^\circ) = 120^\circ$. (98)

But $\angle CAD = \angle ADC$ (104); then $\angle CAD = (\frac{120}{2}) = 60^\circ$.
Therefore $DA = (DC = AC =) 1$.

And as the triangle ADF is right angled at D. (130)

Then $DF = (\sqrt{AF^2 - AD^2} \text{ (113)}) = \sqrt{4 - 1} = \sqrt{3} = 1,7320508075688773$

Therefore the supplemental chord of the arc

AD, or of

$\frac{1}{2}$ of the semicircumference is $FD = \sqrt{3}$	$= 1,7320508075688773$
$\frac{1}{4}$ of the same (190) is $Fd' = \sqrt{2 + FD}$	$= 1,9318516525781366$
$\frac{1}{8}$ $Fd'' = \sqrt{2 + Fd'}$	$= 1,9828897227476208$
$\frac{1}{16}$ $Fd''' = \sqrt{2 + Fd''}$	$= 1,9957178464772070$
$\frac{1}{32}$ $Fd^{iv} = \sqrt{2 + Fd'''} = 1,9989291749527313$	
$\frac{1}{64}$ $Fd^v = \sqrt{2 + Fd^{iv}} = 1,9997322758191236$	
$\frac{1}{128}$ $Fd^{vi} = \sqrt{2 + Fd^v} = 1,9999330678348022$	
$\frac{1}{256}$ $Fd^{vii} = \sqrt{2 + Fd^{vi}} = 1,9999832668887013$	
$\frac{1}{512}$ $Fd^{viii} = \sqrt{2 + Fd^{vii}} = 1,9999958167178004$	
$\frac{1}{1024}$ $Fd^{ix} = \sqrt{2 + Fd^{viii}} = 1,9999989541791767$	

Now Fd^{ix} the supplemental chord of $\frac{1}{1024}$ being known, the chord Ad^{ix} of $\frac{1}{2048}$, part of the semicircumference, or of $\frac{1}{3072}$, part of the whole circumference, is also known.

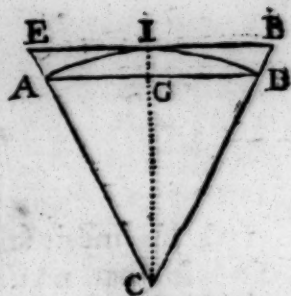
That is $Ad^{ix} = (\sqrt{AF^2 - Fd^{ix2}}) = \sqrt{4 - 2 + \frac{2}{2^{10}}} = 0,0020453073606764$

194. Consequently, the side of a regular polygon of 3072 sides, inscribed in a circle whose diameter is 2, is 0,0020453073606764

$$\sqrt{4 - 2 + Fd^{ix}}$$

195. The

195. The side of a similar polygon circumscribing the same circle, the center of which is C, may be thus found.



Let BE be the side of the circumscribed polygon; and draw BC, EC, cutting the circle in D and A.

Draw DA, and it will be the side of the inscribed polygon; and is parallel to BE. (184)

Draw CI bisecting the angle BCE, and it will bisect BE and DA at right angles (103). And $DG = (\frac{1}{2}DA =) \frac{1}{2}Ad^{ix}$.

Then $CG = \sqrt{CD^2 - DG^2} = \sqrt{1 - \frac{1}{4}Ad^{ix^2}}$.

But $\frac{1}{2}Ad^{ix} = 0,001022653680338$. And its square is 0,00000104582055

Which subtracted from 1, leaves

0,99999895417945

Whose square root, or CG, is equal to

0,99999947708959

Now the triangles CBI, CDG, are similar.

Then $CG : CI :: 2DG : 2BI$.

(167, 151)

Therefore $(2BI =) BE = (\frac{2DG \times CI}{CG} (164) =) \frac{DA}{CG}$; For $IC = I$.

Or $BE = \frac{0,0020453073606764}{0,9999994770895883} = 0,0020453084301895$

which is the side of a regular polygon of 3072 sides, circumscribing a circle the diameter of which is 2.

196. SCHOLIUM. The side of a regular polygon of 3072 sides, inscribed in a circle, the diameter of which is 2, is 0,0020453073606764, (194). Which multiplied by 3072, will give the perimeter of that polygon, which is

6,2831842119979622.

The side of a similar polygon, circumscribing the same circle, is

0,0020453084301895. (195)

Which multiplied by 3072, will give for the perimeter of that polygon

6,2831874973420925.

The sum of these perimeters is

12,5663717093400547.

The half sum is

6,28318585, &c.

Which is very nearly equal to the circumference of a circle, the diameter of which is 2 (187), the difference between it

and the inscribed polygon being only 0,00000164, &c.

and the circumscribed polygon being only 0,00000164, &c.

197. Now the circumferences of circles being in the same proportion as their diameters,

(188)

Therefore the diameter of a circle being 1,

The circumference will be

3,141592, &c. which agrees with the

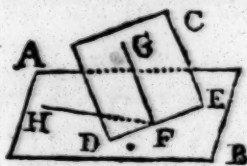
circumference as found by other methods.

SECTION V. *Of Planes and Solids.*

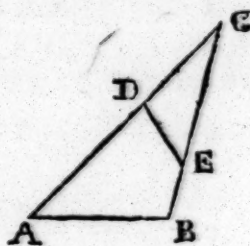
DEFINITIONS and PRINCIPLES.

198. A line is said to be in a plane, when it passes through two or more points in that plane; and the common section of two planes is a line which is in both of them.

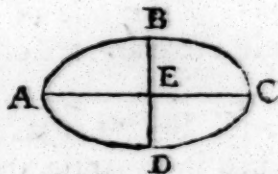
199. The inclination of two meeting planes AB , CD , is measured by an acute angle GFH , made by two right lines FG , FH , one in each plane, and both drawn perpendicular to the common section DE , of these planes from F , some point in it.



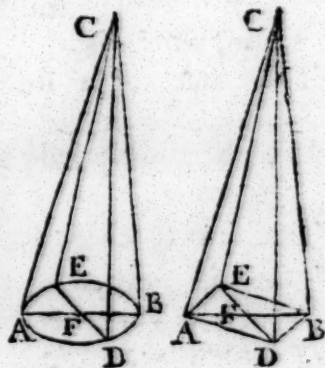
200. A right line DE intersecting two sides AC , BC , of a triangle ABC , so as to make angles CDE , CED , within the figure, equal to the angles CBA , CAB , at the base AB , but with contrary sides of the triangle, is said to be in a subcontrary position to the base.



201. If a circle in an oblique position be viewed, it will appear of an oval form, as $ABCD$; that is, it will seem to be longer one way, as AC , than another, as DB ; nevertheless the radii EA , EB , are to be esteemed as equal. And the same must be understood in viewing any regular figure, when placed obliquely to the eye.



202. If a line be fixed to any point c above the plane of a circle $ADBE$, and this line while stretched be moved round the circle, so as always to touch it; then a solid which would fill the space passed over by the line, between the circle and the point c , is called a CONE.



203. If the figure $ADBE$ had been a polygon, and the stretched line had moved along its sides, the figure which would then have been described, is called a PYRAMID.

So that Cones and Pyramids are solids which regularly taper from a circle, or polygon, to a point.

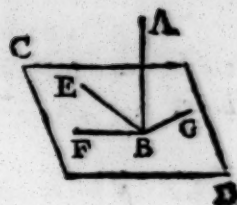
The circle or polygon is called the *Base*; and the point c the *Vertex*.

When the vertex is perpendicularly over the middle or center of the base, then the solid is called a **RIGHT CONE**, or a **RIGHT PYRAMID**; otherwise an **OBLIQUE CONE**, or **OBLIQUE PYRAMID**.

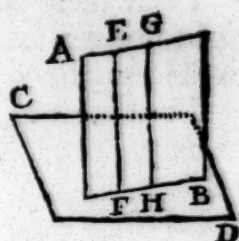
204. If a Cone or Pyramid be cut by a plane passing through the vertex c , and center of the base F , the section ABC , or EDC , is a triangle.

205. A

205. A right line AB , is perpendicular to a plane CD , when it makes right angles $\angle ABE$, $\angle ABF$, $\angle ABG$, with all the right lines BE , BF , BG , drawn in that plane to touch the said right line AB .



206. So that from the same point B , in a plane, only one perpendicular can be drawn to that plane on the same side.

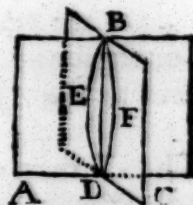


207. A plane AB , is perpendicular to a plane CD , when the right lines EF , GH , drawn in one plane AB , at right angles to FB , the common section of the two planes, are also at right angles to the other plane CD .

208. So that a line EF , perpendicular to a plane CD , is in another plane AB , and at right angles to FB , the common section of the two planes.

209. THEOREM XXXVII.

If two planes AB , CD , cut each other, their common section BD , will be a right line.



DEM. For if it be not, draw a right line DEB in the plane AB , from the point D to the point B ; also draw a right line DFB in the plane BC .

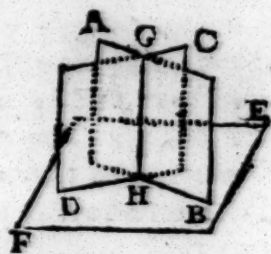
Then two right lines DEB , DFB , have the same terms, and include a space or figure, which is absurd. (7)

Therefore DEB and DFB are not right lines: Neither can any other lines drawn from D to B , besides BD , be right lines.

Consequently the line DB , the common section of the planes, is a right line.

210. THEOREM XXXVIII.

If two planes AB , CD , which are both perpendicular to a third plane EF , cut one another; their intersection HG is at right angles to that third plane EF .



DEM. For the common section of AB and CD is a right line HG . (209) Also HB , HD , are the common sections of AB , CD , with the plane EF .

Now from the point H , a line HG drawn perpendicular to the plane EF , must be at right angles to HB , HD . (205)

But HG must be in both planes AB , CD . (208)

Therefore it must be in the common section of those planes.

Consequently the section HG of the planes AB , CD , is at right angles to the plane EF .

211. THEOREM XXXIX.

The sections *aebd*, of a Cone or Pyramid *CAEBD*, which are parallel to the base *AEBD*, are similar to that base.

DEM. For let *AFBC*, *DFEC*, be sections through the vertex *C*, and center *F* of the base.

Then these sections will cut one another in the right line *FC* (209), and the transverse section *abde*, in the right lines *ab*, and *ed*, intersecting in *f*.

Then are the following sets of triangles similar; namely, *AFC*, *afc*; *BFC*, *bfc*; *DFC*, *dfc*; *EFC*, *efc*.

Wherefore $FC : fc :: FA : fa$
 $FC : fc :: FB : fb$
 $FC : fc :: FD : fd$
 $FC : fc :: FE : fe$ } And the like in any other sections through *C* and *F*. (165)

Now in the Cone, $FA = FB = FD = FE$; therefore $fa = fb = fd = fe$. (152)
 So that all the right lines drawn from *f* to the circumference of the figure *adbe* are equal to one another.

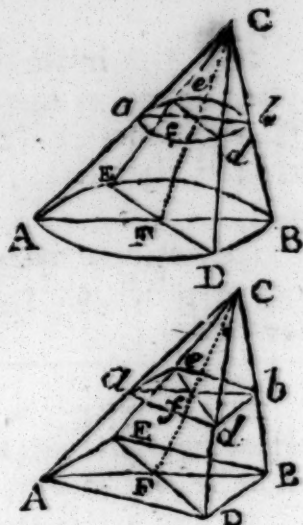
Consequently the figure *adbe* is a circle. (9)

And in the Pyramid, $FC : fc :: DC : dc :: DB : db :: DA : da$.

$FC : fc :: EC : ec :: EA : ea :: EB : eb$.

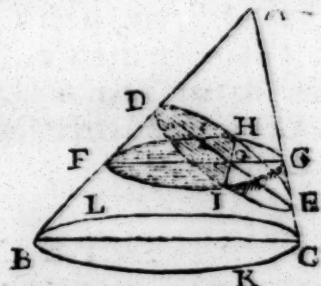
Therefore in each pair of corresponding triangles in the base and transverse section, the sides are respectively proportional.

Consequently, as the base and transverse section are composed of like sets of similar triangles; therefore they are also similar. (183)



212. THEOREM XL.

If a Cone *ABLCK*, the base of which is a circle *CBLCK*, be cut by a plane in a subcontrary position to the base, the section *DIEH* will be a circle.



DEM. Through the vertex *A*, and center of the base, let the triangular section *ABC* be taken, so as to be at right angles to the planes of the base *BKCL*, of the subcontrary section *DIEH*, and of the section *FIGH*, taken parallel to the base, and cutting the subcontrary section in the line *IOH*. Therefore *IOH* is perpendicular to *DE* and *FG* (210) cutting one another in *O*.

Now the section *FIOH* is a circle (211). Therefore $FO \times OG = OI^2$. (171)

Again the triangles *GOE*, *FOD*, are similar.

For $\angle GEO = \angle DFO = \angle ABC$ by constr. And $\angle GOE = \angle DOF$. (93)

Therefore $EO : OG :: FO : DO$ (167.) And $EO \times DO = FO \times OG$ (162) $= OI^2$.

So that *OI* is a mean proportional, either between *FO* and *OG*, or *DO* and *EO*. But as the same would happen wherever *FG* cuts *DE*; therefore all the lines *OI*, both in the sections *FIGH* and *DIEH*, are lines in a circle.

Consequently the section *DIEH* is a circle.

213. If

213. If the section cut both sides of the cone not in a subcontrary position to BC , the diameter of the base, then the section (suppose it still) $DIEH$, is called an ELLIPTIC SECTION, which, though not a circle, will be a bounded curve, longer one way than the other; and, like a circle, return into itself.

The curve $DIEH$ is called an Ellipsis.

214. The line DE , the TRANSVERSE DIAMETER or AXIS.

The line OH or OI , is called an ORDINATE.

215. The ordinate through the middle of DE , is called the CONJUGATE AXIS.

The intersection of the Transverse and Conjugate Axis, is called the CENTER of the Ellipsis.

216. If a circular arc be described, with a radius equal to half the Transverse Axis, from one end of the Conjugate Axis, its intersections with the Transverse Axis, are called FOCI, one on each side of the center of the Ellipsis.

217. Every right line passing through the center of the Ellipsis, and terminated at each end by the curve, is called a DIAMETER.

218. The radius that would describe a circular arc of the same curvature with the ellipsis at any point of it, is called the RADIUS OF CURVATURE.

A TANGENT to any point in the Ellipsis, is a right line perpendicular to the radius of curvature at that point.

219. Two Diameters being so drawn, that one is parallel to a tangent, and the other passes through the point of contact; those two Diameters are said to be CONJUGATE DIAMETERS; and have certain relations to their Ordinates, Tangents, Radii of Curvature, and other lines belonging to the Ellipsis.

220. A third proportional to any two Conjugate Diameters, is called the PARAMETER.

SECTION VI.

Of the Spiral.

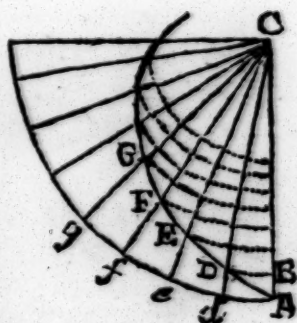
221. Suppose the radius of a circle to revolve with an uniform motion round its center, and while it is so revolving, let a point move along the radius; then will the successive places of that point be in a curve, which is called a spiral.

This will be readily conceived by imagining a fly to move along the spoke of a wheel, while the wheel is turning round.

If while the radius revolves once, the point has moved the length of the radius; then the spiral will have revolved but once round the center, or pole; consequently the motion in the circumference is to the motion in the radius, as the circumference is to the radius; And if the wheel revolves twice, thrice, or in any proportion to the motion in the radius; then the spiral will make so many turns, or parts of a turn, round the center.

222. Now suppose, while the radius revolves equably, a point from the circumference moves towards the center, with a motion decreasing in a geometric progression; then will a spiral be generated, which is called a proportional spiral.

Let the radius CA be divided in any continued decreasing geometric progression (90), as of 10 to 8; then the series of terms will be 10; 8; 6,4; 5,12; 4,096; 3,2768; 2,62144, &c. Also let the circumference be divided into any number of equal parts, in the points *d, e, f, g, &c.* Then if the several divisions of the radius CA be successively transferred from the center C, cutting the other radii in the points D, E, F, G, &c. and a curved line be evenly drawn through those points, it will be a spiral of the kind proposed.



223. From the nature of a decreasing geometric progression, it is easy to conceive that the radius CA may be continually divided; and although each successive division becomes shorter than the next preceding one, yet if ever so great a number of divisions, or terms, be taken, there will still remain a finite magnitude.

224. Hence it follows, that this spiral winds continually round the center, and does not fall into it till after an infinite number of revolutions.

Also, that the number of revolutions decrease, as the number of the equal parts, into which the circumference is divided, increases.

225. THEO-

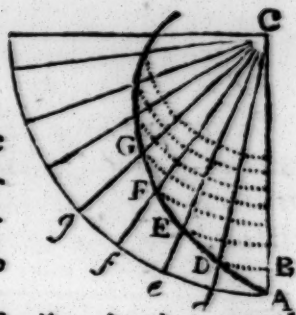
225.

THEOREM XLI.

Any proportional spiral cuts the intercepted radii at equal angles.

DEM. If the divisions Ad , de , ef , fg , &c. of the circumference were very small, then would the several radii be so close to one another, that the intercepted parts AD , DE , EF , FG , &c. of the spiral, might be taken as right lines.

And the triangles CAD , CDE , CEF , &c. would be similar, having equal angles at the point C , and the sides about those angles proportional. (168) Therefore the angles at A , D , E , F , &c. being equal, the spiral must necessarily cut the radii at equal angles.



226.

THEOREM XLII.

If the radii of any proportional spiral be taken as numbers, then will the corresponding arcs of the circle, reckoned from their commencement, be as the logarithms of those numbers.

DEM. As the lines CA , CD , CE , CF , CG , &c. are a series of terms in geometric progression; and the arcs Ad , Ae , Af , Ag , &c. are a series of terms in arithmetic progression; therefore these arcs may serve (l. 66) as the indices to the geometric terms, and be thus placed;

Radii of the spiral CA , CD , CE , CF , CG , &c. Geometric terms.

Corresponding arcs O , Ad , Ae , Af , Ag , &c. Arithm. terms, or indices.

In this disposition, the first term CA is not distant from itself, therefore its index is represented by O .

Then if the distance of the second term CD from the first term CA be expressed by the arc Ad ; the distance of the third term CE , from CA , will be expressed by the arc Ae ; and so of the rest.

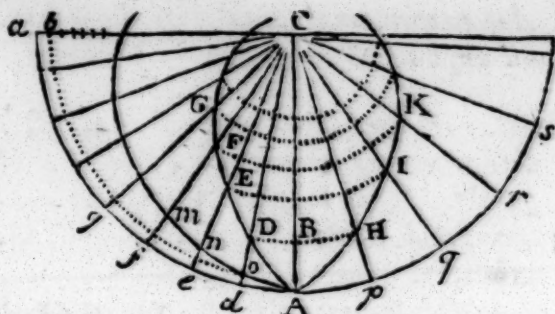
Consequently, if the terms in the geometric series be represented by numbers, taken as parts of the radius, then the numbers of the same kind, expressing the measures of the arcs, or indices, will be as the logarithms of the geometric terms. (l. 73)

227. COROL. If the difference between CA and CB was indefinitely small, or CA and CB were nearly in a ratio of equality; then might the number of proportional lines into which CA could be divided, be so many, that any proposed number might be found among the terms of this series; and if the number of parts in the circumference was increased in like manner, then would every term of the proportional division of the radius CA have its corresponding index among the equal divisions of the circumference; and consequently would exhibit the logarithms of all numbers.

228.

THEOREM XLIII.

There may be almost an infinite variety of proportional spirals, and as many different kinds of logarithms.



DEM. For with the same equal divisions $Ad, de, ef, \&c.$ of the circumference, every variation in the ratio of CA to CB , as of ca to cb , will produce a different spiral, $Aonm$.

And with the same divisions of the radius CA , and different sets of equal parts, $Ad, de, ef, \&c.$ and $Ap, pq, qr, \&c.$ of the circumference, may be formed different spirals $ADEF, AHIK$.

Also, varying at the same time both the divisions of the radius and circumference, different spirals will be produced.

But the variations in these three cases may be almost infinite: Therefore the number of such spirals are almost infinite.

Now it is evident, that there is a peculiar relation between the rays of any spiral, and the corresponding arcs of the circle; that is, between the terms of a geometric progression, and its indices: Therefore there may be as many kinds of logarithms, as there are proportional spirals.

229.

THEOREM XLIV.

That proportional spiral which intersects equidistant rays at an angle of 45 degrees, produces logarithms that are of Napier's kind.

DEM. Suppose AB , the difference between CA and CB , the first and second terms of the geometric progression, to be indefinitely small, and take Ap , the logarithm of CB , equal to AB ; then may the figure $ABHp$ be taken as a square, whose diagonal AH would be part of the spiral $AHIK$, and the angle BAH would be half a right one, or 45 degrees.

Therefore that spiral which cuts its rays $CA, CH, \&c.$ at angles of 45 degrees, has a kind of logarithms belonging to it, so related to their corresponding numbers, that the smallest variation between the first and second numbers is equal to the logarithm of the second number.

But of this kind were the first logarithms made by Lord Napier.

Therefore the logarithms to the spiral which cuts its equidistant rays at an angle of 45 degrees, are of the *Nepierian* kind.

END OF BOOK II.

THE



THE
 ELEMENTS
 OF
 NAVIGATION.

BOOK III.
 OF PLANE TRIGONOMETRY.

SECTION I.

Definitions and Principles.

1. **P**LANE TRIGONOMETRY is an art which shews how to find the measures of the sides and angles of plane Triangles, some of them being already known.

It will be proper for the learner, before he reads the following Articles to turn to the definitions relative to a circle and angle, contained in the Articles 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, and 36, of Book II.

2. A Triangle consists of six parts; namely, three sides and three angles.

The sides of plane triangles are denoted, or estimated by measures of length; such as Feet, Yards, Fathoms, Furlongs, Miles, Leagues, &c.

The angles of triangles are estimated by circular measures, that is, by arcs containing Degrees, Minutes, Seconds, &c. (II. 15); and for convenience these circular measures are represented by right lines, called right lines, tangents, secants, and versed sines.

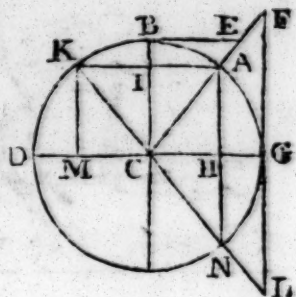
3. The

3. The **RIGHT SINE** of an arc, is a right line drawn from one end of the arc perpendicular to a radius drawn to the other end : Or it is half the chord of the double of that arc.

Thus AH is the right sine of the arc AG ; and also of the arc DBA .

4. The **TANGENT** of an arc, is a right line touching one end of the arc, and continued till it meets a right line drawn from the center through the other end of that arc.

Thus GF is the tangent of the arc GA .



5. The **SECANT** of an arc, is a right line drawn through the center and one end of the arc, and produced till it meets the tangent drawn from the other end.

Thus CF is the secant of the arc AG .

6. The **VERSED SINE** of an arc, is that part of the radius intercepted between the arc and its right sine.

Thus HG is the versed sine of the arc AG .

7. The **COMPLEMENT** of an arc, is what that arc wants of 90 degrees.

Thus if the arc $GB = 90^\circ$. Then AB is called the complement of AG ; and AG is the complement of AB .

8. The **SUPPLEMENT** of an arc, is what that arc wants of 180 degrees.

Thus the arc ABD is the supplement of AG ; and AG of ABD .

9. The **CO-SINE** of an arc, is the right sine of the complement of that arc.

The **CO-TANGENT** of an arc, is the tangent of that arc's complement.

The **CO-SECANT** of an arc, is the secant of its complement.

The **CO-VERSED SINE** of an arc, is the versed sine of its complement.

Thus AI , BE , CE , BI , being respectively the sine, tangent, secant, and versed sine of the arc AB , which is the complement of AG ; therefore AI is called the co-sine, BE the co-tangent, CE the co-secant, BI the co-versed sine, of the arc AG .

The right lines, called sines, tangents, secants, and versed sines, are used as well for the measures of angles, as for the arcs which measure these angles : And it is as common to say the sine, tangent, &c. of an angle, as the sine, tangent, &c. of an arc.

10. The

10. The greatest right line, is the sine of 90° ; and the sines to arcs less than 90° , serve equally for arcs as much greater than 90° .

Thus the sines of 80° and 100° ; of 60° and 120° ; of 40° and 140° , &c. are respectively equal.

11. The same tangent and secant will serve to arcs equally distant from 90 degrees; that is, to any arc and its supplement.

Thus if the arc $BAG = 90^\circ$, and $BK = BA$; then the arcs GN , GA , DK , are equal; and the arcs GAK and GN , or DK , are supplements to one another: Then the sine KM , the tangent GL , the secant CL , of the arc GBK , are respectively equal to the sine AH , the tangent GF , the secant CF of the arc GA .

12. When an arc is greater than 90° , the sine, tangent, secant, of the supplement is to be used.

13. The chord of an arc is equal to twice the co-sine of half the supplemental arc.

Thus AN , the chord of the arc AGN , $= 2CI$, the co-sine of the arc AB , and AB is half of the arc ABK , the supplement of AGN .

14. The versed sine and co-sine together, $HG + CH$ of any arc AG , is equal to the radius; CH being equal to AI .

15. The sines, tangents, secants, or versed sines of similar arcs in different circles, are in the same proportion to one another, as the radii of those circles. (II. 185)

16. The angles of two triangles may be respectively equal, although their sides may be unequal.

Therefore in a triangle among the things given, in order to find the rest, one of them must be a side.

In Trigonometry, the three things given in a triangle must be either,

1st. Two sides and an angle opposite to one of them.

2d. Two angles and a side opposite to one of them.

3d. Two sides and the included angle.

4th. The three sides.

In either case, the other three things may be found by the help of a few Theorems, and a *Triangular Canon*, which is a table where is orderly inserted every degree and minute in a quadrant or arc of 90 degrees; and against them, the measures of the lengths of their corresponding sines, tangents, and secants, estimated in parts of the radius, which is usually supposed to be divided into a number of equal parts, as 10 , 100 , 1000 , 10000 , 100000 , &c.

SECTION II.

Of the Triangular Canon.

PROPOSITION I.

17. To find the lengths of the Chords, Sines, Tangents, and Secants to arcs of a circle of a given radius.

CONSTRUCTION. Through each end of the given radius CD , and at right angles to it (II. 60) draw the lines CF , DG : On C , with the radius CD , describe the quadrantal arc DA , and draw the chord DA .

18. FOR THE CHORDS. Trisect the arc AD (II. 61.), and (by trials) trisect each part; then the arc AD will be divided into 9 equal parts of 10 degrees each; if these arcs are divided each into 10 equal parts, the quadrant will be divided into 90 degrees: But, in this small figure, the divisions to every 10 degrees only are retained, as in (II. 83.)

From D , as a center, with the radius to each division, cut the right line DA ; and it will contain the chords of the several arcs into which the quadrantal arc AD was divided.

For the distances from D to the several divisions of the right line DA , are thus made respectively equal to the distances or chords of the several arcs reckoned from D .

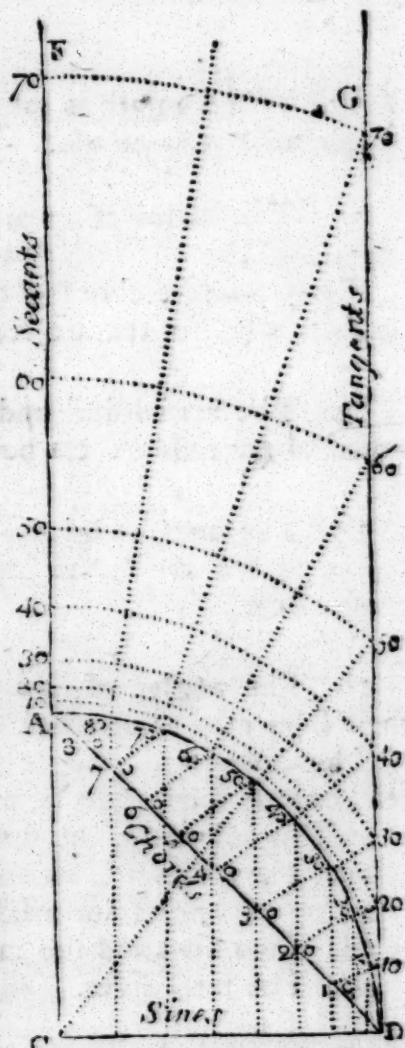
19. FOR THE SINES. Through each of the divisions of the arc AD , draw right lines parallel to the radius AC ; these parallel lines will be the right lines of their respective arcs, and CD will be divided into a line of sines, which are to be numbered from C to D , for the right sines; and from D to C for the versed sines.

For the distance from C to the several divisions of the right line CD , are respectively equal to the sines of the several arcs beginning from A .

20. FOR THE TANGENTS. A ruler on C , and the several divisions of the arc AD , will intersect the line DG ; and the distances from D to the several divisions of DG , will be the lengths of the several tangents.

21. FOR THE SECANTS. From the center C , with radii to the divisions of the tangents DG , cut the line CF ; and the distances from C to the several divisions of CF , will be the lengths of the secants to the several arcs.

For these lengths are made respectively equal to the secants reckoned from C to the several divisions of the tangent DG .



22. If

22. If the figure was so large, that the quadrantal arc could contain every degree and minute of the quadrant, or 5400 equal parts; then the chord, sine, tangent, and secant to each of them could be drawn. Now a scale of equal parts being constructed (II. 81), 1000 of which parts are equal to the radius CD ; then the lengths of the several lines, tangents, and secants may be measured from that scale, and entered in a table called the triangular canon, or the table of sines, tangents, and secants.

But as these measures cannot be taken with sufficient accuracy to serve for the computation to which such tables are applicable; therefore the several lengths have been calculated for a radius divided into a much greater number of equal parts; as is shewn in the following articles.

23. PROP. II.

In any circle the chord of 60 degrees is equal to the radius: and the sine of 30 degrees is equal to half the radius.

DEM. Let the arc CB , or $\angle CAB = 60$ degrees;
and draw the chord CB .

Now since the radii AC and AB are equal; (II. 9)

Therefore $\angle C = \angle B$. (II. 104)

And the $\angle C + \angle B = (180^\circ - (\angle A = 60^\circ)) = 120^\circ$ (II. 96)

Therefore $\angle C$, or $\angle B = (\text{half } 120^\circ; \text{ or } =) 60^\circ = \angle A$.

Consequently $CB = AB = AC$.

From A , draw the radius AE perpendicular to CB .

Then AE bisects the arc CB , and its chord.

(II. 124)

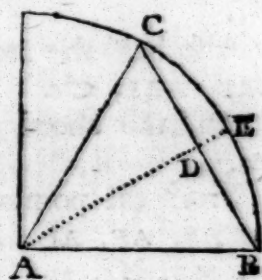
And $CD = \text{sine of (the arc } CE = \text{half of } 60^\circ =) 30^\circ$.

(3)

Consequently CD is equal to half the radius AB .

24. Hence, *Twice the co-sine of 60 degrees is equal to the radius.*

For 30° is the complement of 60° , and twice the sine of 30° is equal to the radius.



25. PROP. III.

To find the sine of one minute of a degree.

It is evident (II. 187), that the less the arc is, the less is the difference between the arc and its sine, or half chord; so that a very small arc, such as that of one minute, may be reckoned to differ from its sine, by so small a quantity, that they may be esteemed as equal; and consequently may be expressed by the same number of such equal parts of which the radius is supposed to contain 1,00000, &c. which is readily found by the following proportion.

As the circumference of the circle in minutes	21600
To the circumf. in equal parts of the radius (II. 196.)	6,283185
So is the arc of one minute	1,
To the corresponding parts of the radius	0,0002908882
So that for the sine of one minute, may be taken	0,0002908882

26. PROP.

26.

PROP. IV.

In a series of arcs in arithmetic progression, the sine of any one of them, taken as a mean, and the sum of the sines of any other two, taken as equidistant extremes, are ever in a constant ratio of radius to twice the co-sine of the common difference of those arcs.

DEM. For in a circumference, the center of which is C , and diameter AB , let there be taken a series of arcs, ARB , ARD , ARE , ARF , ARG , ARH , &c. the common difference of which is the arc BD .

Then drawing the chords AB , AD , AE , AF , AG , AH , &c. their halves will be the sines of half the arcs ARB , ARD , &c. (3)

Also half the arc BD , is the common difference of half the arcs ARB , ARD , ARE , &c. (II. 150)

And the chord AD is twice the co-sine of half the supplemental arc BD . (13)

From the points, D , E , F , G , &c. with the radii DA , EA , FA , GA , &c. cut AE , AF , AG , AH , &c. produced in I , K , L , M , &c. draw ID , KE , LF , MG , &c. and BD , DE , EF , FG , GH , &c.

Then by the first part of the demonstration (II. 189), the following triangles are congruous, namely,

ABD , IED ; ADE , KFE ; AEF , LGF ; AFG , MHG , &c.

Therefore $IE=AB$; $KF=AD$; $LG=AE$; $MH=AF$, &c.

Also the triangles IDA , KEA , LFA , MGA , &c. being each of them isosceles, and their angles respectively equal, are similar to DCA . (II. 167)

Therefore $CA : AD :: (AD : (AI =) AB + AE ::) \frac{1}{2} AD : \frac{1}{2} AB + \frac{1}{2} AE$.
 $:: (AE : (AK =) AD + AF ::) \frac{1}{2} AE : \frac{1}{2} AD + \frac{1}{2} AF$.
 $:: (AF : (AL =) AE + AG ::) \frac{1}{2} AF : \frac{1}{2} AE + \frac{1}{2} AG$.
 &c. &c.

The halves being in the same ratio as the wholes. (II. 150)

27. Consequently, in a series of arcs in arithmetic progression, viz. $\frac{1}{2}ARB$, $\frac{1}{2}ARD$, $\frac{1}{2}ARE$, &c. the common difference of which is half the arc BD , it will be, (II. 164)

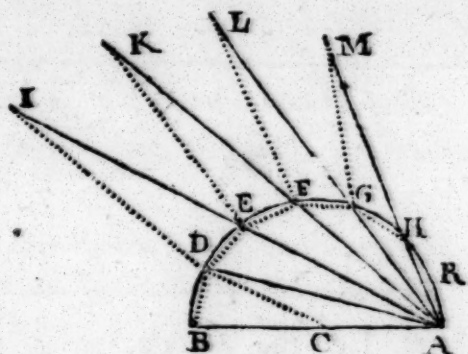
As (AC) radius,

To (AD) twice the co-sine of the common difference;

So is the sine of either arc taken as a mean,

To the sum of the sines of two equidistant extremes.

28. Hence, The sine of either extreme, subtracted from the product of the sine of the mean by twice the co-sine of the common difference, will give the sine of the other extreme. (II. 164)



29. When the common difference of three arcs is 60 degrees; then twice the co-sine of that difference is equal to the radius. (24.)

And with any such three arcs, as 30, 90, 150; or 25, 85, 145; or 20, 80, 140, &c. it will be (27).

$$\begin{array}{l} \text{As } R : (2 \cos. 60 =) R :: s, 90^\circ : (s, 30^\circ + s, 150^\circ =) s, 30^\circ + s, 30^\circ. \\ \quad \quad \quad :: s, 85^\circ : (s, 25^\circ + s, 145^\circ =) s, 25^\circ + s, 35^\circ. \\ \quad \quad \quad :: s, 80^\circ : (s, 20^\circ + s, 140^\circ =) s, 20^\circ + s, 40^\circ. \\ \quad \quad \quad :: s, 75^\circ : (s, 15^\circ + s, 135^\circ =) s, 15^\circ + s, 45^\circ. \\ \quad \quad \quad \text{\&c.} \quad \quad \quad \text{\&c.} \end{array} \left. \vphantom{\begin{array}{l} R \\ R \\ s, 90^\circ \\ s, 85^\circ \\ s, 80^\circ \\ s, 75^\circ \\ \text{\&c.} \end{array}} \right\} (26)$$

Here the first and second terms in the proportions being equal, the third and fourth terms are also equal.

30. Hence, *The sine of an arc greater than 60 degrees, is equal to the sine of an arc as much less than 60 degrees, added to the sine of its difference from 60 degrees.*

Therefore the sines of arcs above 60 degrees are readily obtained from those under 60 degrees.

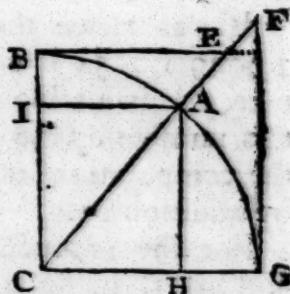
31.

PROP. V.

The right sine of an arc being known, to find its co-sine; and from these to find the tangent, secant, versed sine; and also the co-tangent, co-secant, and co-versed sine.

Let AG be any arc, and let AH be its sine, AI its co-sine; GF the tangent, BE the co-tangent; CF the secant, CE the co-secant; HG the versed sine, BI the co-versed sine.

Now if the sine AH be given, then the co-sine AI or CH, will be known (II. 113:) For $CA^2 - AH^2 = CH^2$. Therefore the square root of the difference between the squares of the radius and sine, will be the co-sine.



32. { Then the versed sine $HG = CG - CH$; and co-versed sine $IB = CB - CI$.
And since the triangles CHA, CGF, CBE, are similar,

33. { Therefore (II. 167) $CH : HA :: CG : GF$, the tangent.

That is, *As the co-sine to the sine, so is radius to the tangent.*

And $CH : CA :: CG : CF$, the secant.

34. { That is, *As the co-sine to the radius, so is radius to the secant.*

And $CI : CA :: CB : CE$, the co-secant.

35. { That is, *As the sine to the radius, so is radius to the co-secant.*

Also $CI : IA :: CB : BE$, the co-tangent.

36. { That is, *As the sine to the co-sine, so is radius to the co-tangent.*

Or $GF : CG :: CB : BE$; that is, *As tangent : rad. :: rad. : co-tangent.*

37. Hence it is evident, that the tangent and co-tangent of an arc of 45° are equal to one another, and to the radius, or sine of 90 degrees.

And as the square of radius is equal to the rectangle of any tangent and its co-tangent,

Therefore $\tan. \times \cot. = \text{rad.} \times \text{rad.}$ Therefore $\tan. : \text{rad.} :: \cot. : \text{rad.}$

(II. 163)

Or the tangents of different arcs are reciprocally as their co-tangents.

38. The

38. The principles by which the lengths of the sines, tangents, secants, &c. may be constructed, being delivered, the following examples are annexed to illustrate this doctrine.

Required the co-sine of one minute.

The sine of 1 minute being	0,0002908882 (25)
Its square is	0,00000008461594
Which subtracted from the square of radius	1,
Leaves	0,99999991538406
Whose square root	0,9999999577 is the

co-sine of 1 minute; or the sine $89^{\circ} 59'$.

Now having the sine and co-sine of 1 minute, the other sines may be found in the following manner. (28)

twice the cos. 1 min. $\times$ sine of 1 m. = sum of the sines of $0'$ & $2'$.
 twice the cos. 1 min. $\times$ sine of 2 m. = sum of the sines of $1'$ & $3'$.
 twice the cos. 1 min. $\times$ sine of 3 m. = sum of the sines of $2'$ & $4'$.
 twice the cos. 1 min. $\times$ sine of 4 m. = sum of the sines of $3'$ & $5'$.
 twice the cos. 1 min. $\times$ sine of 5 m. = sum of the sines of $4'$ & $6'$.

Proceeding thus in a progressive order from each sine to its next, all the sines may be found.

But as twice the co-sine of 1 minute, viz. 1,9999999154 is concerned in each operation, therefore if a table be made of the products of this number by the nine digits, as here annexed, the computations of the sines may be performed by addition only.

For the products by the digits in the given multiplier, being taken from the table, and written in their proper order, will prevent the trouble of multiplication.

And even this operation may be very much shortened, by setting under the right hand place (viz. 4.) of the double co-sine of one minute, the unit place of the sine used as a multiplier, and reversing or placing in a contrary order, all its other figures: then the right-hand figure of each line arising by the multiplication, is to be set under one another; and in these lines, the first figure to be set down, is what arises from the figure standing over the present multiplying one; observing to add what would be carried from the places omitted.

Now if the products of the figures in the multiplier, thus inverted, be taken from the above table of products, it is necessary to remark what number of places will arise from each digit used in the multiplier; then in the products of those digits in the table, take only the like number of places, observing to add 1 to the right-hand place, if the next of the omitted figures exceed 5.

Multipliers.	Products.
1	1,9999999154
2	3,9999998308
3	5,9999997462
4	7,9999996616
5	9,9999995770
6	11,9999994924
7	13,9999994078
8	15,9999993232
9	17,9999992386

Required

Required the sine of two minutes.

The sine of 1 min. placed in an inverted order under the double cos. of 1 min. as in the margin; the right-hand figure 2 stands under the 9 in the 6th decimal place, therefore the first 6 decimal places of the product against 2 in the table, are to be used; but 1 being added, because the 7th place 8, exceeds 5, makes the product 4000000: Also for 9 the next figure in the multiplier, standing under the 5th decimal place, take 17,99999 from the table of products, and 1 being added to the 5th place, because the 6th exceeds 5, make it 18,00000: In like manner the product by 8, adding 1, is 16000, &c. and the sum of these products 0,0005817764 is the sine of 2 minutes, as required.

1.9999999154
2388092000,0
4000000
1800000
16000
1600
160
4
0.0005817764

This kind of operation will be very easily conceived without farther illustration, by comparing the process in this and the following operations, with what has been already said.

Required the sines of 3', 4', 5', and 6 minutes.

For 3 min.	For 4 min.	For 5 min.	For 6 min.
1.9999999154	1.9999999154	1.9999999154	1.9999999154
4677185000,0	5466278000,0	6255361100,0	5044454100,0
10000000	16000000	20000000	20000000
1600000	1400000	2000000	8000000
20000	40000	1200000	1000000
14009	12000	60000	80000
1400	1200	10000	8000
120	80	1000	800
8	10	40	10
		12	
0,0011635528	0,0017453290	0,0023271052	0,0029088810
0,0002908882	0,0005817764	0,0008726645	0,0011635526
0,0008726646	0,0011635526	0,0014544407	0,0017453284

In each example, the sine of an arc which is 2 minutes less than that required, (28) is subtracted.

The sines being made, the tangents, secants, &c. are to be constructed as before shewn. (33, 34)

39. There are many methods by which the triangular canon may be made; but that which is here delivered was chosen as the most easy, the best adapted to this work, and what would give the learner a sufficient notion how these numbers are to be found: For at this time there is no occasion to construct new tables of sines, and rarely to examine those already extant; they having passed through the hands of a great many careful examiners, and for a long time have been received by the learned as a work sufficiently correct.

These lines were first introduced into mathematical computations by *Hipparchus* and *Menelaus*, whose methods of performance were contracted by *Ptolemy*, and afterwards perfected by *Regiomontanus*; and since his time, *Rheticus*, *Clavius*, *Petiscus*, and many other eminent men, have treated largely on this subject, and greatly exemplified the use of this triangular Canon, or Tables; which are now, by way of distinction, called Tables of *natural* sines, tangents, &c. But the greatest improvement ever made in this kind of mathematical learning, was by the Lord *Nepier*, Baron of *Merchiston* in *Scotland*; who, being very fond of such studies, where calculations by the sines, tangents, &c. did frequently occur, judged it would be of vast advantage if these long multiplications and divisions could be avoided; and this he effected by his happy invention of computing by certain numbers, considered as the indices of others (I. 63), which he called *Logarithms*; this was about the year 1614.

The tables now chiefly used in Trigonometrical computations, are the logarithms of those numbers which express the lengths of the sines, tangents, &c. and therefore to distinguish them from the *natural* ones, they are called *Logarithmic* sines, tangents, &c. (or by some artificial sines, &c.) Only those of the logarithmic sines and tangents are annexed to this treatise, because the business of Navigation may be performed by them; neither are these tables carried to more than five places beside the index, that being sufficiently exact for all nautical purposes: But it must be allowed that, for general use, such tables are the most esteemed as consist of most places.

40. These tables are at the end of Book IX. and are so disposed, that each opening of the book contains eight degrees; four of which are numbered at the top, and four at the bottom of the page; and those at the top proceed from left to right, or forward, from 0 degrees to 45; and those at the bottom, from right to left, or backward, from 45 to 90 degrees: To each degree there are four columns, titled sines, co-sines, tangents, co-tangents; and the minutes are in the marginal column of each page, signed with M; those on the left side of the page belong to the degrees which are at the top, and those on the right-hand side, to the degrees which are at the bottom of the page.

41. A sine, tangent, co-sine, co-tangent, to a given number of degrees, is found as follows:

For an arc less than 45 degrees,

Seek the degree at the top, and the minutes in the column signed M at the top; against which, in the column signed at the top with the proposed name, stands the sine, or tangent, &c. required.

But when the arc is greater than 45 degrees,

Seek the degrees at the bottom, the minutes in the column with M at the bottom, and the proposed name at the bottom.

EXAMPLE

EXAMPLE I. Required the logarithmic sine of $28^{\circ} 37'$.

Find 28 deg. at the top of the page ; and in the side column, marked with M at the top, find 37 ; against which, in the column signed at the top with the word sine, stands $9,68029$, the log. sine of $28^{\circ} 37'$, as required.

EXAMPLE II. Required the logarithmic tangent of $67^{\circ} 45'$.

Find 67 deg. at the bottom of the page ; and in the side column, titled M at the bottom, find 45 ; then against this, in the column marked tangent at the bottom, stands $10,38816$, which is the log. tangent required.

42. But when a logarithmic sine or tangent is proposed, to find the degrees and minutes belonging to it, then,

Seek in the table, among the proper columns, for the nearest logarithm to the given one ; and the corresponding degrees and minutes will be found ; observing to reckon them from the top or bottom, according as the column is titled, where the nearest logarithm to the given one is found.

43. It may sometimes happen that a log. sine or log. tang. may be wanted to degrees, minutes, and parts of minutes ; which may be thus found.

Take the difference between the logs. of the degrees and minutes next less, and those next greater than the given number.

Then for $\frac{1}{4}$, take a quarter of this difference ; for $\frac{1}{3}$, take a third ; for $\frac{1}{2}$, take a half ; for $\frac{2}{3}$ take two thirds ; for $\frac{3}{4}$, take three quarters, &c.

Add the parts taken of this difference to the right hand figures of the log. belonging to the deg. and min. next less, and the sum will be the log. to the deg. min. and parts proposed.

EXAMPLE I. Required the log. tang. to $60^{\circ} 56\frac{1}{2}'$.

Log. tang. $60^{\circ} 57'$ is

10,25535

Log. tang. $60^{\circ} 56'$ is

10,25506

The diff. is

29

Its half is

14

Add it to tang. $60^{\circ} 56'$

10,25506

Gives tang. $60^{\circ} 56\frac{1}{2}'$

10,25520

EXAMPLE II. Required the log. sine to $32^{\circ} 15\frac{1}{4}'$.

Log. sine $32^{\circ} 16'$ is

9,72743

Log. sine $32^{\circ} 15'$ is

9,72723

The diff. is

20

Its three fourths is

15

Add it to sine $32^{\circ} 15'$

9,72723

Gives sine $32^{\circ} 15\frac{1}{4}'$

9,72738

In most cases the work may be done by inspection.

44. And if a given log. sine or log. tangent falls between those in the tables : then the degrees and minutes answering may be reckoned $\frac{1}{4}$, or $\frac{1}{3}$, or $\frac{1}{2}$, &c. minutes more than those belonging to the nearest less log. in the tables, according as its difference from the given one is $\frac{1}{4}$, or $\frac{1}{3}$, or $\frac{1}{2}$, &c. of the difference between the logarithm next greater and next less than the given log.

SECTION III.

Of the Solution of Plain Triangles.

45.

PROBLEM I.

In any plain triangle, ABC, if among the things given there be a side and its opposite angle, to find the rest.

Then say, *As a given side,* (AB)
To the sine of its opposite angle; (C)
So is another given side, (AC)
To the sine of its opposite angle. (B)

Therefore, to find an angle, begin with a side opposite to a known angle.

Also, *As the sine of a given angle,* (B)
To its opposite side; (AC)
So is the sine of another given angle, (C)
To its opposite side. (AB)

Therefore, to find a side, begin with an angle opposite to a known side.

DEM. Take $BD = CF =$ radius of the tables.

Draw DE, AH, FG, each perpendicular to BC.

(II. 59)

Then DE and FG are the sines of the angles B and C. (3)

Now the triangles BDE, BAH, are similar, and so are the triangles CFG, CAH.

Therefore $BD : DE :: BA : AH$. (II. 167)

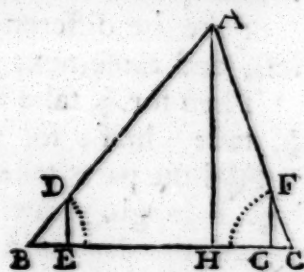
And $(CF =) BD : FG :: CA : AH$. (II. 167)

But $(BD \times AH =) DE \times BA = FG \times CA$.

(II. 162)

Therefore $DE : CA :: FG : BA$. Or, $s, \angle B : AC :: s, \angle C : BA$. (II. 163)

SCHOL. Or, by circumscribing the triangle with a circle, it will readily appear, that the half of each side is the sign of its opposite angle. And halves have the same proportion as the wholes.



46.

PROBLEM II.

In a right-angled plane triangle, ABC, if the two sides containing the right angle B are known, to find the rest.

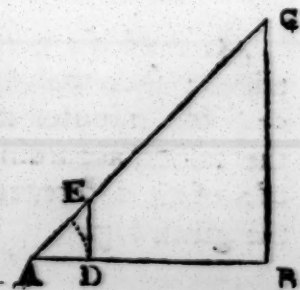
Then, *As one of the known sides,* (AB)
To the radius of the tables (or tangent of 45°); (AD)
So is the other known side, (BC)
To the tangent of its opposite angle. (DE)

DEM. Take AD = radius of the tables.

Then DE, perpendicular to AD, is the tangent of the angle A. (4)

And the triangles ADE, ABC, are similar.

Therefore $AB : AD :: BC : DE$. (II. 167)



47. PRO.

47.

PROBLEM. III.

In any two quantities, their half difference added to their half sum, gives the greater.

The half diff. subtracted from the half sum, gives the less.

And if half the sum be taken from the greater, the remainder will be the half difference of those quantities.

DEM. Let AB be the greater, and BC the less, of two quantities.

Take AD=BC; then BD is their difference.

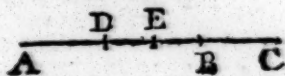
Bisect DB in E; then DE=EB, is the half diff.

And AD+DE=BC+BE (II. 47); therefore AE is the half sum.

Now AE+EB=AB, is the greater.

And AE-ED=(AD=)BC, is the less.

Also AB-AE=BE, is the half diff.



48.

PROBLEM IV.

In any plane triangle, ABC; if the three things known, be two sides, AC, CB, and their contained angle c, to find the rest.

Find the sum and difference of the given sides.

Take half the given angle from 90 degrees, and there remains half the sum of the unknown angles. Then say,

As the sum of the given sides,

To the difference of those sides;

So is the tangent of half the sum of the unknown angles,

To the tangent of half the difference of those angles,

Add the half difference of the angles to the half sum, and it will give the greater angle=B.

Subtract the half difference of the angles from the half sum, and it will give lesser angle=A.

$$AC+CB$$

$$AC-CB$$

$$t. \frac{1}{2}B+A$$

$$t. \frac{1}{2}B-A$$

DEM. On c, with the radius CB, describe a circle, cutting AC, produced, in E and D; draw EB, and BD; also draw DF parallel to EB.

Then AE=AC+CB, is the sum of the sides.

And AD=AC-CB, is the difference of the sides.

Now $\angle CDB=\angle CBD$. (II. 104)

And $(\angle CDB+\angle CBD)=2\angle CDB=\angle CBA+\angle A$. (II. 98)

Therefore $\frac{1}{2}\angle CBA+\angle A=\angle CDB$, is half the sum of the unknown angles,

And BE (II. 123) is the tangent of CDB, to the radius DB. (4)

Also $(\angle CBA-\angle CBD)=\angle DBA=\frac{1}{2}\angle CBA-\frac{1}{2}\angle A$. (47)

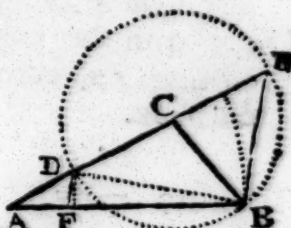
Therefore $\frac{1}{2}\angle CBA-\angle A=\angle DBA$, is half the difference of the unknown angles.

And DF is the tangent of DBA, to the radius DB. (4)

Now the triangles AEB, ADF, are similar, DF being parallel to EB.

Therefore AE : AD :: BE : DF. (II. 167)

Or AC+CB : AC-CB :: t. $\frac{1}{2}\angle CBA+\angle A$: t. $\frac{1}{2}\angle CBA-\angle A$.



49.

PROBLEM V.

In a plane triangle, ABC, if the three sides are known, and the angles required.

From the greatest angle, B, suppose a line BD drawn perpendicular to its opposite side, or base, dividing it into two segments, AD, CD, and the given triangle into two right-angled triangles. ADB, CDB: Then say,

As the base, or sum of the segments,
Is to the sum of the other two sides;
So is the difference of those sides,
To the difference of the segments of the base.

AC
AB + BC
AB - BC
AD - DC

Add half the difference of the segments to half the base, gives the greater segment AD. (47)

Subtract half the difference of the segments from half the base, there remains the lesser segment DC. (47)

Then, in each of the triangles, ADB, CDB, there will be known two sides, and a right angle opposite to one of them; therefore the angles will be found by Problem I. (45)

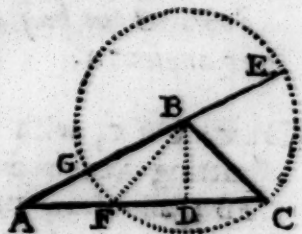
When two of the given sides are equal; then a line drawn from the included angle, perpendicular to the other side, bisects the side. (II. 103)

And the angles being found in one of the right-angled triangles, will also give the angles of the other.

DEM. of the foregoing proportion.

In the triangle ABC, the line BD, perpendicular to AC, divides AC into the segments AD, DC.

On B with the radius BC, describe a circle GCE, cutting AB, continued, in G, E; and AC in F; draw BF.



Then $DC = DF$.

(II. 103)

Now $AC (=AD + DC)$ is the sum of the segments.

And $AF (=AD - FD)$ is the difference of the segments.

Also $AE (=AB + BC)$ is the sum of the other sides.

And $AG (=AB - BC)$ is the difference of those sides.

But $AC \times AF = AE \times AG$.

(II. 172)

Therefore $AC : AE :: AG : AF$.

(II. 163)

Or $AC : AB + BC :: AB - BC : AD - DC$; or $AD - DF$.

55.

PROBLEM VI.

In any plane triangle, ABC, the three sides being known, to find either of the angles.

Put E and F for the sides including the angle sought.

G for the side opposite to that angle.

D for the difference between the sides E and F.

Find half the sum of G and D.

And half the difference of G and D.

Then write these four logarithms under one another, namely,

The Arithmetical complement of the logarithm of E;

(I. 88.)

The Arithmetical complement of the logarithm of F;

The logarithm of the aforesaid half sum of G and D;

The logarithm of the aforesaid half difference of G and D.

Add them together, take half their sum; which seek among the log. sines.

And the degrees and minutes answering, being doubled, will give the measure of the angle sought.

DEM. In the triangle, ABC, let A be the angle sought.

Take AH = AB, draw BH; and through K, the middle of BH, draw AP, which bisects the angle A, and is perpendicular to BH. (II. 103)

Through K draw KL, KQ, parallel, to BC, AC; which will bisect

HC, BC, in L and I; then KL = IC, KI = LC.

(II. 28, 163)

And the difference between AC and AB is HC = D; then KI = $\frac{1}{2}$ D.

From I, with the radius IK, describe a circle cutting AP, BH, KQ, BC, in P, O, Q, M, N, and join CQ; now IQ = IK = LC = LH; therefore KQ = HC, and CQ = KH, as the triangles CQI, KHL, are congruous. (II. 99)

Therefore CQ parallel to KH (II. 28.) being produced, will meet AP at right angles (II. 53), in the point P, by the reverse of (II. 130).

Then PQ = KO, as the triangles KQP, QOK, are congruous. (II. 95, 100)

Now BM = (BI + IM = $\frac{1}{2}$ BC + $\frac{1}{2}$ HC =) $\frac{1}{2}$ G + D: And BN = $\frac{1}{2}$ G - D.

Also BO = CP: For BK = (KH =) CQ, and KO = PQ.

Let Ar = radius of the tables; then rn (parallel to BH) = sine of $\frac{1}{2}$ ∠A. (3)

Then the triangles Anr, AKB, APC, are similar.

And Ar : rn :: AB : BK; also Ar : rn :: AC : (CP =) BO. (II. 167)

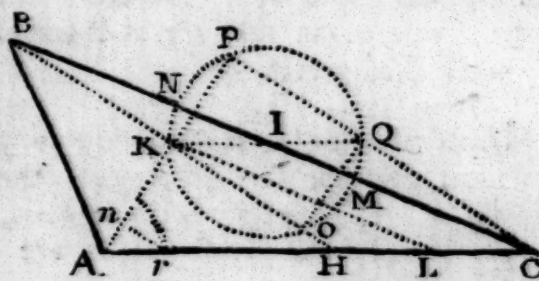
Therefore Ar² : rn² :: AC × AB : (BK × BO =) BM × BN. (II, 161, 172)

Or (sq. rad. = R² : sq. sine $\frac{1}{2}$ ∠A :: AC × AB : $\frac{1}{2}$ G + D × $\frac{1}{2}$ G - D.

Therefore the square of the sine $\frac{1}{2}$ ∠A = $\frac{\frac{1}{2}G + D \times \frac{1}{2}G - D}{AC \times AB} \times R^2$. (II. 164)

Therefore the sine $\frac{1}{2}$ ∠A = $\sqrt{\frac{\frac{1}{2}G + D \times \frac{1}{2}G - D}{E \times F}}$; (II. 113)

And R, the radius of the tables, being 1.



$$\text{Or Log. s, } \frac{1}{2} \angle A = \frac{\text{Log. } \frac{1}{2} G + D + \text{Log. } \frac{1}{2} G - D - \text{Log. E.} - \text{Log. F.}}{2}$$

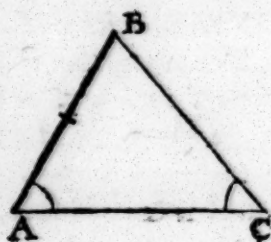
$$\text{Or Log. s, } \frac{1}{2} \angle A = \frac{l. E + l. F + L. \frac{1}{2} G + D + L. \frac{1}{2} G - D}{2} \quad \left. \begin{array}{l} 85 \\ 86 \\ 91 \end{array} \right\} \text{I.}$$

Where L. stands for logarithm, and l. for Arith. comp. of the logarithm,

51. Every possible case in plane Trigonometry may be readily solved by the preceding Problems, observing the following Precepts.

I. Make a rough draught of the triangle, and put the letters A, B, C, at the angles.

II. Let such parts of this triangle be marked, as represent the things which are given in the question. Thus, mark a given side with a scratch across it; and a given angle by a little crooked line; as in the figure; where the side AB, and the angles A and C, are marked as given.



III. If two angles are known, the third is always known.

For if one angle is 90 degrees, the other given angle (which (II. 97) will be acute) taken from 90 degrees, leaves the third angle.

And if both the given angles are oblique; their sum taken from 180 degrees, gives the other angle. (II. 96)

IV. Compare the given things together, and determine to which Problem the question proposed belongs.

V. Then according as the Problem directs, perform the preparatory work; and write down, under one another, in four lines, (or more if necessary), the *literal stating*; expressing each angle by a letter, or by three; each line by two letters; and the sums, or differences of lines, by proper marks.

VI. Against such terms as are known, write their numeral value, as given in the question, or as found in the preparatory work; and against these numbers write their logarithms; those for the lines being found (by I. 81) in the table of the logarithms of numbers; and those for the angles, found (by 41) in the table of logarithmic sines and tangents: Observing that an Arithmetical complement (see I. 88) is always used in the first term: And that when an angle is greater than 90 degrees, its supplement is used.

VII. Add these logarithms together, and seek the sum (I. 81) in the log. numbers, when a line is wanted; or (42) in the log. sines or tangents, when an angle is wanted. Then the number or degree, answering to that logarithm which is the nearest to the said sum, will be the thing required.

A SYNOPSIS.

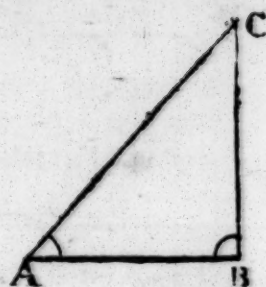
Of the Rules in Plane Trigonometry.

52.

PROB.	Given.	Required.	SOLUTION.
	All the angles and one side.	Either of the other sides	Since two angles are known, the third is known. And, As fin. of $\angle$ opp. to side given, is to that opp. side; So fin. of another angle, to its opp. side.
I. see art. 45	Two sides and an $\angle$ oppof. to one side.	The angle oppof. to the other given side.	As one given side is to the sine of its opp. angle; So is the other given side, to the sine of its opp. angle. Then two angles being known, the third is known. And the other side is found as before.
II. art. 46	Two sides and the included right $\angle$.	Either of the other angles.	As one of the given sides, is to the radius; So is the other given side, to the tangent of its opp. $\angle$. Then two $\angle$ s being known, the third is known. The other side is found by opp. sides and $\angle$ s.
III. art. 48	Two sides and the included oblique angle.	The other angles.	Find the sum and diff. of the given sides. Take $\frac{1}{2}$ given $\angle$ from 90° leaves $\frac{1}{2}$ sum of the other $\angle$ s. Then, As sum sides, is to diff. of sides; So tan. $\frac{1}{2}$ sum other $\angle$ s, to tan. $\frac{1}{2}$ diff. those $\angle$ s. The $\frac{1}{2}$ sum $\angle$ s $\left\{ \begin{array}{l} + \\ - \end{array} \right\} \frac{1}{2}$ diff. $\angle$ s, gives $\left\{ \begin{array}{l} \text{greater } \angle. \\ \text{lesser } \angle. \end{array} \right.$ Find the other side by opp. sides and angles.
IV. art. 49	The three sides.	All the angles.	Draw a line perpend. to the greatest side, from the opp. $\angle$, dividing that side into two parts. Then, As the longest side is to sum other two sides; So is the diff. those sides, to the diff. p ^{ts} of longest. Then $\frac{1}{2}$ long. side $\left\{ \begin{array}{l} + \\ - \end{array} \right\} \frac{1}{2}$ diff. p ^{ts} gives the $\left\{ \begin{array}{l} \text{great.} \\ \text{lesser} \end{array} \right\}$ part Now the said perp. cuts the triangle into 2 right $\angle$ d ones. In both, are known the Hyp. a Leg. and the right $\angle$. The angles are found by Problem I.
V. art. 50	The three sides.	Either angle.	Having chose which angle to find; call the sides including that angle x and y . The side opp. that $\angle$, call g . Put d for the difference between x and y . Find the half sum, and half diff. of g and d . Then write these four Logs. under one another; <i>Viz.</i> $\left\{ \begin{array}{l} \text{The Ar. Co. Log. of } x, \text{ The Ar. Co. Log. of } y, \\ \text{The Log. of } \frac{1}{2} \text{ sum, And the Log. of } \frac{1}{2} \text{ difference.} \end{array} \right.$ Add the four logs. together, take half their sum. Seek it among the log. fines; and the corresponding deg. and min. doubled, is the angle sought.

53. EXAMPLE I. *In the plane Triangle ABC.*Given $AB = 195$ Poles. $\angle B = 90^\circ 00'$ $\angle A = 47^\circ 55'$

Required the other parts.



FOR THE LINEAR SOLUTION.

1st. Draw AB equal to 195 poles, taken from a scale of equal parts.2d. From B , draw BC , making with AB an angle of 90° . (II. 84)3d. From A , draw AC , making with AB an angle of $47^\circ 55'$; and meeting BC in the point C .Then is the triangle ABC such, the parts of which correspond with the things given; and the sides CA , CB , being applied to the scale that AB was taken from, their measures will be found, viz. $AC = 291$; and $BC = 216$.

FOR THE NUMERAL SOLUTION, OR COMPUTATION.

Since two angles are known; Therefore,

From	$90^\circ 00'$	
Take	$47^\circ 55' = \angle A$	
Remains	$42^\circ 05' = \angle C$	

Now in this triangle, there are known all the angles and one side; therefore among the known things, there is a side and its opposite angle; which belongs to the first problem.

Then to find the side AC , begin with the angle c opposite AB .

As the sine of $\angle C$,
 To the opposite side AB ;
 So the sine of the $\angle B$,
 To the opposite side AC .

Or thus, As s, $\angle C = 42^\circ 05'$	0,17379	Ar. Co.
To $AB = 195$ po.	2,29003	
So s, $\angle B = 90^\circ 00'$	10,00000	
To $AC = 291$ po.	2,46382	

And to find the side BC , begin with the angle c opposite AB .

As the sine of the $\angle C$,
 To the opposite side AB ;
 So the sine of the $\angle A$,
 To the opposite side BC .

Or thus, As s, $\angle C = 42^\circ 05'$	0,17379	Ar. Co.
To $AB = 195$ po.	2,29003	
So s, $\angle A = 47^\circ 55'$	9,87050	
To $BC = 216$ po.	2,33432	

So that AC is 291 poles, and BC is 216 poles.The letters *Ar. Co.* standing on the right of the first line, signify the arithmetical complement of the log. sine of $42^\circ 05'$. (I. 88)

54.

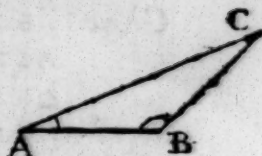
EXAMPLE II. In the plane Triangle ABC.

Given $AB=117$ miles.

$\angle B=134^{\circ} 46'$

$\angle A=22^{\circ} 37'$

Required the other parts.



FOR THE LINEAR SOLUTION, OR CONSTRUCTION.

Make $AB=117$ equal parts; at A make an angle $=22^{\circ} 37'$ (II. 84); and at B make an angle of $134^{\circ} 46'$; then the lines which make with AB those angles, will meet in C, and form the triangle proposed. And the measure of BC will be 117, and of AC 216.

BY COMPUTATION. See art. 45.

Since two angles are known,

Namely, $\angle B=134^{\circ} 46'$	Now from $180^{\circ} 00'$	And from $180^{\circ} 00'$
$\angle A=22^{\circ} 37'$	Take $157^{\circ} 23'$	Take $134^{\circ} 46'$
Their sum $=157^{\circ} 23'$	Leaves $\angle C=22^{\circ} 37'$	The sup ^t $\angle B=45^{\circ} 14'$

Since the angle $c=\angle A$
Therefore $BC=AB$

(II. 104)

To find the side AC.

As s, $\angle C=22^{\circ} 37'$	0,41503	Ar. Co.
To $AB=117$	M. 2,06819	
So s, $\angle B=134^{\circ} 46'$	9,85125	sup.
To $AC=216$	M. 2,33447	

55.

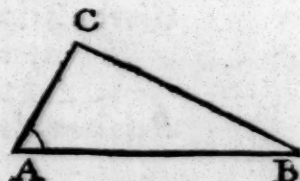
EXAMPLE III. In the plane Triangle ABC.

Given $AB=408$ yards.

$\angle B=22^{\circ} 37'$

$\angle A=58^{\circ} 07'$

Required the other parts.



CONSTRUCTION.

Make $AB=408$ yards, or equal parts; make the angle $A=58^{\circ} 07'$; and the $\angle B=22^{\circ} 37'$; then the lines forming these angles will meet in C; and the measure of AC is 159 yards, and of BC is 351.

COMPUTATION. See art. 45.

Two angles being known, viz. $\angle A=58^{\circ} 07'$	From $180^{\circ} 00'$
$\angle B=22^{\circ} 37'$	Take $80^{\circ} 44'$
Their sum $=80^{\circ} 44'$	Leaves $99^{\circ} 16'=\angle C$.

To find the side AC.

As s, $\angle C=99^{\circ} 16'$	0,00570	Ar. Co.
To $AB=408$ Y.	2,61066	
So s, $\angle B=22^{\circ} 37'$	9,58497	
To $AC=159$ Y.	2,20133	

To find the side BC.

As s, $\angle C=99^{\circ} 16'$	0,00570	Ar. Co.
To $AB=408$ Y.	2,61066	
So s, $\angle A=58^{\circ} 07'$	9,92897	
To $BC=351$ Y.	2,54533	

In these operations the supplement of the angle c is used.

(12)

56. Ex-

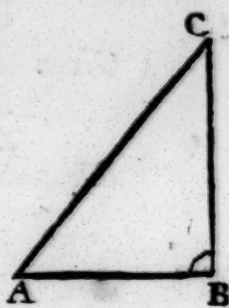
56. EXAMPLE IV. In the plane Triangle ABC.

Given $AB=195$ } Furlongs.
 $AC=291$ }
 $\angle B=90^\circ 00'$.

Required the rest.

CONSTRUCTION.

Make $AB=195$ equal parts; draw BC , making an angle at $B=90^\circ 00'$. From A with 291 equal parts cut BC in c , and draw AC .



Then the $\angle A$ measured on the scale of chords will be about 48 degrees, and $\angle C$ about 42° : Also BC , on the equal parts, measures about 216.

COMPUTATION.

Here being two sides, and an angle opposite to one of them, the solution falls under problem the first. See art. 45.

To find the angle c .

As $AC=291$ F. 7,53611 Ar. Co.
 To s , $\angle B=90^\circ 00'$ 10,00000
 So $AB=195$ F. 2,29003

To s , $\angle C=42^\circ 05'$ 9,82614

From $90^\circ 00'$
 Take $42^\circ 05' = \angle c$.

Leaves $47^\circ 55' = \angle A$.

To find the side BC .

As s , $\angle B=90^\circ 00'$ 10,00000
 To $AC=291$ F. 2,46389
 So s , $\angle A=47^\circ 55'$ 9,87050

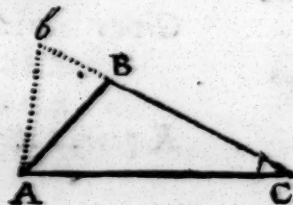
To $BC=216$ F. 2,33439

Here the sine of $90^\circ 00'$ or radius being the first term, its Arith. Comp. being 0, is not taken.

57. EXAMPLE V. In the plane Triangle ABC.

Given $AC=216$ } Yards.
 $AB=117$ }
 $\angle C=22^\circ 37'$.

Required the rest.



CONSTRUCTION.

Make $AC=216$ yards; the $\angle C=22^\circ 37'$; and draw CB : Then from A , with 117 yards, cut CB in b or in B ; and either of the triangles ACb or ACB will answer the conditions proposed: But the triangle to be used is generally determined by some circumstances in the question it belongs to. Thus if the angle opposite to AC is to be obtuse, the triangle is ABC .

COMPUTATION.

The solution belongs to problem the first. See art. 45.

To find the angle B .

As $AB=117$ Y.
 To s , $\angle C=22^\circ 37'$
 So $AC=216$ Y.

7,93181
 9,58497
 2,33445

From $180^\circ 00'$
 Take $157^\circ 23' = \angle c + \angle A$.

Leaves $22^\circ 37' = \angle B$.

To s , $\angle B=134^\circ 46'$

9,85123

$\angle c + \angle B = 157^\circ 23'$

And as $\angle A = \angle c$.
 Therefore $BC=AB$.

(II. 104)

If the angle required be obtuse, subtract the deg. and min. corresponding to the fourth log. from 180; the remainder is the $\angle B$. For the fourth log. gives the $\angle b$, which is the supplement to the angle B . (II. 104, 95).

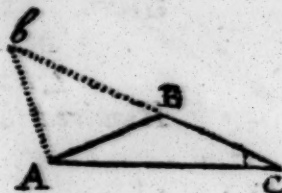
58. Ex.

58.

EXAMPLE VI. In the plane Triangle ABC.

Given $AC=408$ } Fathoms.
 $AB=159$ }
 $\angle C=22^\circ 37'$.

Required the rest.



CONSTRUCTION.

Make $AC=408$ fathoms; the $\angle C=22^\circ 37'$; and draw cb ; from A , with 159 fathoms, cut cb in b , or in B , and draw Ab or AB :

Then if the angle opposite to AC is to be acute, the triangle AcB is that which is required; but if the angle is to be obtuse, ACB is the triangle sought.

COMPUTATION. See art. 45.

Here being a side and its opposite angle known, the solution falls under problem the first; the $\angle B$ is to be obtuse.

To find the angle B obtuse.

As $AB=159$ F. 7,79860
 To $s, \angle C=22^\circ 37'$ 9,58497
 So $AC=408$ F. 2,61066

To $s, \angle B=99^\circ 19'$ 9,99423

$\angle C + \angle B = 121 \quad 56$
 Taken from $180 \quad 00$

Leaves $\angle A=58 \quad 04$

To find BC .

As $s, \angle C=22^\circ 37'$ 0,41503
 To $AB=159$ F. 2,20140
 So $s, \angle A=58^\circ 04'$ 9,92874

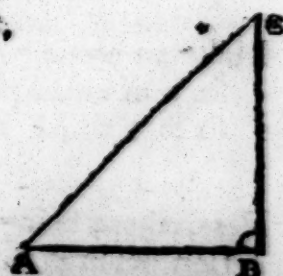
To $BC=350,9$ F. 2,54517

59.

EXAMPLE VII. In the plane Triangle ABC,

Given $AB=195$ } Furlongs.
 $BC=216$ }
 $\angle B=90^\circ 00'$.

Required the rest.



CONSTRUCTION.

Make the angle $ABC=90^\circ$; take $BA=195$ equal parts, and $BC=216$; and draw AC ; then ABC is the triangle proposed; where the parts required may be measured by the proper scales.

COMPUTATION. See art. 46.

As two sides and the contained right angle are known, the solution belongs to problem the second.

To find the angle A .

As $AB=195$ F. 7,70997
 To $\text{rad. or tang. } 45^\circ 00'$ 10,00000
 So $BC=216$ F. 2,33445

To $t. \angle A=47^\circ 55'$ 10,04442

90 00

$\angle C=42 \quad 05$

To find AC .

As $s, \angle A=47^\circ 55'$ 0,12950
 To $BC=216$ F. 2,33445
 So $s, \angle B=90^\circ 00'$ 10,00000

To $AC=291$ F. 2,46395

60.

EXAMPLE VIII. *In the plane Triangle ABC.*

Given $AB=117$ } Yards.
 $BC=117$ }
 $\angle B=134^\circ 46'$.
 Required the rest.



CONSTRUCTION.

Make the $\angle ABC=134^\circ 46'$; take BA and BC, each equal to 117 equal parts, from the same scale, and draw AC; then is the triangle ABC equal to that proposed; and the parts required, measured on their proper scales, will give their values.

COMPUTATION. See art. 48.

Now as AB and BC are equal; therefore the angles A and C are also equal.

From $180^\circ 00'$
 Take $134^\circ 46' = \angle B$.
 Leaves $45^\circ 14' = \angle A + \angle C$.
 The half $22^\circ 37' = \angle A = \angle C$.

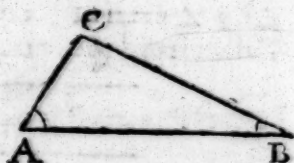
To find AC.

As s, $\angle A=22^\circ 37'$	0,41503
To $BC=117$ Y.	2,06818
So s, $\angle B=134^\circ 46'$	9,85125
To $AC=216$ Y.	2,33446

61.

EXAMPLE IX. *In the plane Triangle ABC.*

Given $AB=408$ } Yards.
 $AC=159$ }
 $\angle A=58^\circ 07'$.
 Required the rest.



CONSTRUCTION.

Make an angle CAB= $58^\circ 07'$; take AC=159, AB=408, from the same scale of equal parts; and draw CB; then will the triangle ACB be equal to that which was proposed.

COMPUTATION.

Here, there being two sides and their contained angle known, the solution belongs to art. 48.

$AB=408$
 $AC=159$
 $AB+AC=567$ =sum of sides.
 $AB-AC=249$ =diff. of sides.

The half of $58^\circ 07'$
 Is $29^\circ 03\frac{1}{2}'$, which
 Taken from $90^\circ 00'$

Leaves $60^\circ 56\frac{1}{2}' = \frac{1}{2}\angle C + \frac{1}{2}\angle B$.

To find the angles.

As $AB+AC=567$ 7,24643
 To $AB-AC=249$ 2,39620
 So t. $\frac{1}{2}\angle C + \angle B=60^\circ 56\frac{1}{2}'$ (See 43) 10,25520
 To t. $\frac{1}{2}\angle C - \angle B=38^\circ 19\frac{1}{2}'$ 9,09781
 Then (47) $99^\circ 16' = \angle C$. (Il. 105)
 And $22^\circ 37' = \angle B$.

To find BC.

As s, $\angle C=99^\circ 16'$	0,00570
To $AB=408$ Y.	2,61066
So s, $\angle A=58^\circ 07'$	9,92897
To $BC=351$ Y.	2,54533

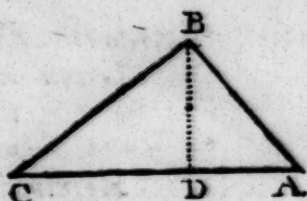
62. EXAMPLE X. In the plane Triangle ABC.

Given $AB=195$

$BC=216$

$AC=291$

Required the angles.



CONSTRUCTION.

Make $CA=291$ equal parts; from C , with 216 , describe an arc B ; from A with 195 cut the arc B in B ; draw BC , BA , and the triangle is constructed, then the angles may be measured by the help of a scale of chords.

COMPUTATION.

The three sides being given, the solution falls under either Problem V. or Problem VI. But that the use of these Problems may be sufficiently illustrated, the solutions according to both of them are here annexed.

Solution by Problem V. (49)

From the angle B , draw BD perpendicular to CA , which will be divided into the segments CD , DA , the sum of which AC is known.

Now $BC=216$

And $AB=195$

$BC+AB=411$ =sum of the sides.

$BC-AB=21$ =diff. of the sides.

Now the half of 291 is $145,5$
And the half of $29,66$ is $14,83$

Therefore (47) the sum $160,33=CD$; or $CD=160,3$

the difference $130,67=AD$; or $AD=130,7$.

Then in the triangle CDB .

As $BC=216$ — $7,66555$

To s , $\angle CDB=90^\circ 00'$ — $10,00000$

So $CD=160,3$ — $2,20493$

To s , $\angle CBD=47^\circ 55'$ — $9,87048$

Wh. taken from $90^\circ 00'$

Leaves $\angle C=42^\circ 05'$

To find the diff. of the segments.

As $AC=291$ — $7,53611$

To $BC+AB=411$ — $2,61384$

So $BC-AB=21$ — $1,32222$

To $CD-AD=29,66$ — $1,47217$

And in the triangle ADB .

As $AB=195$ — $7,70997$

To s , $\angle ADB=90^\circ 00'$ — $10,00000$

So $AD=130,7$ — $2,11628$

To s , $\angle ABD=42^\circ 05'$ — $9,82625$

Wh. taken from $90^\circ 00'$

Leaves $\angle A=47^\circ 55'$, & $\angle B=90^\circ 00'$.

Solution by Problem VI. (50)

To find the angle c .

Put $E=291=AC$

$F=216=BC$

$D=75=E-F$

$G=195=AB$

$2)270)135=\frac{1}{2}G+D$

$2)120)60=\frac{1}{2}G-D$

Then, to Ar. Co. log. $E=291$ — $7,53611$

Add Ar. Co. log. $F=216$ — $7,66555$

And the log. $\frac{1}{2}G+D=135$ — $2,13013$

Also the log. $\frac{1}{2}G-D=60$ — $1,77815$

The half of this sum $2)19,11014$

Is the log. sine of $21^\circ 02' \frac{1}{2}(43)$ — $9,55507$

Which doubled, gives $42^\circ 05'=\angle c$.

The angle c being known, the other angles may be found by Prob. I.

63. Ex.

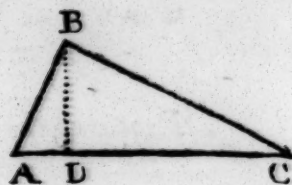
63. EXAMPLE XI. In the plane Triangle ABC.

Given $AC=408$

$BC=351$

$AB=159$

Required the angles.



CONSTRUCTION.

The construction and mensuration is performed as in the last EXAMPLE.

COMPUTATION BY PROB. V. art. 49.

From the $\angle B$ draw the perpendicular BD ; and find the segments AD , DC ; which may be done without logarithms.

Thus $BC=351$ Then (I. 46), As $408 : 510 :: 192 : 240 = DC - DA$.
 $AB=159$ For $192 \times 510 = 97920$; which divided by 408 gives 240 .

$BC + AB = 510$ Now half of $408 = 204$; and half of 240 is 120 .

$BC - AB = 192$ Then $204 + 120 = 324 = DC$; and $204 - 120 = 84 = DA$.

In the triangle ADB .

As $AB=159$
 To $s, \angle D=90^\circ 00'$
 So $AD=84$

To $s, \angle ABD=31^\circ 53'$

And $\angle A = 58^\circ 07'$

7,79860
 10,00000
 1,92428

9,72288

In the triangle BDC .

As $BC=351$
 To $s, \angle D=90^\circ 00'$
 So $DC=324$

To $s, \angle CBD=67^\circ 23'$

And $\angle C = 22^\circ 37'$

7,45469
 10,00000
 2,51054

9,96523

Then $\angle ABD + \angle CBD = \angle ABC = 99^\circ 16'$

COMPUTATION BY PROB. VI. art. 50.

To find the angle C .

Put $E=408=AC$

$F=351=BC$

$D=57=E-F$

$C=159=AB$

$2) 216 (108 = \frac{1}{2}G + D$

$2) 102 (51 = \frac{1}{2}G - D$

Then, to Ar. Co. log. $E = 408$

Add Ar. Co. log. $F = 351$

And the log. $\frac{1}{2}G + D = 108$

Also the log. $\frac{1}{2}G - D = 51$

The half of this sum $2) 18,58502$

Is the log. sine of $11^\circ 18' \frac{1}{2}$ $9,29251$

Which doubled, is $22^\circ 37' = \angle C$.

Now the angle C being known, the other angles may be found by Prob. I. But for a farther illustration of Prob. VI. the work for another angle is here repeated.

To find the angle B .

Put $E=351=BC$

$F=159=AB$

$D=192=E-F$

$C=408=AC$

$2) 600 (300 = \frac{1}{2}G + D$

$2) 216 (108 = \frac{1}{2}G - D$

Then, to Ar. Co. log. $E = 351$

Add Ar. Co. log. $F = 159$

And the log. $\frac{1}{2}G + D = 300$

Also the log. $\frac{1}{2}G - D = 108$

The half of this sum $2) 19,76383$

Is the log. sine of $49^\circ 38'$ $9,88191$

Which doubled, is $99^\circ 16' = \angle B$.

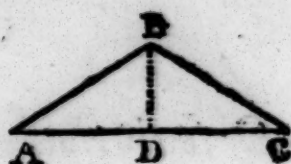
64. Ex.

64.

EXAMPLE XII. *In the plane Triangle ABC.*

Given $AB=117$
 $BC=117$ } Miles.
 $AC=216$

Required the angles.



CONSTRUCTION.

The construction of this triangle, and the measuring of the angles, is performed as in the Xth and XIth Examples:

COMPUTATION.

In the triangle ABC, as $AB=BC$; therefore the angles A and c are equal (II. 104); and the perpendicular BD bisects the side AC; so that the right angled triangles ADB, CDB, are congruous; consequently, the angles being found in one triangle, will give those of the other.

Now in the triangle ADB, the side $AB=117$; the side $AD=\frac{1}{2}AC$, is 108; and the $\angle D$ is $90^\circ 00'$: Here, therefore, being a side and its opposite angle given, the solution belongs to Prob. I.

To find the angle ABD.

As $AB=117$

7.93181

To s, $\angle D=90^\circ 00'$

10.00000

So $AD=108$

2.03342

To s, $\angle ABD=67^\circ 23'$

9.96523

Wh. taken from $90^\circ 00'$

Leaves

 $22^\circ 37' = \angle A = \angle C.$ And $67^\circ 23'$ doubledGives $134^\circ 46' = \angle ABC.$

A like process is to be used in every triangle, in which are two equal sides.

The foregoing examples contain all the variety that can possibly happen in the solutions of plane triangles, considered only with regard to their sides and angles; but besides the methods shewn of resolving such triangles by construction and computation, there is another way to find these solutions, called Instrumental; and this is of two kinds, *viz.* either by a ruler called a Sector, or by one called the Gunter's Scale: The method by the sector, the curious reader may see in many books, particularly in a treatise on Mathematical Instruments, published in the year 1775, 3d edition*: But the other method by the Gunter's scale being in great use at sea, it will be proper in this place to treat of it.

* By the author of these Elements.

SECTION IV.

Description and Construction of the Gunter's Scale.

65. Mr. *Edmund Gunter*, Professor of Astronomy at *Gresham College*, some time about the year 1624, applied the Logarithms of Numbers to a flat ruler : This he effected by taking the lengths expressed by the figures in those logarithms from a scale of equal parts, and transferring them to a line, or scale, drawn on such a ruler ; and this is the line which, from his name, is called the *Gunter's Line* : He also, in like manner, constructed lines containing the logarithms of the sines and tangents ; and since his time there have been contrived other logarithmic scales adapted to various purposes.

The Gunter's scale is a ruler, commonly two feet long ; having on one of its flat sides several lines or logarithmic scales ; and on the other side various other scales ; which, to distinguish them from the former, may be called natural scales.

While the reader is perusing what follows, it is proper he should have a Gunter's scale before him.

66. *Of the Natural Scales.*

The half of one side is filled with different scales of equal parts, for the convenience of constructing a larger, or smaller figure : The other half contains scales of Rhumbs, marked *Rhu* ; Chords, marked *Ch* ; Sines, marked *Sin* ; Tangents, marked *Tan* ; Secants, marked *Sec* ; Semi-tangents, denoted by *S. T.* and Longitude distinguished by *M. L.* The descriptions and uses of these scales will be considered hereafter, in the places where they will be wanted.

67. *Of the Logarithmic Scales.*

On the other side of the scale are the following lines.

- I. A line marked *s. R.* (sine rhumbs), which contains the logarithmic sines of the degrees to each point and quarter point of the compass.
- II. A line signed *r. R.* (tan rhumbs), the divisions of which correspond to the logarithmic tangents of the said points and quarters.
- III. A line marked *Num.* (numbers), where the logarithms of numbers are laid down.
- IV. A line marked *Sin.* containing the log. sines.
- V. A line of log. versed sines, marked *v. s.*
- VI. A line of log. tangents, marked *Tan.*
- VII. A meridional line signed *Mer.*
- VIII. A line of equal parts, marked *E. P.*

68.

I. *Of the Line of Numbers.*

The whole length of this line, or scale, is divided into two equal spaces, or intervals: the beginning, or left-hand end of the first, is marked 1; the end of the first interval, and beginning of the second, is also marked 1; and the end of the second interval, or end of the scale is marked with 10: Both these distances are alike divided, beginning at the left hand ends, by laying down in each the lengths of the logarithms of the numbers 20, 30, 40, 50, 60, 70, 80, 90; taken from a scale of equal parts, such that 10 of its primary divisions make the length of one interval: And the intermediate divisions are found, by taking the logarithms of like intermediate numbers.

From this construction it is evident, that when the first 1 stands for 1, the second 1 stands for 10, and the end 10 denotes 100;

$$\begin{array}{l} \text{And if the first} \\ \text{1 is called} \end{array} \left\{ \begin{array}{l} 10, \\ 100, \\ \text{\textit{Ec.}} \\ \frac{1}{10}, \\ \frac{1}{100}, \\ \text{\textit{Ec.}} \end{array} \right. \quad \begin{array}{l} \text{the 2d 1 stands} \\ \text{for} \end{array} \left\{ \begin{array}{l} 100, \\ 1000, \\ \text{\textit{Ec.}} \\ 1, \\ \frac{1}{10}, \\ \text{\textit{Ec.}} \end{array} \right. \quad \begin{array}{l} \text{And the 10 at} \\ \text{the end stands} \\ \text{for} \end{array} \left\{ \begin{array}{l} 1000; \\ 10000; \\ \text{\textit{Ec.}} \\ 10; \\ 1; \\ \text{\textit{Ec.}} \end{array} \right.$$

And the primary and intermediate divisions in each interval must be estimated according to the values set on their extremities, *viz.* at the beginning, middle, and end of the scale.

Now the examples most proper to be worked by this scale, are such where the numbers concerned do not exceed 1000, and then the first 1 stands for 10, the middle 1 for 100, and the 10 at the end for 1000: The primary divisions in the first interval, *viz.* 2, 3, 4, 5, 6, 7, 8, 9, stands for 20, 30, 40, 50, 60, 70, 80, 90, and the intermediate divisions stand for units. In the second interval, the primary divisions signed 2, 3, 4, 5, 6, 7, 8, 9, stand for 200, 300, 400, 500, 600, 700, 800, 900; each of these divisions are also divided into ten parts, which represent the intermediate tens. Between 100 and 200 the divisions for tens are each subdivided into five parts; so that each of these lesser divisions stand for two units. The tens between 200 and 500 are divided into two parts, each standing for five units: The units between the tens from 500 to 1000 are to be estimated by the eye; which by a little practice is readily done.

From this description it will be easy to find the division representing a given number not exceeding 1000: Thus the number 62 is the second small division from the 6, between the 6 and 7 in the first interval: The number 435 is thus reckoned; from the 4 in the second interval, count toward the 5 on the right, three of the larger divisions, and one of the smaller; and that will be the division expressing 435. And the like of other numbers.

69.

II. *For the Line of Sines.*

This scale terminates at 90 degrees, just against the 10 at the end of the line of numbers; and from this termination the degrees are laid backward, or from thence toward the left: Now seeking in a table of logarithmic sines, for the numbers expressing their arithmetic complements, without the index, take those numbers from the scale of equal parts the logs. of the numbers were taken from, and apply them to the scale of sines from 90°, and they will give the several divisions of this scale.

Thus the arith. comp. of the log. sines (or the co-secants) abating the index, of 10°, 20°, 30°, 40°, &c. are the numbers 76033, 46595, 30103, 19193, &c. then the equal parts to those numbers, laid from 90°, will give the divisions for 10°, 20°, 30°, 40°, &c. and the like for the intermediate degrees.

Proceeding in this manner, the arith. comp. of the sine of 5° 45' will be about equal to 10 of the primary divisions of the scale of equal parts, or to one interval in the log. scale; so that a decrease of the index by unity, answers to one interval; then a decrease of the index by 2 answers to two intervals, or the whole length of the log. scale; and this happens about the sine of 35 min. and the divisions answering to the sine of a little above 3 min. viz. 3' 26'' will be equal to 3 intervals; and the sine of about 26'' will be 4 intervals, &c. so that the sine of 90° being fixed, the beginning of the scale is vastly distant from it.

It is usual to insert the divisions to every 5 minutes, as far as 10 degrees; from 10° to 30°, the small divisions are of 15 minutes each; from 30° to 50°, contains every half degree; from 50° to 70°, are only whole degrees; the rest are easily reckoned.

70.

III. *For the Line of Tangents.*

As the tangent of 45 degrees is equal to the radius, or sine of 90°; therefore 45° on this scale, is terminated directly opposite to 90 on the sines; and the several divisions of this scale of log. tangents are constructed in the same manner as those of the sines, by applying their arith. comp. backward from 45°, or toward the left-hand.

The degrees above 45, are to be counted backward on the scale: Thus the division at 40° represents both 40° and 50°; the division 30 serves for 30° and 60°; and the like of the other divisions, and their intermediates.

71.

IV. *For the Line of Versed Sines.*

This line begins at the termination of the numbers, sines, and tangents: But as the numbers on those lines descend from the right to the left, so these ascend in the same direction: Now having a table of logarithmic versed sines to 180 degrees, let each log. versed sine be subtracted from that of 180 degrees; then the remainders being successively taken from the said scale of equal parts, and laid on the ruler backward from the common termination, the several divisions of this scale will be obtained.

The

The numbers for each 10 deg. are in the following table :

D.	Numb.	D.	Numb.	D.	Numb.	Deg.	Numb.	Deg.	Numb.	Deg.	Numb.
10	0,0033	40	0,0540	70	0,1733	100	0,3839	130	0,7481	160	1,5207
20	0,0133	50	0,0854	80	0,2315	110	0,4328	140	0,9319	170	2,2194
30	0,0301	60	0,1219	90	0,3010	120	0,6021	150	1,1740	180	10,3010

The other scales will be described in their proper places.

72. *Demonstration of the foregoing constructions.*
That of the log. numbers, is evident from the nature of logarithms.

For the Sines and Tangents.

Now co-sine : rad. :: rad. : secant (34). Then co-s. $\times$ secant = 1
 fine : rad. :: rad. : co-sec. (35). fine $\times$ co-sec. = 1
 tan. : rad. :: rad. : co-tan. (36). tan. $\times$ co-tan. = 1 } II. 162

The radius of the tables being supposed equal to 1.

Hence it is evident, that in either case, one of the quantities will be equal to the quotient of unity divided by the other.

But division is performed by subtraction with logarithms.

And to subtract a log. is the same as to add its arith. comp.

Consequently the logarithmic co-sine and secant of the same degrees are the arithmetical complements of one another.

And so are the logarithmic fines and co-secants : Also the logarithmic tangents and co-tangents are the arith. comp. of one another.

73. Now as the arith. comp. of any number is what that number wants of unity in the next superior place.

Therefore every natural fine and its arith. comp. together make the radius.

And the fines begin at one end of a radius, and end in 90° at the other end.

Therefore in a scale of fines, the arith. comp. of any fine, or its co-secant, laid backward from 90°, gives the division for that fine. And the like must happen in a scale of log. fines.

74. Also, as the logarithmic tangents and co-tangents are the arith. comp. of one another ; therefore in a scale of log. tangents, the divisions to the degrees both under and above 45, are equally distant from the division of 45°.

Consequently the divisions serving to the degrees under 45, will serve, by reckoning backward, for those above 45.

75. *For the Versed Sines.*

Although the numbers in the line of versed fines ascend from right to left, yet they are only the supplements of the real versed fines, which are numbered in the same order as the fines, that is, from left to right: But as the beginning of the versed fines falls without the ruler, therefore it is most convenient to lay down the divisions from the point where the versed fines terminate at 180 degrees, that is, against 90° on the fines.

Now it is evident that the divisions laid off from this termination must be the differences between the log. versed fines of the several degrees, &c. and that of 180 degrees.

SECTION V.

The use of the Gunter's Scale in Plane Trigonometry.

76. When a Trigonometrical Question is to be solved by the Gunter's scale, it must first be stated by the precepts to that problem under which the question falls, whether it be by opposite sides and angles, or by two sides and their included angle, or by three sides.

77. In all proportions wrought by the Gunter's scale, when the first and second terms are of the same kind, then,

The extent from the first term to the second will reach from the third term to the fourth.

Or, when the first and third terms are of the same kind,

The extent from the first term to the third will reach from the second term to the fourth.

That is, set one point of the compasses on the division expressing the first term, and extend the other point to the division expressing the second (or third) term; then, without altering the opening of the compasses, set one point on the division representing the third term (or second term), and the other point will fall on the division shewing the fourth term or answer.

In working by these directions, it is proper to observe,

78. First. The extent from one side to another side, is to be taken from the scale of numbers; and the extent from one angle to another is to be taken from the scale of sines, in working by opposite sides and angles; or from the scale of tangents, in working by two sides and the included angle.

Secondly. When the extent from the first term to the second (or third) is decreasing, or is from the right to the left, then the extent from the third term (or second) must be also decreasing; that is, applied from the right toward the left: And the like caution is necessary when the extent is from the left toward the right.

These precepts being carefully attended to, what follows will be readily understood.

79. In EXAMPLE I. See Article 53.

As $s, \angle C : AB :: s, \angle B : AC$. Or $s, 42^\circ 05' : 195 :: s, 90^\circ 00' : Q$,
where Q stands for the number sought.

Now the extent from $42^\circ 05'$ to $90^\circ 00'$, taken on a scale of sines, and applied to the scale of numbers, will reach from 195 to 291. See art. 69.

Also. As $s, \angle C : AB :: s, \angle A : BC$. Or $s, 42^\circ 05' : 195 :: s, 47^\circ 55' : Q$.

Then the extent from $42^\circ 05'$ to $47^\circ 55'$ on the sines, being applied to the numbers, will reach from 195 to 216. See art. 68.

In each of these operations, the first extent was from the left to the right, or increasing; therefore the second extent must be from left to right also.

80. In EXAMPLE IV. See Art. 56.

As $AC : s, \angle B :: AB : s, \angle C$. Or $291 : s, 90^\circ 00' :: 195 : Q$.

Here the extent from 291 to 195, taken on the numbers, and applied to the sines, will reach from $90^\circ 00'$ to $42^\circ 05'$.

The first extent being from the right toward the left, or decreasing; therefore the second extent must be also from the right to the left.

In EXAMPLE VII. See Art. 59.

As $AB : \text{Rad.} :: BC : t, \angle A$. Or $195 : t, 45^\circ 00' :: 216 : Q$.

Then the extent from 195 to 216 on the numbers, will reach from $45^\circ 00'$ to $47^\circ 55'$ on the tangents.

Here the first extent being from left to right, or increasing, therefore the second extent must also be increasing: Now on the tangents, this increase above 45° does not proceed from left to right, but from right to left, the same way that the decrease proceeds (70); consequently the division which the point falls on for the fourth term, must be estimated according as the first extent is increasing or decreasing.

Thus had the proportion been,

As $BC : \text{Rad.} :: AB : t, \angle C$. Or, $216 : t, 45^\circ 00' :: 195 : Q$.

Then the extent from 216 to 195 on the numbers, will reach from $45^\circ 00'$ to $42^\circ 05'$, estimated as decreasing.

81. When two sides and the included angle are given, and the tangent of half the difference of the unknown angles is required.

Then, on the line of numbers take the extent from the sum of the given sides to their difference; and on the line of tangents apply this extent from 45° downward, or to the left; let the point of the compasses rest where it falls, and bring the other point (from 45°) to the division answering to the half sum of the unknown angles; then this extent applied from 45° downward, will give the half difference of the unknown angles: Whence the angles may be found.

(47)

In EXAMPLE IX. See the Art. 61.

$$\begin{array}{l} \text{As } AB+AC : AB-AC :: t. \frac{1}{2} \angle C + \angle B : t. \frac{1}{2} \angle C - \angle B. \\ \text{Or } 567 : 249 :: t. 60^\circ 56' : Q. \end{array}$$

Now the extent from 567 to 249 on the numbers, being applied to the tangents, will reach from 45° to about $23^\circ 40'$: Let one point of the compasses rest on this division, and bring the other to $60^\circ 56'$; then this extent will reach from 45° to $38^\circ 19'$, the half difference sought.

And this method will always give the half difference, whether the half sum of the angles is greater or less than 45° .

82. But when the half sum and half difference are greater than 45° ; then the extent from the sum of the sides to their difference on the scale of numbers, will (on the tangents) reach from the half sum of the angles to their half difference, reckoning from left to right.

And when the half sum and half difference are both less than 45° ; then the extent from the sum of the sides to their difference, taken from the numbers and applied to the tangents, will reach from the half sum of the angles downward to their half difference.

83. When the three sides are given to find an angle, and a perpendicular is drawn from an angle to its opposite side. See Ex. X. art. 62.

$$\begin{array}{l} \text{As } AC : BC+AB :: BC-AB : CD-AD. \\ \text{Or } 291 : 411 :: 21 : Q. \end{array}$$

Now the extent from 291 to 411 on the scale of numbers, will reach from 21 to 29,6 on the numbers also.

Then the extent for the angles is performed in the same manner as shewn in Ex. I. (78)

84. Or an angle may be found by Problem VI. as follows.

In the scale of numbers, take the extent from the half sum (of G and D) to either of the containing sides (as E); apply this extent from the other containing side (as F), to a fourth term: Let one point of the compasses rest on this fourth term, and extend the other to the half difference (of G and D); then this extent applied to the versed sines from the beginning, will give the supplement of the angle sought.

In EXAMPLE X. See Art. 62.

$$E = 291; F = 216; \text{ half sum} = 135; \text{ half diff.} = 60.$$

Then on the numbers, the extent from 135 to 291, will reach from 216 to 465; let the point rest there, and extend the other to 60; then this extent applied to the versed sines, will reach from the beginning to $137^\circ 56'$; which taken from 180° , leaves $42^\circ 04'$ for the angle sought.

85. In

85.

In EXAMPLE XI. See Art. 63.

Here $E=408$; $F=351$; half sum of G and $D=108$; half diff. $=51$. Then on the numbers, the extent from 108 to 408, will reach from 351 in the second interval, to a fourth number: But as the point of the compasses falls beyond the end of the scale, therefore let the extent from 108 to 408 be applied in the first interval, which will reach from 35,1 to 132,6; let one point rest on 132,6, and extend the other point of the compasses to 51. Now as this extent of the compasses is less than it ought to be, by one interval, or half the length of the scale of numbers; therefore the last extent, when applied to the versed lines, must be from that division, on the versed lines, opposite to the middle of the scale of numbers, which is nearly at 143° ; and it will reach from thence to the versed sine of $157^\circ 23'$; which taken from 180° , leaves $22^\circ 37'$ for the angle sought.

86. Most of the writers on Plane Trigonometry treat of right angled, and of oblique angled triangles separately; making seven cases in the former, and six cases in the latter: But as every one of these thirteen cases fall under one or other of the foregoing Problems, therefore such distinctions are here avoided, it being conceived, that they rather tend to perplex than instruct a learner: Also in the generality of the treatises on this subject, it is usually shewn how the solutions of right angled triangles are performed, by making (as it is called) each side radius; that is, by comparing each side of the triangle with the radius of the tables: And although these considerations are here omitted, yet the inquisitive reader will find them in Book VII. near the beginning.

87. Beside the demonstration of Problem IV. at art. 48. it has been thought proper to give another demonstration; because there arises from it a Theorem useful on some occasions: Moreover, there is also added methods of deriving other rules for the solution of the case where the three sides are given to find an angle, which, if they should be found of no other use, will perhaps be agreeable exercises of Geometry to those who are delighted with these studies.

90. In any plane triangle ABC, where the three sides are known, the measure of either angle (as the angle A, included between the sides AB, AC) may be found several ways, as shewn in the following articles.

The letters s, t, v , stand for fine, tangent, versed sine.

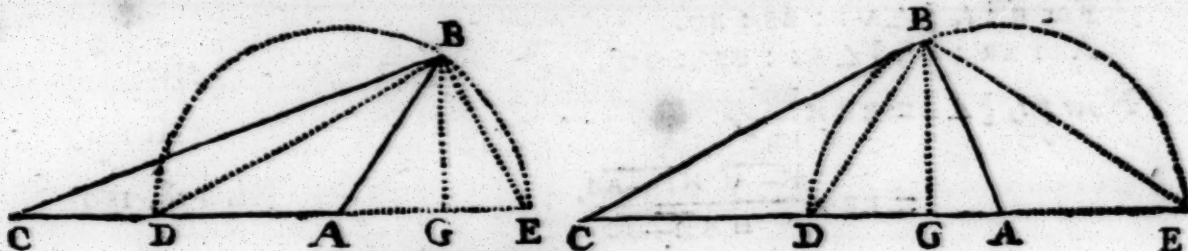
$\dot{s}, \dot{t}$, stand for co-sine, co-tangent.

$\dot{v}$, stands for the versed sine of the supplement.

Also ss, tt , stand for the squares of the fine and tangent.

Also $\dot{s}\dot{s}, \dot{t}\dot{t}$, stand for the squares of the co-sine and co-tangent.

Radius $= R = AB = AE = AD$; and $H = \frac{1}{2}AB + \frac{1}{2}AC + \frac{1}{2}BC$.



$$91. s, \frac{1}{2}\angle A = R \times \sqrt{\frac{H-AC \times H-BC}{AC \times AB}}. \text{ Here } \angle E = \frac{1}{2}\angle A.$$

For $R : s, \frac{1}{2}\angle A :: (DE =) 2AB : BD.$ (45)

And $RR : ss, \frac{1}{2}\angle A :: 4AB^2 : BD^2.$ (II. 161)

Now $ss, \frac{1}{2}\angle A = RR \times \frac{BD^2}{4AB^2}.$ (II. 164)

$$= \frac{2AB}{4AB^2} \times \frac{2H-2AC \times 2H-2BC}{2AC} \times RR. \text{ (II. 179)}$$

$$\text{Then } s, \frac{1}{2}\angle A = R \times \sqrt{\frac{H-AC \times H-AB}{AC \times AB}}.$$

$$92. s, \angle A = \frac{2R}{AB \times AC} \times \sqrt{H \times H-CB \times H-AC \times H-AB}.$$

For $AB : BG :: R : s, \angle A.$ (45)

And $AB^2 : BG^2 :: RR : ss, \angle A.$ (II. 161)

Now $ss, \angle A = \frac{RR}{AB^2} \times BG^2.$ (II. 164)

$$= \frac{RR}{AB^2} \times \frac{4}{AC^2} \times H \times H-CB \times H-AC \times H-AB. \text{ (II. 180)}$$

$$93. \dot{s}, \frac{1}{2}\angle A = R \times \sqrt{\frac{H \times H-CB}{AC \times AB}}. \text{ Here } \angle E = \frac{1}{2}\angle A; \text{ and } \angle D = \text{comp. } \angle E.$$

For $R : \dot{s}, \frac{1}{2}\angle A :: (DE =) 2AB : BE.$ (45)

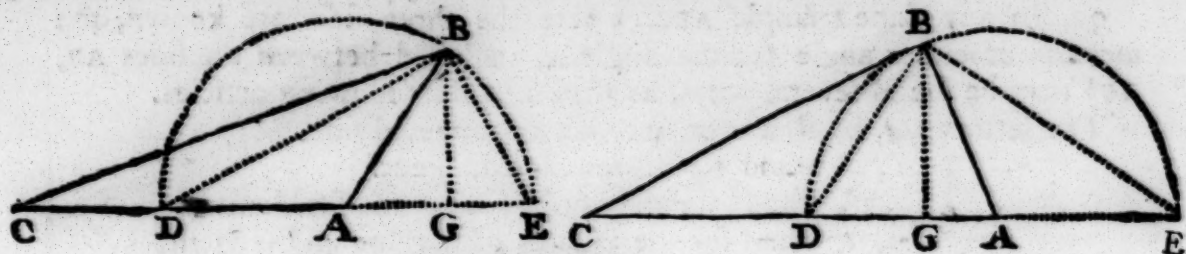
And $RR : \dot{s}\dot{s}, \frac{1}{2}\angle A :: 4AB^2 : BE^2.$ (II. 161)

Now $\dot{s}\dot{s}, \frac{1}{2}\angle A = RR \times \frac{BE^2}{4AB^2}.$ (II. 164)

$$= RR \times \frac{2AB \times 4 \times H \times H-CB}{4AB^2 \times 2AC}.$$

(II. 178)

94. $t, \frac{1}{2}$



$$94. \quad t, \frac{1}{2} \angle A = R \times \frac{\sqrt{H-AC \times H-AB}}{H \times H-CB}.$$

For $R : t, \frac{1}{2} \angle A :: BE : BD.$

And $RR : t t, \frac{1}{2} \angle A :: BE^2 : BD^2.$

Now $t t, \frac{1}{2} \angle A = RR \times \frac{BD^2}{BE^2}.$

$$= RR \times \frac{H-AC \times H-AB}{H \times H-CB}.$$

(46)

(II. 161)

(II. 164)

(II. 178, 179)

$$95. \quad t', \frac{1}{2} \angle A = R \times \sqrt{\frac{H \times H-CB}{H-AC \times H-AB}}.$$

For $BD^2 : BE^2 :: (t t, \frac{1}{2} \angle A : RR :: t' t', \frac{1}{2} \angle A.$

(36)

Then $t' t', \frac{1}{2} \angle A = RR \times \frac{BE^2}{BD^2}.$

(II. 178, 179)

$$96. \quad v, \angle A = 2R \times \frac{H-AC \times H-AB}{AC \times AB}.$$

For $R : v, \angle A :: AB : GD.$

(45)

Then $v, \angle A = R \times \frac{GB}{AB} = R \times \frac{2 \times H-AC \times H-AB}{AC \times AB}.$

(II. 177)

$$97. \quad v', \angle A = 2R \times \frac{H \times H-CB}{AC \times AB}.$$

For $R : v', \angle A :: AB : GE.$

(45)

Then $v', \angle A = R \times \frac{GE}{AB} = R \times \frac{2 \times H \times H-CB}{AC \times AB}.$

(II. 176)

$$98. \quad s', \angle A = \frac{1}{2} R \times \frac{CB^2 - AC^2 - AB^2}{AC \times AB}.$$

For $R : s', \angle A :: AB : AG.$

(45)

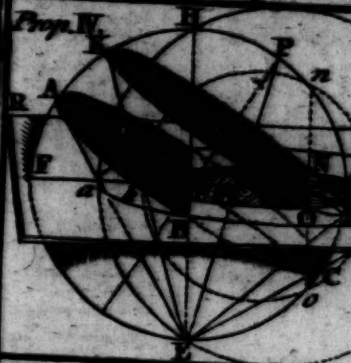
Then $s', \angle A = R \times \frac{AG}{AB} = R \times \frac{BC^2 - AC^2 - AB^2}{2AC \times AB}.$

(II. 174)

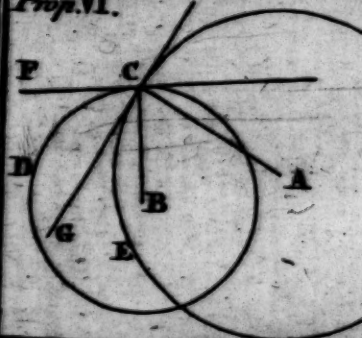
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Vol. I.

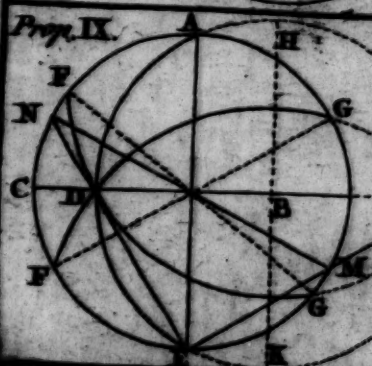
Definitions. P. 125 Prop.



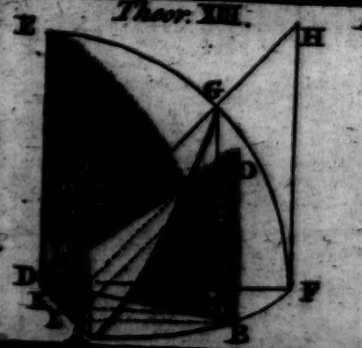
Prop. VI.



Prop. IX.



Theor. III.



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SPHERICS.

H. I.

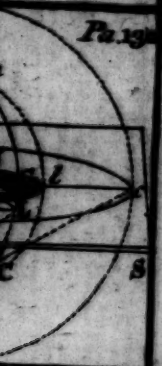
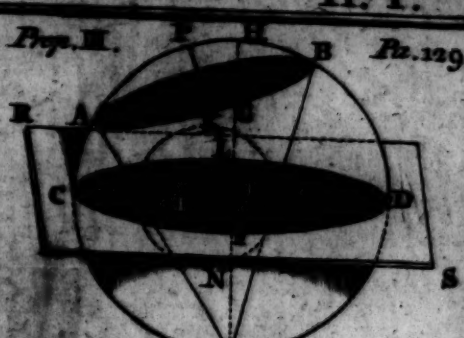
P. I. P. 128

Prop. II.

P. 129

Prop. III.

P. 129

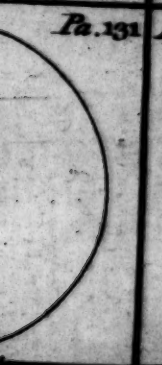
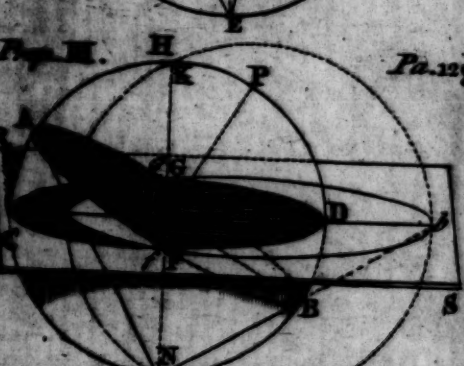
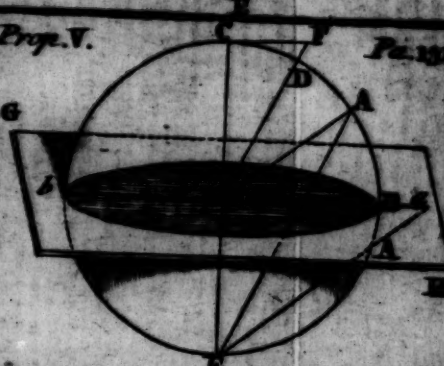


Prop. V.

P. 131

Prop. VI.

P. 129

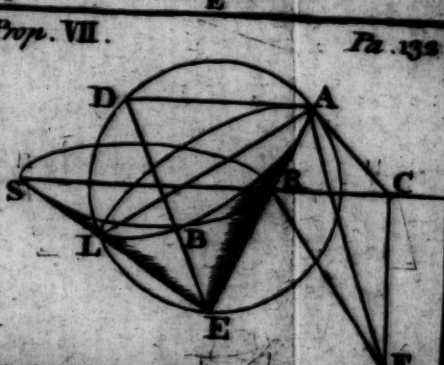


Prop. VII.

P. 132

Prop. VIII.

P. 132

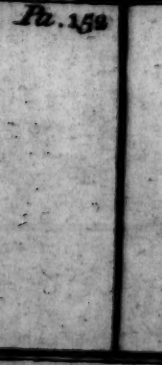
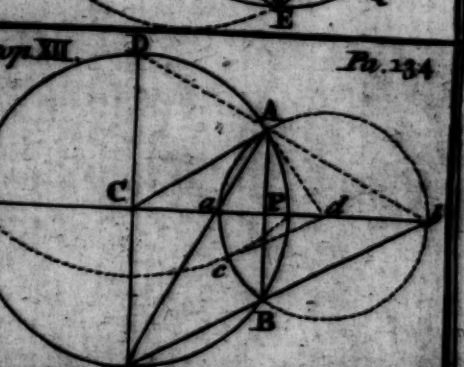


Prop. IX.

P. 133

Prop. X.

P. 134

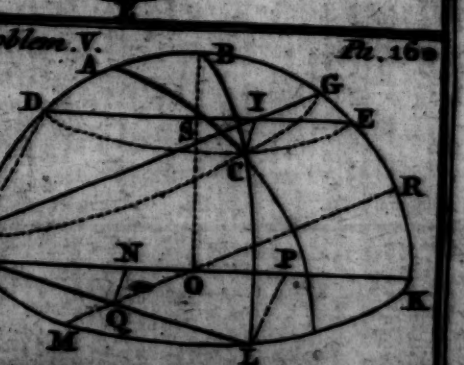
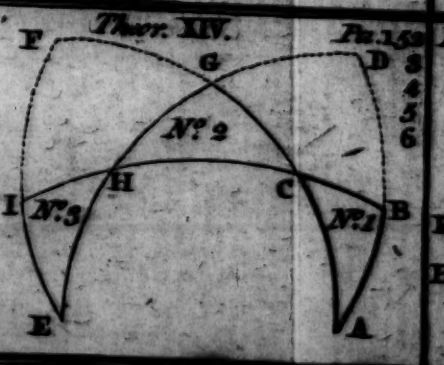


Theor. XIV.

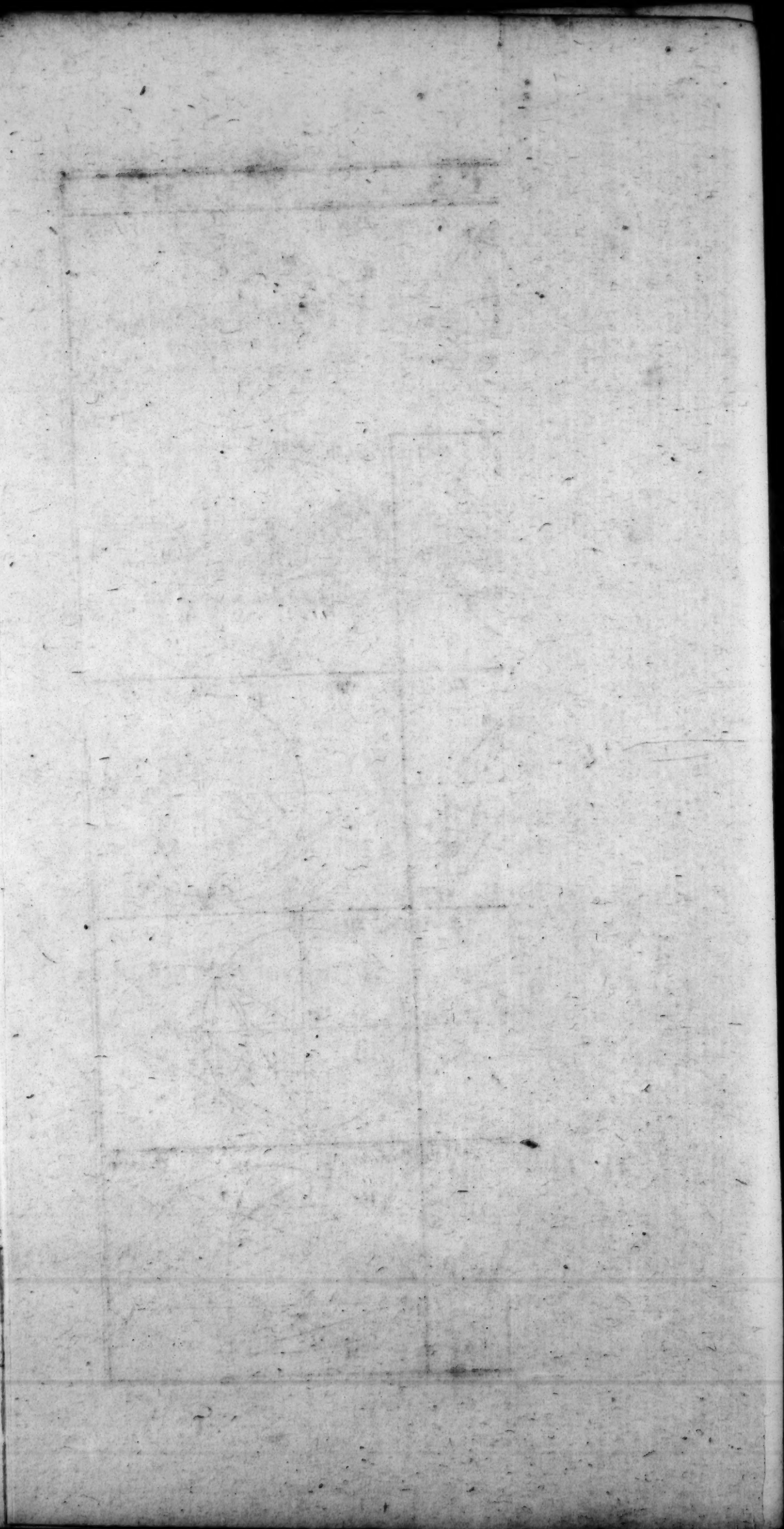
P. 159

Problem. V.

P. 160



against page 125



201

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THE
ELEMENTS
OF
NAVIGATION.

BOOK IV.
OF SPHERICS.

SECTION I.
Definitions and Principles.

1. **SPHERICS** is that part of the Mathematics which treats of the position and magnitude of arcs of circles described on the surface of a sphere.

2. A **SPHERE** is a solid contained under one uniform round surface, such as would be formed by the revolution of a circle about its diameter, that diameter being immoveable during the motion of the circle.

Thus the circle AEBD revolving about the diameter AB, will generate a sphere, the surface of which will be formed by the circumference AEBD. See Plate I.

3. The **CENTER** and **AXIS** of a sphere are the same as the center and diameter of a generating circle: And as a circle has an indefinite number of diameters, so a sphere may be considered as having also an indefinite number of axes, round any one of which the sphere may be conceived to be generated.

4. **CIRCLES OF THE SPHERE** are those circles described on its surface by the motion of the extremities of such chords in the generating circle as are at right angles to the diameter, or to the axis of the sphere.

Thus by the motion of the circle AEBD about the diameter AB, the extremities of the chords ED, GF, IH, at right angles to AB, will describe circles the diameters of which are equal to those chords respectively. Plate I.

5. The

5. The **POLES** of a circle on the sphere, are those points on its surface equally distant from the circumference of that circle.

Thus A and B are the poles of the circles described on the sphere by the ends of the chords ED, GF, IH. Plate I.

6. A **GREAT CIRCLE** of the sphere, is that circle which is equally distant from both its poles.

Thus the circle described by the extremities E, D, of the diameter ED, at right angles to AB, being equally distant from its poles A and B, is called a great circle.

7. **LESSER CIRCLES** of the sphere, or small circles, are those circles which are unequally distant from both their poles.

Thus the circles of which FG, HI, are diameters, having their poles A and B unequally distant from them, are called lesser circles.

8. **PARALLEL CIRCLES** of the sphere, are those circles, the planes of which are considered as parallel to the plane of some great circle.

Thus the circles having the diameters FG, HI, are called parallel circles in respect of the great circle of which ED is the diameter.

9. A **SPHERIC ANGLE** is the inclination of two great circles of the sphere meeting one another.

10. A **SPHERIC TRIANGLE** is a figure formed on the surface of a sphere by the mutual intersections of three great circles.

11. The **STEREOGRAPHIC PROJECTION** of the sphere, is such a representation of its circles, upon the plane of one of them passing through the center, and called the **PLANE OF PROJECTION**, as would appear to an eye placed in one of the poles of that great circle, and thence viewing the circles on the sphere.

12. The place of the Eye is called the **PROJECTING POINT**, or lower pole: and the point diametrically opposite is called the remotest, or opposite, or upper pole.

Also, the projection of any point on the sphere, is that point in the plane of projection, through which the visual ray passes to the eye.

13. The **PRIMITIVE CIRCLE** is that great circle, on the plane of which the representations of all the other circles are supposed to be drawn.

14. An **OBLIQUE CIRCLE** is one which has its plane oblique to the eye.

15. A **RIGHT CIRCLE** is that which is perpendicular to the plane of the primitive circle, and if it be a great circle, its plane passes through the eye, and it is seen edgewise; consequently it is represented by a straight line drawn through the center of the primitive circle.

AXIOMS.

A X I O M S.

16. The diameter of every great circle passes through the center of the sphere; but the diameters of small circles do not pass through the same center: Also the center of the sphere is the common center of all its great circles.

17. Every section of a sphere, by a plane passing through its circumference, is a circle.

18. A sphere is divided into two equal parts by the plane of every great circle; and into two unequal parts by the plane of every small circle.

19. The pole of every great circle is at 90 degrees distance from it on the surface of the sphere: And no two great circles can have a common pole.

20. The poles of a great circle are the extremities of that diameter, or axis, of the sphere, which is perpendicular to the plane of that circle.

21. Lines flowing to the projecting point, or place of the eye, from every point in the circumference of a circle which it views, form the convex surface of a Cone.

22. A plane passing through three points on the surface of a sphere, equally distant from the pole of a great circle, will be parallel to the plane of that circle.

23. The shortest distance between two points on the surface of a sphere, is the arc of a great circle passing through those points.

24. If one great circle meet another, the angles on either side are supplements to one another; and every spheric angle is less than 180 degrees.

25. A spheric angle is measured by an arc of a great circle intercepted between the legs of that angle, 90 degrees distant from the angular point.

26. If two circles intersect one another, the opposite angles are equal.

27. Two spheric triangles are congruous, if two sides and their contained angle in one, are equal to two sides and their contained angle in the other, each to each: Or if two angles and the contained side in the one, are equal to two angles and their contained side in the other, each to each: Or if the three sides in the one are respectively equal to the three sides in the other.

28. All parallel circles have the same pole, and may be conceived to be concentric to the great circle which they are parallel to.

29. All parallel circles on the sphere, having the same pole, are cut into similar arcs by two great circles passing through that pole.

SECTION

SECTION II.

Stereographic Propositions.

30. PROPOSITION I.

Great circles of a sphere mutually cut one another into two equal parts.

DEM. Any two great circles have the same common center. (16)
And their planes intersect in a right line. (II. 209)

Now the center must lie in the line of their intersection.

Therefore this right line is a diameter common to both.

But every circle is bisected by its diameter.

Therefore the circles mutually bisect one another.

31. COROL. I. The circumferences of any two circles intersecting one another twice, make the angles at both sections equal.

For the planes of those circles have the same inclination at both ends of their intersection, or where the circumferences intersect.

32. COROL. II. Two great circles of the sphere will cut each other twice at the distance of 180 degrees, or in opposite points of the sphere.

33. PROP. II.

The distance of the poles of two great circles, is equal to the angle formed by the circumferences of those circles. Plate I.

DEM. Let AEB, CED, be two great circles of the sphere, their planes passing through its center F; and let a, b , be the poles of the circle AEB, and c, d , the poles of the circle CED.

Then is the arc $Aa = \text{arc } Cc = 90^\circ$. (19)

And the arc ca is common to both the arcs Aa and Cc .

Therefore the arc AC , measuring the inclination CFA of the circles, is equal to the arc ac measuring the distance of the poles. (II. 48.)

34. COROL. I. Two great circles are at right angles to one another, when they pass through each other's poles.

35. COROL. II. The pole of a great circle is 90 degrees distant from it, taken in another great circle, or in an arc of it, drawn perpendicular to the former circle.

36. COROL. III. Two or more great circles, at right angles to another great circle, intersect one another 90° distant from it, or in the pole of the latter circle. And the like of arcs of great circles.

37. COROL. IV. If several great circles intersect one another in the pole of another great circle; then are the former circles perpendicular to the latter.

38. PROP.

38.

PROP. III.

In the stereographic projection of the sphere, the representations of all circles, not passing through the projecting point, will be circles. Plate I. three figures.

Let $ACEDB$ represent a sphere, cut by a plane RS , passing through the center I , at right angles to the diameter EH , drawn from E , the place of the eye.

And let the section of the sphere (17) by the plane RS , be the circle $CFDL$, its poles being H and E .

Suppose AGB is a circle on the sphere to be projected, its pole, most remote from the eye, being P : And the visual rays from the circle ABG meeting in E , form the cone $AGBE$ (21) of which the triangle AEB is a section through the vertex E , and diameter of the base AB . (II. 204)

Then will the figure $agbf$, which is the projection of the circle BGA , be a circle.

DEMONSTRATION. Since the $\angle Eab$ is measured by $\frac{1}{2}$ arc $AC + (\frac{1}{2}$ arc $ED =) \frac{1}{2}$ arc CE , (II. 137)

And the $\angle EBA$ is measured by $\frac{1}{2}$ arc $AC + \frac{1}{2}$ arc CE . (II. 128)

Therefore the angle $EBA = \text{angle } Eab$. (II. 50)

And so the triangles EAB , Eba are similar, the $\angle E$ being common.

Therefore ab cuts the sides EA and EB of the cone, in a subcontrary position to AB ; and consequently the section $afbg$ is a circle. (II. 213)

Now suppose the plane RS to revolve on the line CD , till it coincides with the plane of the circle $ACEB$;

Then it is evident, that the point L will fall in H , the point F in E , and the circle $CFDL$ will coincide with the circle $CEDH$, which now becomes the primitive circle, where the point F , or E , is the projecting point: Also the projected circle $afbg$ will become the circle $anbk$.

39. COROL. I. Hence the middle of the projected diameter is the center of the projected circle, whether it be a great circle or a small one.

40. COROL. II. Hence in all circles parallel to the plane of projection, their centers and poles will fall in the center of the projection.

41. COROL. III. The centers and poles of circles, inclined to the plane of projection, fall in that diameter of the primitive circle which is at right angles to the diameter drawn through the projecting point; but at different distances from its center.

42. COROL. IV. All oblique great circles cut the primitive circle in two points diametrically opposite.

43.

PROP. IV.

The measure of the angle which the projected diameter of any circle subtends at the eye, is equal to the distance of that circle from its pole, which is most remote from the projecting point, taken on the surface of the sphere. And that angle is bisected by a right line joining the projecting point and that pole. Plate I.

Let the plane RS cut the sphere $HFEG$, as in the last.

And let ABC be any oblique great circle, the diameter of which AC is projected into ac ; and KOL any small circle parallel to ABC , the diameter of which KL is projected into kl .

The distances of those circles from the pole P , being the arcs AHP , KHP , and the angles aEc , kEl , are angles at the eye subtended by their projected diameters ac , kl .

Then is the angle aEc measured by the arc AHP , the angle kEl is measured by the arc KHP ; and those angles are bisected by EP .

DEM. For arc $PHA = \text{arc } PC$; and arc $PHK = \text{arc } PL$.

(5)

And the $\angle AEC$ is measured by $\frac{1}{2}$ arc $AFC = \text{arc } PHA$.

(II. 128)

Also the $\angle KEL$ is measured by $\frac{1}{2}$ arc $KPL = \text{arc } PHK$.

(II. 128)

Therefore the angles AEC , KEL , are respectively measured by the arcs PHA , PHK .

And it is evident those angles are bisected by the line EP .

44. COROL. I. Hence as the line EP projects the pole P in p ; so the same line refers a projected pole to its place on the sphere, in the circumference of the primitive circle.

45. COROL. II. Hence, on the plane of the primitive circle, may be described the representation of any circle whose distance from its pole, and the projected place of that pole, are given.

For PA and PC are projected into pa and pc ; and the bisection of ac gives the center of the circle sought.

46. COROL. III. Hence every projected oblique great circle cuts the primitive circle in an angle equal to the inclination of the plane of that oblique circle to the plane of projection.

For fa is equivalent to FA the inclination.

And fa measures the angle FHa , since FH , Ha , are each 90° . (9)

47. COROL. IV. The distance between the projections of a great circle and any of its parallels is equivalent to their distance on the sphere.

Thus the projection ak is equivalent to AK .

48. PROP.

48.

PROP. V.

Any point of a sphere stereographically projected, is distant from the center of projection, by the tangent of half the arc intercepted between that point and the pole opposite to the eye: The semidiameter of the sphere being made radius. Plate I.

Let $cbeb$ be a great circle of the sphere, the center of which is c , GH the plane of projection cutting the diameter of the sphere in b , B ; E , C , the poles of the section by that plane; and a the projection of A . Then is ca equal to the tangent of half the arc AC .

DEM. Draw CF , a tangent to the arc $CD = \frac{1}{2}$ arc CA , and join CF . Now the triangles cfc , cae , are congruous: For $cc = ce$, $\angle c = \angle eca$, $=$ right $\angle$, $\angle ccf = cea$ (II. 128): Therefore $ca = CF$. Consequently ca is equal to the tangent of half the arc CA .

49.

PROP. VI.

The angle made by the intersection of the circumferences of two circles in the same plane, is equal to the angle made by tangents to those circles in the point of section; and also is equal to the angle made by their radii drawn to that point. Plate I.

Let CE , CD , be two arcs of circles in the same plane cutting in the point C ; AC , BC , their radii; GC , FC , tangents at the point C . Then is the curve-lined angle $ECD = \angle GCF = \angle ACB$.

DEM. Since the radii AC , BC , are at right angles to their tangents GC , FC (II. 126); and are also at right angles to the arcs CE , CD . (II. 136) Therefore the position of the tangents and arcs at the point C are the same; and consequently the $\angle ECD = \angle GCF$. Also the $\angle ACB + \angle BCG = (\text{right } \angle =) \angle FCG + \angle BCG$. Therefore the angle ACB is equal to the angle FCG , by taking away the common angle BCG . (II. 48.) Consequently $\angle ECD = \angle GCF = \angle ACB$.

50. SCHOLIUM. If the arcs CE , CD , were in different planes, the same would hold true with regard to their tangents.

For suppose the circle CD to revolve on the fixed radius BC , still cutting the circle CE in C : Then the tangent CF revolving with it, has still the same inclination to BC : And as the inclination of the planes of the circles vary, so much will the inclination of the tangents vary.

Therefore the angle made by the tangents, in all positions of the circular planes, is the same as the angle made by their circumferences.

51. COROL. Hence if a plane touches a sphere, at the point where two circles of it intersect one another, the tangents to both circles will lie in that plane; and consequently, in all oblique positions, a right line perpendicular to one tangent will cut the other tangent.

52.

PROP. VII.

The angle which any two circles make, when stereographically projected, is equal to the angle which those circles make on the sphere. Plate I.

Suppose DAEL a sphere to be projected on the plane SBRCF, and ALDE a great circle passing through the projecting point E. Let LBA be any other circle, cutting the former in L and A, under the angle BAE, which will be represented by the circle SBR, (38) as the circle ELDA is by the right line SC (15). The angle BAE is equal to the angle BRC.

From the point A draw AC, AF, to touch the circles AEL, ABS in A, and meet the plane of projection in C and F; also draw RF and CF, which will be in that plane; and, in the plane of the great circle AEL, draw AD parallel to SC, and join ED.

DEM. The angular point A is projected into R; (12) consequently AC is projected into RC, and AF into RF. And since SC is the common section of the plane of projection with that of the great circle ELDA (II. 210) the lines AC, SC, AD, ED, AE, lie all in the plane of that circle: Also because AD is parallel to SC, the $\angle ARC = \angle DAE = \angle ADE = \angle RAC$ (II. 94, 104, 132) consequently $AC = RC$ (II. 104). Now the plane passing through AC and AF touches the sphere in A, (51) it is therefore perpendicular to the plane of the circle AED; and FC its common section with the plane of projection, is at right angles to that plane (II. 210); FC is therefore at right angles both to the lines AC and CR (II. 205): Hence, the triangles ACF, RCF, being right angled at C, having the side FC common, and $AC = CR$, are congruous (II. 99), and the $\angle CAF = \angle CRF$. Consequently the $\angle EAB = \angle FAC$ (51) $= \angle FRC$. Now it is manifest that as AF touches the base, ABL, of the cone EABL, in the point A, a plane passing through AF and AE will touch the side of the cone in the line AE; but AF is also in that plane (II. 198); therefore AR touches the cone in the line AE; and as AR lies also in the plane of the circle SBR, it must touch that circle also; consequently (50) $BRC = \angle FRC = \angle FAC = \angle ABE$.

53.

PROP. VIII.

The distance between the poles of the primitive circle and an oblique great circle, in stereographic projections, is equal to the tangent of half the inclination of those circles; and the distance of their centers is equal to the tangent of their inclination: The semidiameter of the primitive circle being made radius. Plate I.

Let AC be the diameter of a circle, the poles of which are P and Q, and inclined to the plane of projection in the angle AIF.

And let a, c, p , be the projections of the points A, C, P.

Also let HAE be the projected oblique circle, the center of which is q .

Now when the plane of projection becomes the primitive circle, the pole of which is I.

Then is $Ip =$ tangent of half $\angle AIF$, or of half the arc AF.

And $Iq =$ tangent of AF, or of the $\angle FHa = AIF$.

DEM. For $AH + HP = AH + AF$. Therefore $HP = AF$.

But $Ip =$ tangent of half HP, or of half AF.

(48)
Again,

Again. As AC is projected in ac , then q , the middle of ac , is the center of the projected circle, and of its representative HAE . (45)

Draw eq produced to r : Then as $qa=qe$; the $\angle qEA=\angle qae$. (II. 104)

But the $\angle qae$ is measured by half the arc EFA . (II. 137)

Therefore the arc $Ahr=\text{arc } AFE$ (II. 50): And as the arc $AHP=FQE$;

Therefore $Pr=AF=HP$; and $HPr=\text{twice the arc } AF$.

Therefore (II. 127) the $\angle IEq=AIF$, the inclination of the circles.

But Iq is the tangent of the $\angle IEq$, EI being the radius.

54. COROL. Hence the radius of an oblique circle is equal to the secant of the obliquity of that circle to the primitive.

For Eq is the secant of the angle IEq , to the radius EI .

55.

P R O P. IX.

If through any given point in the primitive circle an oblique circle be described; then the centers of all other oblique circles passing through that point, will be in a right line drawn through the center of the first oblique circle at right angles to a line passing through the given point, and the center of the primitive.

Let $GACE$ be the primitive circle, $ADEI$ a great circle described through D , its center being B .

HK is a right line drawn through B , perpendicular to a right line CI passing through D , and the center of the primitive circle.

Then the centers of all other great circles FDG passing through D will fall in the line HK .

DEM. For if E be the projecting point, the circle $EDAI$ will be the projection of a circle, the diameter of which is NM . (38)

Therefore D and I are the projections of N , M , which are opposite points on the sphere; or of points at a semicircle's distance.

Therefore all circles passing through D and I must be the projections of great circles on the sphere.

But DI is a chord in every circle passing through the points D , I .

Consequently the centers of all those circles will be found in HK drawn perpendicularly through B , the middle of DI . (II. 125)

56.

P R O P. X.

Equal arcs of any two great circles of the sphere, will be intercepted between two other circles drawn on the sphere through the remotest poles of those great circles. Plate I.

Let $PBEA$ be a sphere, on which AGB , CFD , are two great circles, the remotest poles of which are E , P ; and through these poles let the great circle $PBEC$, and small circle PGE , be drawn, intersecting the great circles AGB , CFD , in the points B , G , and D , F .

Then are the intercepted arcs BG and DF equal to one another.

DEM. For the arcs $ED+DB=\text{arcs } PB+DB$; therefore $ED=PB$.

And the arcs $EF+FG=\text{arcs } PG+FG$ (19); therefore $EF=PG$.

For the points F and G are equally distant from their poles P , E .

Also the $\angle DEF=BPG$; for intersecting circles make equal angles at the sections. (31)

Therefore the triangles EFD and PGB are congruous. (27)

Therefore the arc $BG=\text{arc } DF$.

57.

PROP. XI.

If lines be drawn from the projected pole of any great circle, cutting the peripheries of the projected circle and plane of projection, the intercepted arcs of those circumferences are equal. Plate I.

On the plane of projection, AGB, let the great circle CFD be projected into $cf d$, and its pole P in p ; moreover, draw the lines pd , pf : the arcs GB and fd are equal.

Since pd lies both in the plane AGB and APBE it is their common section. (II. 198)

But the point B is in their common section: (56)

Therefore pd passes through the point B.

And in this manner it may be proved that pf passes through G.

Now the points D and F are projected into d and f . (38)

Therefore the arc fd is equivalent to the arc FD.

But the arc FD is equal to the arc GB: (56)

Therefore the arc GB is equivalent to the arc fd . (II. 46.)

58.

PROP. XII.

The radius of any small circle, the plane of which is perpendicular to that of the primitive circle, is equal to the tangent of that lesser circle's distance from its pole; and the secant of that distance, is equal to the distance of the centers of the primitive and lesser circle. Plate I.

Let P be the pole, and AB the diameter of a lesser circle, the plane being perpendicular to the plane of the primitive circle, the center of which is C: Then d being the center of the projected lesser circle, dA is equal to the tangent of the arc PA, and dc = secant of PA.

DEM. Draw the diameter ED parallel to AB, and through P draw cb . Now E being the projecting point, the diameter AB is projected in ab . (22)

And d , the middle of ab , is the center of a circle on ab . (39)

Then a right line drawn from D through A, will meet b : (II. 130)

And draw CA, dA .

Now the right-angled triangles DCB, DAE, having the angle D common; the $\angle DBC = \angle DEA$. (II. 98)

But $\angle DEA = \frac{1}{2} \angle DCA$; and $\angle DBC = \frac{1}{2} \angle ADC$: (II. 127)

Therefore $\angle DCA = \angle ADC$.

Now $\angle DCA + \angle ACD =$ a right angle.

Then $\angle ADC + \angle ACD =$ a right angle: Therefore $\angle CAD$ is right. (II. 96)

Consequently dA = radius of the circle AaB, is the tangent of the arc PA, to the radius CA. (II. 126)

And dc , the distance of the centers, is the secant of the arc AP. (III. 5.)

59. COROL. Hence the tangent and secant of any arc of the primitive circle, belongs also to an equal arc of any oblique circle; those arcs being reckoned from their intersection.

For the arc Pc of every oblique circle intercepted between P and the arc of the small circle AaB, is equivalent to the arc PA of the primitive circle; Because the arc AaB is equally distant from its pole P. (5)

SECTION II.

Spherical Geometry.

Spheric Geometry, or spheric projection, is the art of describing, or representing, such circles or arcs of circles as are usually drawn upon a sphere on the plane of any one of them; and of measuring such arcs, and their positions to one another, when projected.

60.

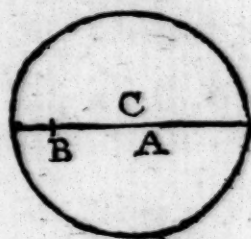
PROBLEM I.

To describe a great circle that shall pass through two given points in the primitive circle, or plane of projection.

Let the given points be A, B; and c the center of the prim. circle,

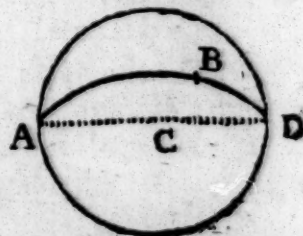
CASE 1. *When one point, A, is the center of the primitive circle.*

CONST. A diameter drawn through the given points A, B, will be the great circle required. (15)



CASE 2. *When one point, A, is in the circumference of the primitive circle.*

CONST. Through A draw a diameter AD. Then an oblique circle described through the three points A, B, D (II. 72) will be the great circle required. (42)



61. CASE 3. *When neither point is at the center or circumference of the primitive circle.*

CONST. Through one point A, and the center c, draw AG, and draw CE at right angles to AG.

A ruler by E and A gives D; by D and c gives F; and by E and F gives G, in AC continued.

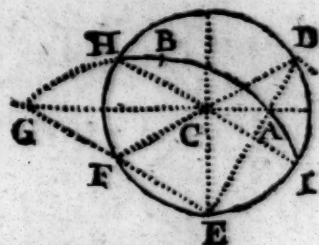
Through the three points G, B, A, describe a circumference (II. 72) cutting the primitive circle in H and I.

Then the oblique circle HBAI will be the great circle required.

For AG may be taken as the projection of the great circle FD. (12)

Therefore A and G are the projections of opposite points on the sphere. (32)

Consequently all circles passing through G and A, will be the representatives of great circles on the sphere.



62.

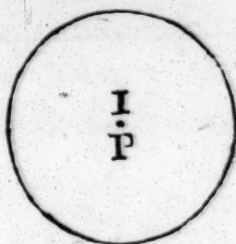
PROBLEM II.

About any given point as a pole, to describe a great circle in a given primitive circle.

Let P be the given point, and I the center of the primitive circle.

CASE I. When the given pole, P , is in the center of the primitive circle.

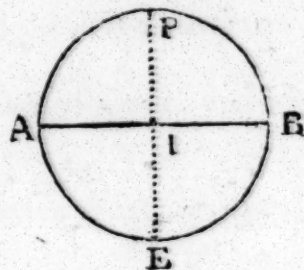
CONST. The primitive circle will be the great circle required. (13)



CASE 2. When the given pole P is in the circumference of the primitive circle.

CONST. Through the given pole P , draw PE a diameter to the primitive circle.

Then another diam. AB , drawn at right angles to PE , will be the great circle required. (20, 15)



63. CASE 3. When the given pole P is neither in the center or circumference of the primitive circle.

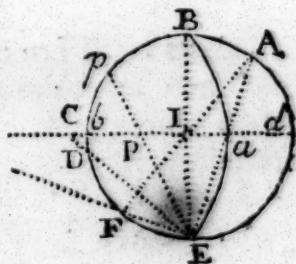
CONST. Through P draw a diameter bd , and another BE at right angles to bd ; then a ruler by E and P gives p .

Make the arc $pA = 90^\circ$; a ruler by E and A gives a in the diameter bd .

Then a circle described through the three points E , a , E , is the great circle required.

Or thus: Make the arc $pD = \text{arc } pB$; a ruler on E and D gives c in db produced.

Then on c , with the radius ca describe BAE .



For, As E is the projecting point, and P the projected pole

Therefore p is the pole of the circle AF to be projected. (44)

And BAE is the projection of the circle AF . (38)

Now $\angle CAE$ is measured by half the arc AdE . (IL 137)

But arc $ABD = \text{arc } AdE$: For $Ap = (Bd =) dE$; and $pD = Ad$ by construction.

Therefore $\angle AEC = \angle CAE$; and $CE = ca$.

Consequently c is the center required. (II. 104)

64.

PROBLEM III.

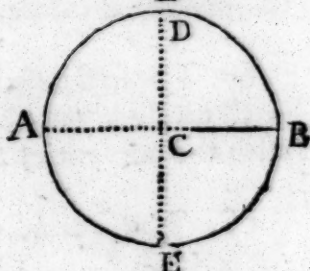
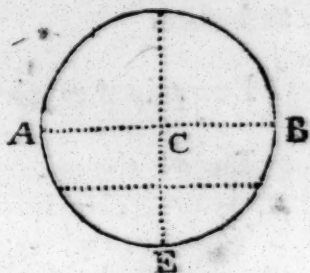
A projected circle being given; to find its poles.

CASE 1. *When the given circle AEB is the primitive.*

CONST. Find the center c , (II. 70) and it is the pole sought.

CASE 2. *When the given circle ACB is a right circle.*

CONST. Draw a diameter ED at right angles to AB , and the ends or points, D , E , of that diameter are the poles required.



65. CASE 3. *When the given circle ABE is oblique.*

CONST. Through the intersections of the primitive and oblique circles draw a diameter AE , and another at right angles to AE , cutting the given oblique circle in B .

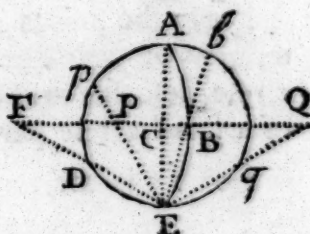
A ruler by E and B gives b ; make bp , bq , each = an arc of 90° .

A ruler by E and p gives, in the diameter through B , the point P , which is the pole required.

And a ruler by E and q gives, in CB continued, the point Q for the other, or opposite or exterior pole.

Make $pD = pA$; then a ruler by E and D gives, in BC continued, the point F , which is the center of the oblique circle ABE .

The reason of this operation is evident from that of the last Problem.



66:

PROBLEM IV.

About any given projected pole, to describe a circle at a given distance from that pole.

Or, at a proposed distance from a given great circle, to describe a parallel circle.

Let P be the given pole, belonging to the given great circle DPE .

GENERAL SOLUTION. Through the given pole P , and c the center of the primitive circle, draw a diameter, and another LE at right angles to it.

A ruler on E and P gives p in the primitive circle.

Make pA and pB , each equal to the proposed distance from the pole.

A ruler on E and A , and then on E and B , will cut the diameter CP in a and b .

Bisect ab in c ; and on c as a center describe a circle passing through a and b , which will be the circle required.

But

But when the parallel circle is to be at a proposed distance from the given great circle DPE ,

Find p as before; and make $pA = pB$, equal to the complement of the proposed distance; the rest as before.

For p is the pole, the projection of which is P . (44)

But p is the pole of a circle, the diameter of which AB is projected in ab . (12)

Therefore c , the middle of ab , is the center of the projected circle. (39)

67. The first case is readily done, by describing the small circle about the center of the primitive circle with the tangent of half its distance from the pole P .

68. The second case is soonest performed thus.

From the points A, B , (found as above) with the tangent of their distance from P , the pole of the right circle, describe arcs cutting in c , which is the center of a small circle parallel to the right circle DPE .

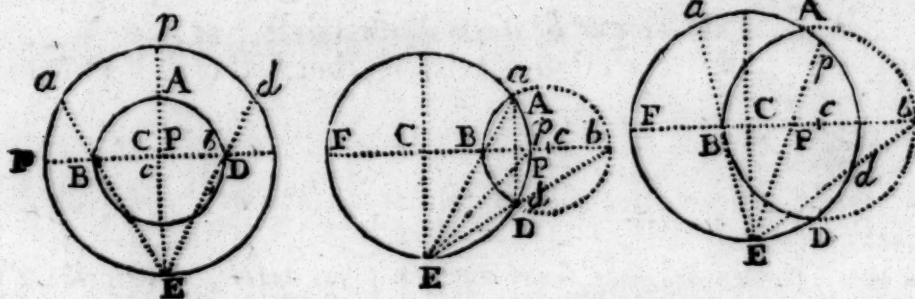
For Ap is the tangent of the arc AP .

(58)

69.

PROBLEM V.

The primitive circle, and the projection of a small circle, being given; to find the pole of that small circle.



Let c be the center of the primitive circle, and ABD a projected small circle, the center of which is c , and radius CB .

GENERAL SOLUTION. Through c the center of the small circle, and c the center of the primitive, draw a diameter CF , and another, CE , at right angles to it.

Find the projected diameter $Bb = 2CB$.

Lines drawn from E through B and b , cut the primitive circle in a, d ; then bisect the arc ad in p .

A ruler by E and p cuts the diameter Bb in P , the pole sought.

The truth of this construction is evident by that of the last Prob.

70. PRO-

PROBLEM VI.

70.

To measure any arc of a projected great circle: Or, in a given projected great circle, to take an arc of a given number of degrees.

GENERAL SOLUTION. Find the pole of the given circle. (64)

From that pole draw lines through the ends of the proposed arc, cutting the primitive circle.

Then the intercepted arc of the primitive circle applied to the scale of chords will give the measure sought.

Thus, if AB be the arc to be measured, and P the pole of the given circle DAF .

Then lines drawn from P through A and B , give the arc ab in the primitive circle, corresponding to AB in the projected circle.

Now if an arc of a given number of degrees was to be taken from a given point A , in the given projected circle DAF .

Draw, from the pole P , through A , the line pa to the primitive circle.

Apply the given number of degrees from a to b .

Draw Pb , and the intercepted arc AB will contain the degrees proposed.

71. Any number of degrees is readily applied to a right circle by the scale of half-tangents. Thus

When the distance of the point A from the center C is known, and the given quantity of the arc is to be laid from A toward F ;

To the known distance CA add the proposed arc AB , the degrees in the sum taken from the scale of half-tangents, and laid from c to B , will make the arc AB equal to the degrees proposed.

But when the arc AB is to be laid from A toward D ;

Then the difference between the arcs AB and AC , taken from the scale of half-tangents and laid toward D from c to B , will make the arc AB equal to the degrees proposed.

The reason of all these operations is evident from art. 57.

Note, The half, or semi-tangents, are only the tangents of half the arcs the scale of tangents is made to; their construction depends on art. 48. On the *plane scale* they are put under the Tangents, and marked *s. t.*

72.

PROBLEM VII.

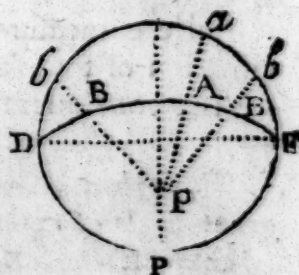
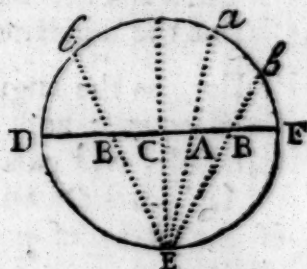
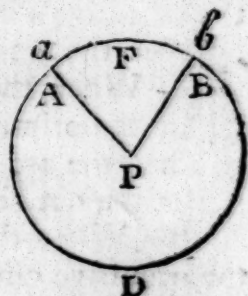
To measure any projected spherical angle.

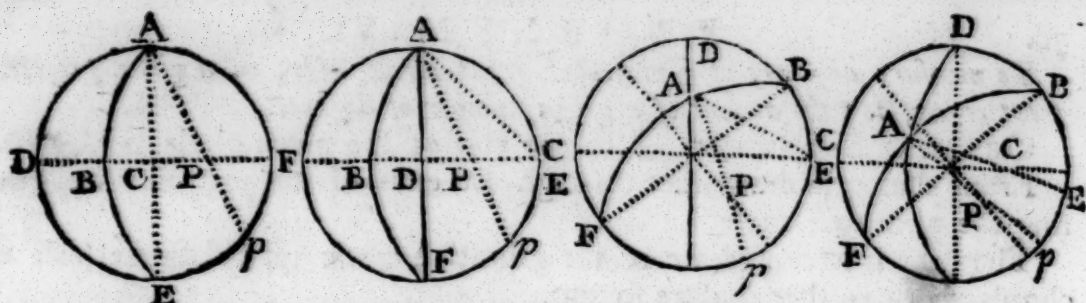
GENERAL SOLUTION. Find the poles of the two circles which form the angle (64); and from the angular point draw lines through those poles to cut the primitive circle.

Then the measure of that angle, if acute, will be the intercepted arc of the primitive circle; or the supplement of that arc will be the measure of the angle when obtuse.

Let the proposed angle DAB , formed by the great circles AD , AB , the poles of which are c and P ; and lines drawn from the angular point A , through the poles c and P , cut the primitive circle in E and p .

1st. When





1st. When the angle is formed by the primitive and oblique circles ;
Then the arc pE measures the acute angle DAB .

But the obtuse angle BAF is measured by the supplement of pE .

2d. When the angle is formed by right and oblique circles meeting in the primitive's circumference ;

Then the arc pE measures the angle DAB .

3d. When the angle is formed by right and oblique circles meeting within the primitive circle ;

Then the arc pE measures the acute angle DAB .

But the obtuse angle DAF is measured by the supplement of pE .

4th. When the angle is formed by two oblique circles meeting within the primitive circle ;

Then the acute angle DAB is measured by the arc pE .

But the supplement of pE measures the obtuse angle DAF .

For, as the angular point A is in both circles, and 90° distant from their poles c and P (19). Therefore a great circle described about A , as a pole, will pass through the poles c and P .

And lines drawn from A through c and P , cut off, in the circumference of the plane of projection, an arc equal to the distance of the poles c and P . (57)

But the measure of the distance of the poles c , P , is equal to the inclination of the planes of the circles AD , AB ; (33)

And consequently measures the angle DAB .

73.

PROBLEM VIII.

Through a given point in any projected great circle, to describe another great circle at right angles to the given one.

GENERAL SOLUTION. Find the pole of the given circle. (64)

Then a great circle described through that pole and the given point will be at right angles to the given circle.

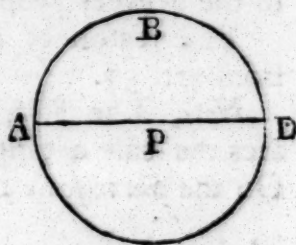
Let the given projected great circle be BAD ; and A the given point.

1st. When BAD is the primitive circle, the pole of which is P .

A diameter through A will be perpendicular to BAD . (II. 136)

2d. When BAD is a right circle, the poles of which are P and C .

An oblique circle described through the points C , A , P , (II. 72) will be at right angles to BAD .

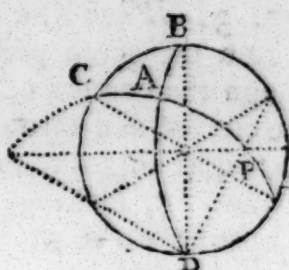


3d. When

3d. When BAD is an oblique circle, the pole of which is P .

Through the points P and A , a great circle PAC being described (61), will be at right angles to BAD .

The truth of these operations is evident from art. 34.



74.

PROBLEM IX.

Through any assigned point in a given projected great circle, to describe another great circle cutting the former in an angle of a given number of degrees.

Let P be a given point in any great circle APB .

1st. When APB is the primitive circle.

Through the given point P draw a diameter PE , and draw the diameter AB at right angles to PE .

Draw PD cutting AB in D , so that the angle CPD be equal to the angle proposed.

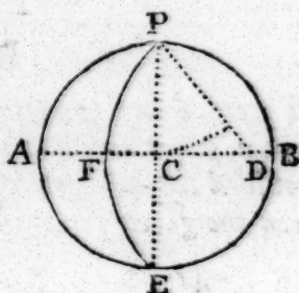
On D with the radius DP describe the great circle PFE .

Then will the angle APF contain the given degrees.

For the $\angle FPA =$ angle made by the radii PC, PD .

And D being equally distant from P and E , is the center sought.

(49)



75. Or thus: Make CD equal to the tangent of the given angle to the radius CP .

Or, Make PD equal to the secant of that angle.

76. 2d. When APB is a right circle.

Draw a diameter GH at right angles to APB .

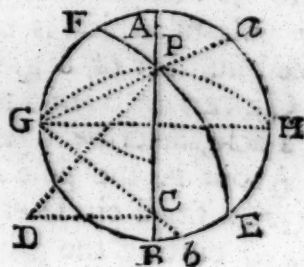
Then a ruler by G and P gives a in the primitive circle.

Make $Hb = 2Aa$; a ruler by G and b gives c in AB .

Draw CD at right angles to AB .

Draw PD cutting CD in D , so that the $\angle CPD =$ complement of the degrees given. (II. 84)

On D with the radius DP describe a circle FPE , which will be a great circle, making with APB the angle APF , as required.



For c is the center of a great circle GPH , by the demonstration to art. 63.

And the centers of all great circles through P , will be in CD . (55)

Now $\angle DPE = 90^\circ$.

(II. 136)

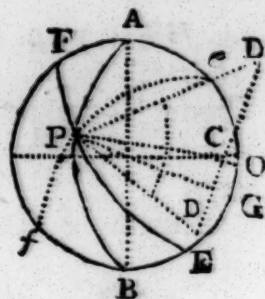
Therefore $\angle APF$ or $\angle BPE$ (26), the compl. of CPD , is the angle sought.

77. 3^d. *When APB is an oblique circle.*

From the given point P, draw the lines PG, PC, through the centers of the primitive and given oblique circles, and through c the center of APB draw CD at right angles to PG. (II. 59)

Draw PD, making the $\angle CPD =$ given degrees, and cutting CD in D. (II. 84)

From D with the radius DP, a circle FPE being described, will be a great circle cutting APB in the angle proposed.



For c, the center of APB, is in a line perpendicular to PG, drawn through P and the center of the primitive, by construction.

And the centers of all great circles through P will be in CD. (55)

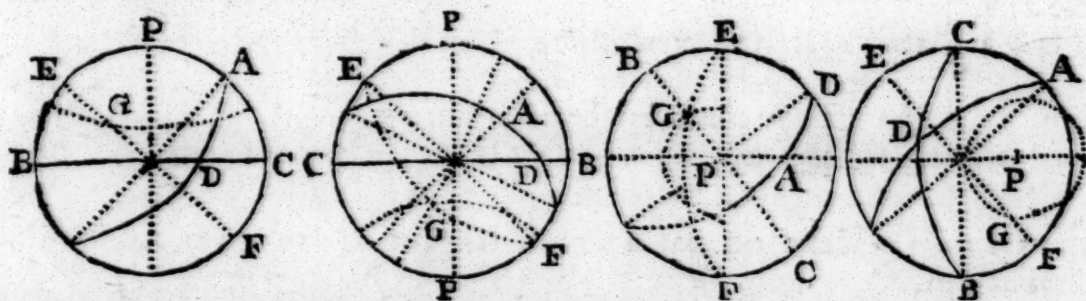
Now the $\angle CPD$ made by the radii PC, PD, contains the given degrees.

Therefore the angle APF is equal to the angle required. (49)

78.

PROBLEM X.

Through any point in the plane of projection, or primitive circle, to describe a great circle that shall cut a given great circle in any angle proposed: Provided the measure of that proposed angle is not less than the distance between the given point and circle.

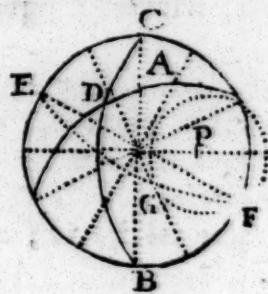


Let the given point be A, through which a circle is to be described to cut a given great circle BDC, the pole of which is P, in an angle equal to a proposed number of degrees.

GENERAL SOLUTION. About the given point A, as a pole, describe a great circle EGF. (62)

About P, the pole of the given circle BDC, describe a small circle at a distance equal to the given angle, cutting the great circle EGF in G. (66)

About the point G, as a pole (62,) describe a great circle cutting the given circle BDC in D. Then will ADC be the angle required.



Note, When the given angle is equal to the distance between the given point and circle, the problem is limited to one answer only: When the measure of the angle is greater, the problem has two solutions by the circle described cutting the given one in two points: But when the measure of the angle is less, the problem is impossible. This construction is thus proved.

P and G are the poles of BC and AD.

And

And the distance of P and G is equal to the degrees in the proposed angle, by construction. But $\angle ADC = \text{distance of P and G.}$ (33)
Therefore the $\angle ADC$ is the angle required.

79. *When the required circle is to make a given angle with the primitive.*

Then, from the center of the primitive, with the tang. of the given angle, describe an arc; and from the given point A, with the secant of the given angle, cut the former arc.

On this intersection, a circle being described through the given point A, will cut the primitive circle in the angle proposed.

This depends on art. 75.

80.

PROBLEM XI.

Any great circle, cutting the primitive, being given, to describe another great circle, which shall cut the given one in a proposed angle, and have a given arc intercepted between the primitive and given circles.

Let ABC be the primitive circle, the center of which is P; and the given great circle be ADC, the center of which is E.

SOLUTION. Draw a diameter EBD at right angles to ADC; and make the angle BDF equal to the complement of the given angle; suppose = complement of 35° .

Make DF equal to the tangent of the given arc (suppose 58°); and from P, with the secant of that arc, describe an arc Gg.

Now when ADC is an oblique circle; from E the center of ADC, with the radius EF, cut the arc Gg in G.

But when ADC is a right circle; through F draw FG parallel to ADC, cutting the arc Gg in G.

From G, with the tangent GF, describe an arc, πo , cutting ADC in I; and draw GI.

Through G and the center P draw GK, cutting the primitive circle in H, K; draw PL perpendicular to GK; and IL at right angles to IG, cutting PL in L.

And L will be the center of a circle passing through H, I, K, which will be the great circle required.

Then the $\angle AIH = 35^\circ$; and arc $IH = 58^\circ$, as was proposed.

For GP is the secant, and GI is the tang. of the arc HI.

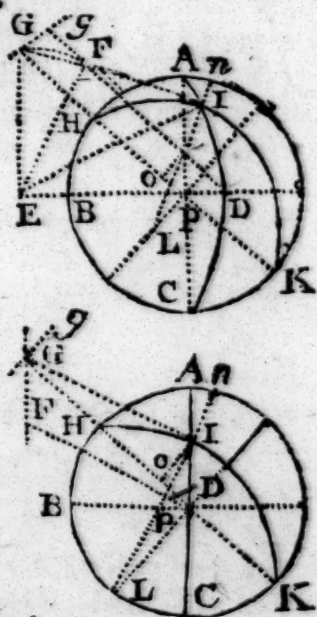
And as the triangles EGI, EFD, are congruous; the $\angle EIG = \angle EDF$. (59)
(II. 101)

But the $\angle EIG$ made by the tangent of the arc HI and the radius of the arc AI, is the complement of the angle made by those arcs. (49)

Consequently the $\angle AIH$ is the complement of the $\angle EDF$.

The center of the right circle AC being supposed at an infinite distance, therefore any circle FG described from that center, will be parallel to AC.

When



When the given arc is more than 90° , the tangent and secant of its supplement is to be applied on the line DF the contrary way, or toward the right; the former construction being reckoned to the left.

81.

PROBLEM XII.

Any great circle in the plane of projection being given; to describe another great circle, which shall make given angles with the primitive and given circles.

Let the given great circle be ADC , and its pole Q .

SOLUTION. About P , the pole of the primitive circle, describe an arc mn , at the distance of as many degrees as are in the angle which the required circle is to make with the primitive: Suppose 62° . (67)

About Q , the pole of the other given circle, and at a distance equal to the measure of the angle which the required circle is to make with the given circle ADC (suppose 48°), describe an arc on , cutting mn in n . (66)

About n , as a pole, describe the great circle EDF , cutting the given circles in E and D . (62)

Then is the angle $AED = 62^\circ$; and $ADE = 48^\circ$.

For the distance of the poles of any two great circles, is equal to the angle which those circles make with one another. (33)

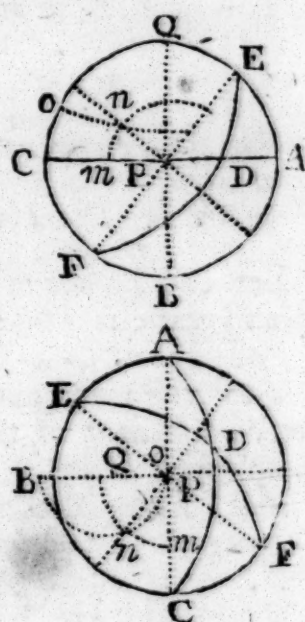
REMARK. The 11th Problem, which is particularly useful in constructing a spherical triangle, in which are given two angles and a side opposite to one of them, includes only two cases of a more general Problem, viz.

Any two great circles being given in position; to describe a third, which shall cut one of those given in an angle proposed, and have a given arc intercepted between the given circles.

Also the 12th Problem, used when the three angles are given, contains only two cases of another Problem; viz.

Any two great circles being given in position, to describe a third which shall cut the given circles in given angles.

The solution of these two general Problems not being wanted in any part of this work; it was not thought necessary here to annex them; more having been already delivered in the preceding pages than it is usual to meet with on this subject. However, their solution is recommended as exercises to speculative learners.



SECTION IV.

Spheric Trigonometry.

DEFINITIONS.

82. SPHERIC TRIGONOMETRY is the art of computing the measures of the sides and angles of such triangles as are formed on the surface of a sphere, by the mutual intersections of three great circles described thereon.

83. A SPHERIC TRIANGLE consists of three sides and three angles.

The measures of unknown sides or angles of spheric triangles are estimated by the relations between the sines, or the tangents, or the secants, of the sides or angles known, and of those that are unknown:

84. A RIGHT ANGLED SPHERIC TRIANGLE has one right angle: The sides about the right angle are called Legs, and the side opposite to the right angle is called the Hypothenufe.

85. A QUADRANTAL SPHERIC TRIANGLE has one side equal to ninety degrees.

86. An OBLIQUE SPHERIC TRIANGLE has all its angles oblique.

87. The CIRCULAR PARTS of a triangle, are the arcs which measure its sides and angles.

88. Two spheric triangles are said to be supplements to one another, when the sides and angles of the one are respective supplements of the angles and sides of the other: And one, in regard to the other, is called the supplemental triangle.

89. Two arcs or angles, when compared together, are said to be alike, or of the same kind, when both are acute, or less than 90° , or when both are obtuse, or greater than 90° : But when one is greater and the other less than 90° , they are said to be unlike.

The lesser circles of the sphere do not enter into Trigonometrical computations, because of the diversity of their radii.

SECTION V.

Spherical Theorems.

90.

THEOREM I.

In every spheric triangle, ABC, equal angles, B, C, are opposite to equal sides, AC, AB: And equal sides, AB, AC, are opposite to equal angles, C, B.

DEM. Since $AB=AC$, make $AE=AD$; and draw BD, CE.

Then is $ED=CE$; and $\angle AEC=\angle ADB$. (27)

For the triangles AEC, ADB, are congruous,

Since $AB=AC$; $AD=AE$; $\angle A$ common.

Also, the triangles BEC, CDB are congruous; (27)

Therefore $\angle EBC=\angle DCB$.

For $EC=BD$; $EB=(AB-AE)=DC(=AC-AD)$. (II. 48)

And $\angle BEC=\angle CDB$, they being the suppl. of equal angles AEC, ADB.

Again, if $\angle ABC=\angle ACB$: Then is $AB=AC$.

For take $BE=CD$; and describe the arcs CE, BD.

Then is $EC=DB$, $\angle BEC=\angle CDB$; $\angle BCE=\angle CBD$. (27)

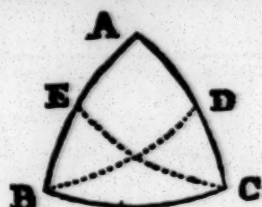
For $\triangle BCE=\triangle CBD$; since BC is common, $BE=CD$, $\angle EBC=\angle DCB$.

Also the triangles ABD, ACE, are congruous;

Since $EC=DB$, $\angle ACE=(\angle BCA-BCE)=\angle ABD(=\angle CBA-CBD)$. (II. 48)

And $\angle AEC=(\text{sup. } \angle BEC)=\angle ADB(=\text{sup. } \angle CDB)$. (II. 48)

Therefore $AE=AD$; and $AB=(AE+EB)=AC(=AD+DC)$. (II. 47)



91. COROL. A line drawn from the vertex of an isosceles spheric triangle, to the middle of the base, is perpendicular to the base.

This is easily proved from article 90, 27.

92.

THEOREM II.

Either side of a spheric triangle is less than the sum of the other two sides.

DEM. For on the surface of the sphere, the shortest distance between two points, is an arc of a great circle passing through those points. (23)

But each side of a spheric triangle is an arc of a great circle. (10)

Therefore either side being the shortest distance between its extremities, is less than the sum of the other two sides.

93.

THEOREM III.

Each side of a spheric triangle is less than a semicircle, or 180 degrees.

DEM. Two great circles intersect each other twice at the distance of 180 degrees. (32)

The sides about any spheric angle are arcs of two great circles. (10)

But a spheric triangle has three sides.

Therefore every two sides before their second meeting must be intersected by the third side.

Consequently each side is less than a semicircle.

* The mark $\triangle$ stands for the word triangle.

THEOREM IV.

94.

In every spheric triangle, ABC , the greatest side, BC , is opposite the greatest angle, A .

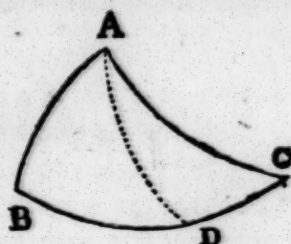
DEM. Make $\angle BAD = \angle ABC$.

Then $AD = BD$ (90); and $BC = AD + DC$.

But $AD + DC$ is greater than AC .

Therefore BC is greater than AC .

(92)



95.

THEOREM V.

If from the three angles of a spheric triangle, ABC , as poles, be described three arcs of great circles, forming another spheric triangle, EDF ; then will the sides of the latter, and the opposite angles of the former, be the supplements of one another: Also the angles in the latter, and their opposite sides in the former, are the supplements of one another.

That is, FE and $\angle CAB$, FD and $\angle ABC$, DE and $\angle ACB$, are supplements to one another.

Also $\angle E$ and AC , $\angle D$ and CB , $\angle F$ and AB , are the supplements to one another.

DEM. The intersection E of the arcs about the poles A and C , being 90° distant from them, is the pole of the arc AC .

(19)

And for the same reason, D is the pole of CB , and F of AB .

Let the sides of the triangle ABC be produced to meet the sides of the triangle DEF in G and H , I and L , M and N .

Then $FI = DL = 90^\circ$: Therefore $(DL + FI = DL + FL + LI =) DF + LI = 180^\circ$.

(II. 47)

Therefore DF and LI are supplements to one another.

But LI measures the angle ABC .

(9)

Therefore $\angle ABC$ and DF are the supplements to one another.

And in the same manner it may be demonstrated, that the $\angle BAC$ and FE , $\angle ACB$ and DE , are the supplements of one another.

Again, since $BI = AH = 90$ degrees;

(19)

Therefore $(IB + AH = IB + BH + AB =) IH + AB = 180$ degrees.

But IH measures the angle F ;

(9)

Therefore AB and $\angle F$ are the supplements of one another.

And the same may be shewn of AC and $\angle E$, CB and $\angle D$.

96.

THEOREM VI.

The sum of the three sides of every spheric triangle, ABC , is less than a circumference, or 360 degrees.

DEM. Continue the sides AC , AB , till they meet in D .

Then the arcs ACD , ABD , are each 180° . (32)

But $DC + DB$ is greater than BC .

(92)

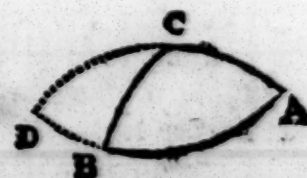
Therefore $AC + AB + DC + DB$ is greater than $AC + AB + BC$.

Or the semicircles $ACD + ABD$ is greater than $AC + AB + BC$.

That is, 360° is greater than the three sides of the triangle ABC .

L 2

97. THEO.



97.

THEOREM VII.

The sum of the three angles of a spheric triangle, ABC , is greater than two right angles, and less than six; or will always fall between 180 and 540 degrees.

DEM. Since $\angle A$ and FE , $\angle B$ and FD , $\angle C$ and DE , are supplements to one another; (95)

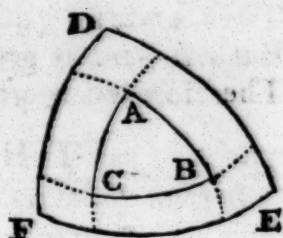
Therefore the three angles A, B, C , together with the three sides FE, FD, DE , make thrice 180° , or 540° .

Now the sum of the three sides $FE + FD + DE$, is less than twice 180° . (96)

Therefore the sum of the three angles $A + B + C$ is greater than 180° .

Again, as a spheric angle is always less than 180° ; (24)

Therefore the sum of any three spheric angles is ever less than thrice 180° , or 540 degrees.



98.

THEOREM VIII.

If one side, AB , of a spheric triangle, ABC , be produced, then the outward angle, CBD , is either equal to, less, or greater than the inward opposite angle A , adjacent to that side; according as the sum of the other two sides, $CA + CB$, is equal to, greater, or less than 180 degrees.

DEM. Produce AC, AB , to meet in D .

Then arc $ACD = \text{arc } ABD = 180^\circ$. (32)

And $\angle D = \angle A$. (31)

Now if $AC + CB$ is equal to 180° ; then $CB = CD$.

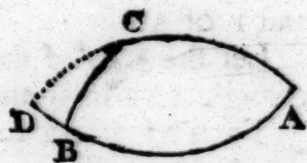
And $\angle CBD = (\angle D =) \angle A$. (90)

If $AC + CB$ is greater than 180° ; then CB is greater than CD .

And $\angle CBD$ is less than $(\angle D =) \angle A$. (94)

If $AC + CB$ is less than 180° ; then CB is less than CD . (94)

And $\angle CBD$ is greater than $(\angle D =) \angle A$.



99.

THEOREM IX.

In right angled spheric triangles, the oblique angles and their opposite sides are of the same kind: That is, if a leg is less or greater than 90° , its opposite angle is also less or greater than 90° .

In the right angled spheric triangle ABC , right angled at A .

If AC is greater than 90° ; then $\angle ABC$ is greater than 90° .

If AC is less than 90° ; then $\angle ABC$ is less than 90° .

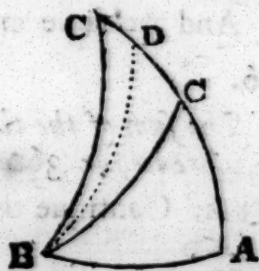
DEM. Let the leg AC be less, AD equal, AC greater, than 90° , and describe the arc DB .

Now D being the pole of AB (37). Therefore $\angle DBA$ is right.

Consequently if AC is less than AD , the $\angle CBA$ is less than $\angle DBA$.

But if AC is greater than AD , the $\angle CBA$ is greater than $\angle DBA$.

And the same may be proved of the leg AB and its opposite angle.



100. THEO-

100.

THEOREM X.

In right angled spheric triangles, BAC , the hypotenuse, BC , is less than 90° , when the legs, AB , AC , are of a like kind: But the hypotenuse is greater than 90° , when the legs are of different kinds.

1st. When the legs AB , AC , are both less than 90° .

DEM. In BA , AC , produced, take BD , AF , equal to quadrants; through F and D describe an arc FD meeting BC , produced in E .

Now F being the pole of BD (19). Therefore B is the pole of ED .

Consequently BC is less than $(BE =) 90^\circ$.

2d. When the legs AB , AC , are both greater than 90° .

Produce AC , AB , till they meet in D .

Now the hypotenuse CB is common to both the right angled triangles BAC and BDC .

And the legs DC , DB , being both less than 90° .

Therefore the hypotenuse BC is less than 90° by the first case of this Theorem.

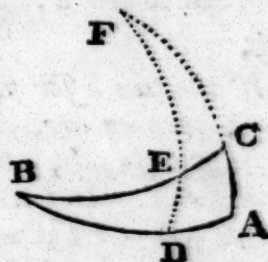
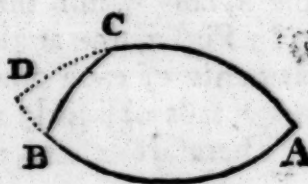
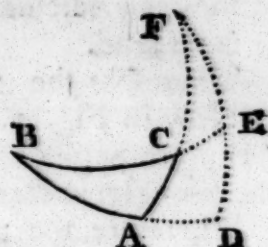
3d. When the legs AB , AC , are one greater, the other less than 90° .

In AB , and AC produced, take BD , AF , each of 90° , and describe the arc FED .

Then B is the pole of FD ; and since F is the pole of BA , and FD is at right angles to BD . Therefore $BE = 90^\circ$.

(37)

Consequently BC is greater than 90 degrees.



101. COROL. I. The hypotenuse is less or greater than 90° , according as the oblique angles are of a like or different kinds.

For if legs are like or unlike, the angles are like or unlike. (99)

And if legs are like or unlike, the hypoth. is acute or obtuse. (100)

Therefore if the angles are like, the hypotenuse is acute, or less than 90° ; but if unlike, the hypotenuse is obtuse, or greater than 90° .

102. COROL. II. The legs and their adjacent angles are like or unlike, as the hypotenuse is less or greater than 90 degrees.

For like legs or like angles, make the hypotenuse acute (by 1st and 2d of 100).

And unlike legs or unlike angles, make the hypotenuse obtuse (by 3d of 100 and by 101).

103. COROL. III. A leg and its opposite angle are both acute or both obtuse, according as the hypotenuse and other leg are like or unlike.

This is evident from the three cases of this Theorem.

104. COROL. IV. Either angle is acute or obtuse, as the hypotenuse and the other angle are like or unlike.

This follows from case 1st and 2d of this Theorem.

105.

THEOREM XI.

In every spheric triangle, ABC, if the angles adjacent to either side, AB, be alike, then a perpendicular, CD, drawn to that side from the other angle, will fall within the triangle: But the perpendicular CD falls without the triangle, when the angles adjacent to the side AB it falls on are unlike.

DEMONST. Since in all right angled triangles, the perpendicular and its opposite angle are of the same kind. (99)

Therefore the $\angle$ s CAD, CBD, are each like CD.

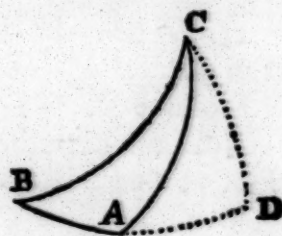
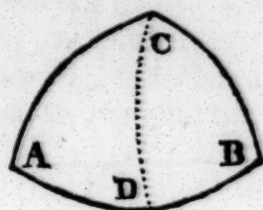
Now in Fig. 1. the angles CAD, CBD, or CAB, CBA, are angles adjacent to the base AB within the triangle, and are therefore alike.

Therefore the perpendicular falling between A and B, falls within the triangle.

In Fig. 2. the angles CAD and CAB are the supplements of each other, and are therefore unlike, as CA falls obliquely on AB.

Therefore $\angle$ CAB is unlike to $\angle$ CBA.

Consequently the perpendicular CD cannot fall between A and B: Therefore it must fall without.



106.

THEOREM XII.

If the two lesser sides, CA, CB, of a spheric triangle, ABC, are of the same kind; then an arc, CD, drawn from their included angle, ACB, perpendicular to the opposite side, AB, will fall within the triangle.

DEMONST. In AB take AF=AC; draw CF and AH at right angles to CF.

Then CH=HF (91) are each less than 90° . (93)

Also take BE=BC; draw CE and EG at right angles to CE.

Then CG=GE (91) are each less than 90° . (93)

Now in the right angled triangles FHA, EGB; if the hypotenuse AF (=AC), and BE (=BC), are acute, or like FH and EG;

Then the angles AFH and BEG are acute, and like AC and BC; (103)

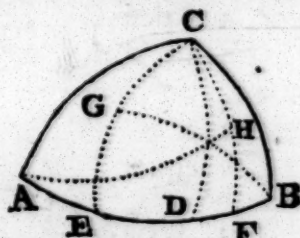
Therefore the perpendicular CD falls on EF, within the triangle. (105)

Also if the hypotenuses AF and BE are obtuse, or unlike to FH and GE;

Then the angles AFH and BEG are obtuse, and also like CA and CB. (103)

Consequently the perpendicular will fall on EF. (105)

Therefore in either case the perpendicular falls within the triangle.



107.

THEOREM XIII.

In all right angled spheric triangles,

As sine hypoth. : Rad :: sine of a leg : sine of its opposite angle.

108. *And sine a leg : Rad. :: tan. other leg : tan. of its opposite angle. Pl. I.*

DEMONST. Let EDAFG represent the eighth part of a sphere, where the quadrantal planes EDFG, EDBC, are both perpendicular to the quadrantal plane ADFB; and the quadrantal plane ADGC is perpendicular to the quadrantal plane EDFG; and the spheric triangle ABC is right angled at B, where CA is the hypotenuse, and BA, BC, are the legs.

To the arcs GF, CB, draw the tangents HF, OB, and the sines GM, CI, on the radii DF, DB; also draw BL, the sine of the arc AB, and CK, the sine of AC; then join IK and OL.

Now

Now HF, OB, GM, CI, are all perpendicular to the plane ADFB.

And HD, CK, OL, lie all in the same plane ADGC.

Also FD, IK, BL, lie all in the same plane ADFB.

Therefore the right angled triangles HFD, CIK, OBL, having the equal angles HDF, CKI, OLB, (II. 199) are similar. (II. 167)

Therefore $CK : DG :: CI : GM$.

That is, As sin. hyp. : Rad. :: sin. of a leg : sin. opp. angle.

For GM is the sine of the arc GF, which measures the angle CAB. (9)

Also, As $LB : DF :: BO : FH$.

That is, As sin. of a leg. : Rad. :: tan. of other leg : tan. opp. angle.

109.

THEOREM XIV.

In right angled spheric triangles, ABC, if about the oblique angles, A, C, as poles, at 90° distance, there be described arcs, DE, FE, cutting one another in E; and the sides AB, AC, BC, of the triangle be produced to cut those arcs in D; G, F; H, I; there will be constituted two other triangles, CGH, HIE, the parts of which are either equal to, or are the complements of, the parts of the given triangle, ABC. Pl. I.

DEMONST. Now since A is the pole of ED (19). Therefore AD, AG, are at right angles to ED; and so is ED to AD. (37)

And since BI and DE are at right angles to AD, their intersection H is the pole of AD (36). Therefore HB, HD are quadrants (35)

Then in the triangle CGH, right angled at G.

CG = complement of AC.

HG = comp. $\angle A$; For HG is the comp. of GD, which mea. $\angle A$. (9)

HC the hypth. is the comp. of CB.

The $\angle HCG = \angle ACB$. (26)

The $\angle CHG = \text{comp. } AB$: For BD, the comp. of AB, measures $\angle CHG$.

Also in the triangle EIH, right angled at I: Because CF, CI, are at right angles to EF; and EF, EG, being also at right angles to AF; therefore E is the pole of AF; (36) consequently EF and EG are quadrants. (35)

Then the hypth. EH = $\angle A$; For GH = comp. of EH and GD; and GD measures the angle A.

HI = CB; for HC = comp. of HI and CB.

EI = comp. $\angle C$; For EI = comp. of IE, which measures $\angle C$.

The $\angle H = \text{comp. } AB$; For BD, the comp. of AB, measures $\angle H$.

The $\angle E = AC$; For GF, the measure of $\angle E$, is equal to $\angle AC$.

110.

THEOREM XV.

In every spheric triangle, it will be,

As the sine of either angle, is to the sine of its opposite side;

So is the sine of another angle, to the sine of its opposite side.

Let ABC be a spheric triangle, where BD is perpendicular to AC produced; forming the two right angled triangles ADB, CDB.

DEM. Now sin. AB : rad. :: sin. BD : sin. $\angle A$. (107)

And sin. BC : rad. :: sin. BD : sin. $\angle C$.

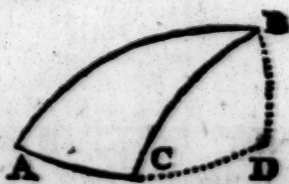
Therefore sin. AB $\times$ sin. $\angle A = \text{rad.} \times \text{sin. BD}$.

And sin. BC $\times$ sin. $\angle C = \text{rad.} \times \text{sin. BD}$.

Therefore sin. AB $\times$ sin. $\angle A = \text{sin. BC} \times \text{sin. } \angle C$.

Therefore sin. $\angle A : \text{sin. BC} :: \text{sin. } \angle C : \text{sin. AB}$.

L 4



(II. 162)

(II. 162)

(II. 46)

(II. 162)

SECTION

SECTION VI.

Of the Solution of right angled spheric Triangles.

In every case of right angled spheric triangles, three things beside the radius enter the proportion, of which two are given, and the third is sought.

Now the solution of every case will be obtained by the application of the two following rules to Theorem XIII. and XIV. (107, 108, 109.)

III. RULE I. If of the three things concerned, or their complements, two are opposite to one another, and the third is opposite to the right angle, in one of the triangles marked 1, 2, 3, in the fig. to Theo. XIV. Pl. I. Then the thing sought will be found by the first proportion (107) either directly, or by inversion.

III.2. RULE II. If of the three things concerned, or their complements, two are sides, and the third is an oblique angle, in either of the three triangles marked 1, 2, 3, in fig. to Theo. XIV. Pl. I. Then the thing sought will be found by the second proportion (108) either directly, or by inversion.

III.3.

PROBLEM I.

In the right angled spheric triangle ABC, Plate I. Theorem XIV.

Given the hypotenuse AC } Required the rest.
and one of the legs AB }

1st. To find the angle ACB opposite the given leg AB.

Here the things concerned are AC, $\angle B$, AB, $\angle C$; which are found in the triangle, N° 1, to be opposite; and so fall under Rule I. (III)

Then $\sin. AC : \text{rad.} :: \sin. AB : \sin. \angle ACB.$ (107)

Or $\sin. \text{hyp.} : \text{rad.} :: \sin. \text{g. leg} : \sin. \text{op. } \angle$ Like the g. leg. (99)

2d. To find the angle CAB adjacent to the given leg AB.

Here the things concerned are AC, $\angle B$, AB, $\angle A$.

Now trying in the triangle, N° 1, I find the things concerned will fall under neither of the Rules.

But trying in the triangle, N° 2, the things concerned, or their complements, fall under Rule II. (III.2)

Then $\sin. HG : \text{rad.} :: \tan. GC : \tan. \angle CHG.$ (108)

Or $\text{co-f. } \angle CAB : \text{rad.} :: \text{co-t. } AC : \text{co-t. } AB.$

Or $\text{co-f. } \angle CAB : \text{co-t. } AC :: (\text{rad.} : \text{co-t. } AB) :: \tan. AB : \text{rad.}$ (III. 36)

Therefore $\text{rad.} : \text{co-t. hyp.} :: \tan. \text{g. leg.} : \text{co-f. adj. angle.}$ (II. 145)

Like or unlike the given leg; as the hyp. is acute or obtuse. (102)

3d. To find the other leg BC.

Here the things concerned are AC, $\angle B$, AB, BC; which in the triangle, N° 1, do not fall under either Rule: But in N° 2 they will be found to fall under the first Rule. (III)

Then $\sin. HE : \text{rad.} :: \sin. CG : \sin. \angle CHG.$ (107)

Or $\text{co-f. } CB : \text{rad.} :: \text{co-f. } AC : \text{co-f. } AB.$

Therefore $\text{co-f. g. leg, AB,} : \text{rad.} :: \text{co-f. hyp. AC,} : \text{co-f. req. leg CB.}$

And is acute, if hyp. and given leg are like; but obtuse, if unlike. (103)

III.4. PRO-

114.

PROBLEM II.

In the right angled spheric triangle ABC. Pl. I. Theorem XIV.

Given the hypotenuse AC
 And one of the oblique angles A } Required the rest.

1st. To find the leg CB opposite to the given angle A.

In the triangle, N° 1. the things concerned fall under Rule I. (111)

Then rad. : sin. AC : : sin. $\angle$ CAB : sin. CB. (107)

Or rad. : sin. hyp. : : sin. given angle : sin. opp. side.

And is like the given angle. (99)

2d. To find the leg AB adjacent to the given angle A.

In the triangle, N° 2. the things concerned fall under Rule II. (112)

Then sin. HG : rad. : : tan. CG : tan. $\angle$ CHG. (108)Or co-f. $\angle$ BAC : rad. : : (co-t. AC : co-t. AB : :) tan. AB : tan. AC.

(III. 37)

Therefore rad. : tan. AC : : co-f. $\angle$ BAC : tan. AB.

Or rad. : tan. hyp. : : co-f. given angle : tan. adjacent leg.

And is acute, if hyp. and given angle are alike ; but obtuse if unlike. (104)

3d. To find the other angle ACB.

In the triangle, N° 2, the things concerned fall under Rule II. (112)

Then sin. CG : rad. : : tan. GH. : tan. $\angle$ HCG. (108)Or co-f. AC : rad. : : co-t. $\angle$ BAC : tan. $\angle$ BCA : :) co-t. $\angle$ BCA. : tan. $\angle$ BAC. (III. 37)Therefore rad. : tan. $\angle$ BAC : : co-f. AC : co-t. $\angle$ BCA. (II. 145)

Or rad. : co-f. hyp. : : tan. given angle : co-t. req. angle.

And is acute, if hyp. and given angle are alike ; but obtuse, if unlike.

(104)

115.

PROBLEM III.

In the right angled spheric triangle ABC. Plate I. Theorem XIV.

Given one of the legs AB
 And its opposite angle ACB } Required the rest.

1st. To find the hypotenuse AC.

In the triangle, N° 1. the things concerned fall under Rule I. (111)

Then sin. $\angle$ ACB : sin. AB : : rad. : sin. AC. (107)

Or sin. given angle : sin. given leg : : rad. : sin. hyp.

And is either acute or obtuse.

2d. To find the other leg CB.

In the triangle, N° 1, the things concerned fall under Rule II. (112)

Then sin. CB : (rad. : :) tan. AB : (tan. $\angle$ ACB) : : co-t. $\angle$ ACB : rad. (III. 36)

Or rad. : co-t. given angle : : tan. given leg : sin. req. leg.

And is either acute or obtuse.

3d. To find the other angle CAB.

In the triangle, N° 3. the things concerned fall under Rule I. (111)

Then sin. EH : rad. : : sin. EI : sin. $\angle$ IHE. (107)Or sin. $\angle$ BAC : rad. : : co-f. $\angle$ ACB : co-f. AB.

Or co-f. given leg : co-f. given angle : : rad. : sin. required angle.

And is either acute or obtuse.

116. PRO-

116.

PROBLEM IV.

In the right angled spheric triangle ABC, Plate I. Theorem XIV.

Given one of the legs AB } Required the rest.
 And its adjacent angle BAC }

1st. *To find the other angle BCA.*

In the triangle N° 3. the things concerned fall under Rule I. (111)

Then rad. : fin. EH :: fin. $\angle$ EHI : fin. EI. (107)Or rad. : fin. $\angle$ BAC :: co-f. AB : co-f. $\angle$ ACB

Therefore rad. : co-f. given leg :: fin. given angle : co-f. req. angle.

And is like the given leg. (99)

2d. *To find the other leg BC.*

In the triangle, N° 1. the things concerned fall under Rule II. (112)

Then fin. AB : rad. :: tan. BC : tan. $\angle$ CAB. (108)Or rad. : fin. AB :: tan. $\angle$ CAB : tan. BC.

Therefore rad. : fin. given leg :: tan. given angle : tan. req. leg.

And is like the given angle. (99)

3d. *To find the hypotenuse AC.*

In the triangle, N° 2. the things concerned fall under Rule II. (112)

Then fin. GH : rad. :: tan. CG : tan. $\angle$ CHG. (108)Or co-f. $\angle$ CAB : rad. :: co-t. AC : co-t. AB.

Therefore rad. : co-f. given angle :: co-t. given leg : co-t. hypotenuse.

And is acute, if the given leg and angle are alike ; but obtuse, if unlike. (102)

117.

PROBLEM V.

In the right angled spheric triangle ABC, Plate I. Theorem XIV.

Given both the legs AB, BC.

Required the rest.

1st. *To find either of the oblique angles, as BAC.*

In the triangle, N° 1. the things concerned fall under Rule II. (112)

Then, as fin. AB : rad. :: tan. BC : tan. $\angle$ BAC. (108)Or rad. : fin. AB :: (tan. $\angle$ BAC : tan. BC ::) co-t. BC : co-t. $\angle$ BAC. (III. 37)

Therefore rad. : fin. one leg :: co-t. oth. leg : co-t. opp. angle.

And is like its opposite leg. (99)

2d. *To find the hypotenuse AC.*

In the triangle, N° 2. the things concerned fall under Rule I. (111)

Then, As rad. : fin. HC :: fin. $\angle$ CHG : fin. CG. (107)

Or rad. : co-f. BC :: co-f. AB : co-f. AC,

Therefore rad. : co-f. one leg :: co-f. oth. leg : co-f. hypotenuse.

And is acute, if the legs are alike ; but obtuse, if unlike. (100)

118. PRO

118.

PROBLEM VI.

In the right angled spherical triangle ABC, Plate I. Theorem XIV.

Given both the angles BAC, BCA.

Required the rest.

1st. To find either of the legs, as BC.

In the triangle, N^o 2 or 3, the things concerned fall under Rule I. (111)

Then, $\text{rad.} : \sin. HC :: \sin. \angle HCG : \sin. HG.$ (107)

Or $\text{rad.} : \text{co-f. BC} :: \sin. \angle ACB : \text{co-f. } \angle BAC.$

Therefore sine of one angle : $\text{rad.} :: \text{co-f. oth. angle} : \text{co-f. opposite side.}$

And is like its opposite angle. (99)

2d. To find the hypotenuse AC.

In the triangle, N^o 2. the things concerned fall under Rule II. (112)

Then, $\text{As } \sin. CG : \text{rad.} :: \tan. GH : \tan. \angle HCG.$ (108)

Or $\text{co-f. AC} : (\text{rad.} ::) \text{co-t. } \angle BAC (\tan. \angle BCA) :: \text{co-t. } \angle BCA : \text{rad.}$ (III. 36)

Therefore $\text{rad.} : \text{co-t. one angle} :: \text{co-t. oth. angle} : \text{co-f. hypotenuse.}$

And is acute, if the angles are like. (101)

But obtuse, if unlike.

In these six Problems are contained sixteen proportions, which are applicable to the like number of cases usually given to right angled spheric triangles; and these proportions being collected and disposed in a Table, will readily shew, by inspection, how any of the cases are to be solved.

The celebrated Lord NEPIER, the inventor of logarithms, contrived a general rule, easy to be remembered, by which the solution of every case in right angled spheric triangles is readily obtained, where the Table of proportions is wanting; which rule is as follows.

GENERAL RULE.

119. Radius multiplied by the sine of the middle part, is either equal to the product of the tangents of extremes conjunct.

Or to the product of the co-sines of extremes disjunct.

Observing ever to use the complements of the hypoth. and angles.

Lord Nepier called the five parts of every right angled spheric triangle, omitting the right angle, circular parts; which he thus distinguished; the two legs, the complements of the two angles, and the complement of the hypotenuse; and any two of these circular parts being given, the others are to be found by this rule, as is shewn in what follows.

Now, In all the proportions about right angled spheric triangles, there are, besides the radius, three things concerned; one of which may be called the middle term in respect of the other two; and these two, in respect of the middle term, may be called extremes.

When the two extremes are joined to the middle, they are called extremes conjunct: But when each of them is disjoined from the middle, by an intermediate term (not concerned), they are then called extremes disjunct; taking notice that the right angle does not disjoin the legs.

If the three parts under consideration do all join, the middle one of those three is readily seen, and the other two are extremes conjunct.

But if only two of the three parts are joined, those two are extremes disjunct, and the other term is the middle part.

These

These things duly observed, the practice of the Rule will appear in the following examples.

EXAMPLE I. *When the hypotenuse and the angles are concerned.*

The hypoth. is the middle term, and the two angles are extremes conjunct; then by the rule.

Rad. $\times$ sin. hyp. = tan. one angle $\times$ tan. other angle.

But the comp. of the hypoth. and angles are always to be used.

Therefore rad. $\times$ co-f. hyp. = co-t. one angle $\times$ co-t. other angle.

Hence rad. : co-t. one angle :: co-t. other angle : co-f. hypoth. (II. 163)

From whence are deduced the 6th and 15th cases.

EXAM. II. *When the hypotenuse and legs are under consideration.*

The hypotenuse is the middle term, and the two legs are extremes disjunct, having the angles between them and the hypotenuse.

Then by the rule. Rad. $\times$ sin. hyp. = co-f. one leg $\times$ co-f. other leg.

But the complement of the hypotenuse is to be used.

Therefore rad. $\times$ co-f. hypoth. = co-f. one leg $\times$ co-f. other leg.

Hence rad. : co-f. one leg :: co-f. other leg : co-f. hypoth. (II. 163)

From whence are deduced the 3d and 13th cases.

EXAM. III. *The legs and an angle under consideration.*

Here the angle and its opposite leg are extremes conjunct; and the other leg is the middle part.

And these being resolved into a proportion by the rule, will produce the 8th, 11th, and 14th cases.

EXAM. IV. *The angles and a leg under consideration.*

Here one angle is the middle, and the other angle and leg are extremes disjunct, the hypotenuse and other leg intervening.

Now these being resolved into a proportion, give the 9th, 12th, and 16th cases.

EXAM. V. *The hypotenuse, a leg, and the angle between them, being under consideration.*

Here the angle is the middle term, and the hypotenuse and leg are extremes conjunct.

And these being resolved into a proportion, will give the 2d, 4th, and 10th cases.

EXAM. VI. *The hypotenuse, a leg, and its opposite angle, being under consideration.*

Here the leg is the middle term, and the hypotenuse and angle are extremes disjunct, the other leg and other angle falling between them and the middle.

And these being converted into a proportion, from thence the 1st, 5th, and 7th cases are deduced.

SECTION

SECTION VII.

Of the Solution of oblique angled spheric Triangles.

120. All the cases in oblique angled spheric triangles, except where the three sides, or the three angles, are given, are most conveniently resolved by drawing a perpendicular from one of the angles to its opposite side, continued if necessary; which perpendicular will either divide the given triangle into two right angled triangles, or make two that are right angled, by joining a right angled one to the given triangle.

In drawing this perpendicular, observe,

1st. It must be drawn from the end of a given side, and opposite to a given angle.

2d. It must be so drawn, that two of the given things in the oblique triangle may remain known in one of the right-angled triangles.

3d. This perpendicular is to be used as a known quantity; and being drawn as here directed, will either fall within or without the triangle, as the angles, next the side on which it falls, are of the same or of different kinds. (105)

121.

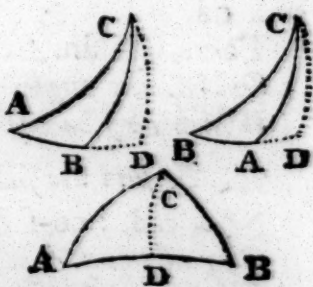
PROBLEM I.

In the oblique angled spheric triangle ABC.

Given two sides.

CA, CB

And the angle opp. to one, $\angle CAB$ } Requir. the rest.



1st. To find the angle opposite to the other given side ($\angle CBA$).

As $\sin. BC : \sin. \angle CAB :: \sin. AC : \sin. \angle CBA$ (110)

Or, As $\sin. one\ side : \sin. opposite\ angle :: \sin. other\ side : \sin. opposite\ angle.$
Which may be either acute or obtuse.

2d. To find the angle between the given sides ($\angle ACB$).

Now $\text{rad.} : \tan. \angle CAB :: \text{co-f. AC} : \text{co-t. (ACD, call it) m.}$ (3d 114)

Or $\text{rad.} : \tan. \text{given } \angle :: \text{co-f. adj. side} : \text{co-t. (of a fourth =) m.}$

And is acute, if AC and $\angle CAB$ are alike; but obtuse, if unlike.

But $\text{rad.} : \tan. CD :: \text{co-t. AC} : \text{co-f. (ACD =) m.}$ (2d 113)

$\text{rad.} : \tan. CD :: \text{co-t. CB} : \text{co-f. (BCD, call it) n.}$

Therefore $\text{co-t. AC} : \text{co-t. CB} :: \text{co-f. m} : \text{co-f. n.}$ (11. 155)

Or $\text{co-t. side adj. given } \angle : \text{co-t. other side} :: \text{co-f. m} : \text{co-f. n.}$

And is like the side opposite the given angle, if that angle is acute.

But unlike that side, if the given angle is obtuse.

Then the angle sought, viz. $\angle ACB = \begin{cases} \text{sum of } m \text{ and } n, \text{ if } \perp^* \text{ falls within.} \\ \text{diff. of } m \text{ and } n, \text{ if } \perp \text{ falls without.} \end{cases}$

* The mark $\perp$ signifies the perpendicular.

3d. To find the other side AB.

Now rad. co-f. $\angle CAB :: \tan. AC : \tan. (AD, \text{ call it } M. \quad (2d \text{ II4})$

Or rad. : co-f. given angle : : tan. adj. side : tan. (of a fourth =) M.

Acute, if the angle and its adj. side are alike; but obtuse, if unlike.

But co-f. CD : rad. : : co-f. AC : co-f. (AD =) M.

co-f. CD : rad. : : co-f. CB : co-f. (DB call it) N. (3d II3)

Therefore co-f. AC : co-f. CB : : co-f. M. : co-f. N. (II. 155)

Or co-f. side adj. given angle : co-f. other side : : co-f. M. : co-f. N.

Like the side opposite the given angle, if that angle be acute;

But unlike that side, if the angle be obtuse.

Then the side sought AB = $\begin{cases} \text{sum of M and N, if the } \perp \text{ falls within.} \\ \text{diff. of M and N, if the } \perp \text{ falls without.} \end{cases}$

But if CA = CB, or if CA = $180^\circ - CB$, or if CA is between BC and $180^\circ - BC$;

Then $\angle B$ is like BC only.

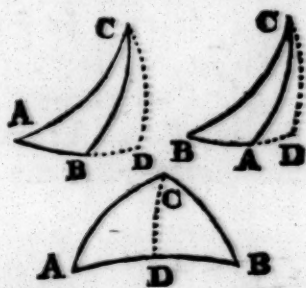
And if BC is $\begin{cases} \text{like} \\ \text{unlike} \end{cases} \angle A$; $\begin{cases} \text{Then } \angle ACB = m \pm n \text{ only; and } AB = M \\ \pm N \text{ only.} \end{cases}$

122. PROBLEM II.

In the oblique angled spheric triangle ABC.

Given two angles CAB, CBA } Required.

And a side opposite one of them AC } the rest.



1st. To find the side opposite the other given angle, viz. CB.

Then, As sin. $\angle CBA : \sin. AC :: \sin. \angle CAB : \sin. CB. \quad (110)$

Or sin. one angle : sin. opposite side : : sin. other angle : sin. opposite side.

Which may be either acute or obtuse.

2d. To find the side included by the given angles, viz. AB.

Now rad. : co-f. $\angle CAB :: \tan. AC : \tan. (AD, \text{ call it } M. \quad (II. 114)$

Or rad. : co-f. $\angle \text{adj. given side} :: \tan. \text{the given side} : \tan. (of a fourth =) M.$

Like the angle adj. the side given, if that side is acute; but unlike, if obtuse.

But rad. : tan. CD : : co-t. $\angle CAB : \sin. (AD =) M.$

rad. : tan. CD : : co-t. $\angle CBD : \sin. (DB, \text{ call it } N. \quad (2d \text{ II5})$

Therefore co-t. $\angle CAB : co-t. \angle CBD :: \sin. M : \sin. N. \quad (II. 155)$

Or co-t. $\angle \text{adj. given side} : co-t. \text{ other angle} :: \sin. M : \sin. N.$

Which may be either acute or obtuse.

Then the side sought AB = $\begin{cases} \text{sum of M and N, if the given angles are alike.} \\ \text{diff. of M and N, if the given angles are unlike.} \end{cases}$

3d. To find the other angle, viz. $\angle ACB$.

Now rad. : tan. $\angle CAD :: co-f. AC : co-t. (\angle ACD, \text{ call it } m. \quad (3d \text{ II4})$

Or rad. : tan. $\angle \text{adj. side given} :: co-f. \text{ of given side} : co-t. (of a fourth =) m.$

Like $\angle \text{adj. side given}$, if that side is acute; but unlike, if obtuse.

But co-f. CD : rad. : : co-f. $\angle CAB : \sin. (\angle ACD =) m.$

co-f. CD : rad. : : co-f. $\angle ABC : \sin. (\angle BCD, \text{ call it } n. \quad (3d \text{ II5})$

Therefore co-f. $\angle CAB : co-f. \angle ABC :: \sin. m. \text{ to } \sin. n. \quad (II. 155)$

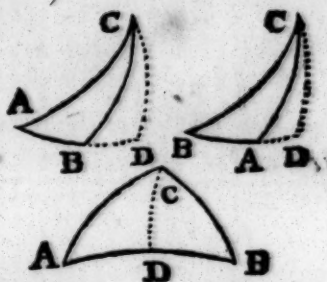
Or co-f. $\angle \text{adj. side given} : co-f. \text{ other angle} :: \sin. m : \sin. n.$

Which may be either acute or obtuse.

Then $\angle$ sought $ABC = \begin{cases} \text{sum of } m \text{ and } n, \text{ if the given angles are alike.} \\ \text{diff. of } m \text{ and } n, \text{ if the given angles are unlike.} \end{cases}$
 But if $AC = BC$, or to $180^\circ - BC$, or is between BC and $180^\circ - BC$.
 Then BC cannot be unlike its opposite angle.
 Neither can DB , or the $\angle BCD$ be obtuse.

123. PROBLEM III.

In the oblique angled spheric triangle ABC .
 Given two sides AC, AB } Required the rest.
 And their contained angle BAC



1st. To find either of the other angles, as $\angle ABC$.

As $\text{rad.} : \text{co-f. } \angle CAB :: \tan. AC : \tan. (AD, \text{ call it } M). \quad (2d \text{ II } 14)$

Or $\text{rad.} : \text{co-f. given } \angle :: \tan. \text{ side opposite } \angle \text{ sought} : \tan. (\text{of a fourth} =) M.$

Like the side opposite $\angle$ sought, if the given $\angle$ is acute;

But unlike that side, if the given $\angle$ is obtuse.

Take the diff. between AB , side adj. $\angle$ sought, and $(AD =) M$; call it N .

Now $\text{rad.} : \text{co-t. } CD :: \sin. (AD =) M : \text{co-t. } \angle CAB. \quad (1st \text{ II } 17)$

$\text{rad.} : \text{co-t. } CD :: \sin. (DB =) N : \text{co-t. } \angle ABC.$

Therefore $\sin. N : \sin. M :: \text{co-t. } \angle ABC : \text{co-t. } \angle CAB. \quad (II. 155)$

$:: \tan. \angle CAB : \tan. \angle CBA. \quad (III. 37)$

Or $\sin. N : \sin. M :: \tan. \text{ given } \angle : \tan. \angle \text{ sought.}$

Like the given angle, BAC , if M is less than AB , the side adjacent the angle sought; but unlike, if M is greater.

2d. To find the other side CB .

As $\text{rad.} : \text{co-f. } \angle CAB :: \tan. AC : \tan. (AD, \text{ call it } M). \quad (2d \text{ II } 14)$

Or $\text{rad.} : \text{co-f. given } \angle :: \tan. \text{ of either given side} : \tan. (\text{of a fourth} =) M.$

Like the side used in this proportion, if the given $\angle$ is acute;

But unlike that side, if the angle is obtuse.

Take the difference between the other side, AB , and $(AD =) M$; call it N .

Now $\text{rad.} : \text{co-f. } CD :: \text{co-f. } (AD =) M : \text{co-f. } AC. \quad (2d \text{ II } 17)$

$\text{rad.} : \text{co-f. } CD :: \text{co-f. } (DB =) N : \text{co-f. } CB.$

Therefore $\text{co-f. } M : \text{co-f. } N :: \text{co-f. } AC : \text{co-f. } CB. \quad (II. 155)$

Or $\text{co-f. } M : \text{co-f. } N :: \text{co-f. side used in first proportion} : \text{co-f. side required.}$

Like N , if the given $\angle$ is acute; but unlike N , if that $\angle$ is obtuse.

124. PROBLEM IV.

In the oblique angled spheric triangle ABC .

Given two angles $\angle CAB, \angle ACB$ } Required the rest.
 And their included side AC

1st. To find either of the other sides, as CB .

As $\text{rad.} : \text{co-f. } AC :: \tan. \angle CAB : \text{co-t. } (\angle ACD, \text{ call it } m). \quad (3d \text{ II } 14)$

Or $\text{rad.} : \text{co-f. given side} :: \tan. \angle \text{ opposite side sought} : \text{co-t. } (\text{of a fourth} =) m.$

Like the angle opposite side sought, if the given side is acute;

But unlike that angle, if the given side be obtuse.

Take the diff. between $\angle ACB$, adj. side sought, and $(\angle ACD =) m$, call it n .

Then $\text{rad.} : \text{co-t. } CD :: \text{co-f. } (\angle ACD =) m : \text{co-t. } AC. \quad (3d \text{ II } 16)$

$\text{rad.} : \text{co-t. } CD :: \text{co-f. } (\angle BCD =) n : \text{co-t. } CB.$

Therefore

Therefore $\text{co-f. } n : \text{co-f. } m :: \text{co-t. } CB : \text{co-t. } AC.$

(II. 155)

$:: \tan. AC : \tan. CB.$

(III. 37)

Or $\text{co-f. } n : \text{co-f. } m :: \tan. \text{ given side} : \tan. \text{ side required.}$

Like n , if the angle opposite the side sought be acute;

But unlike n , if the angle is obtuse.

2d. To find the other angle ABC .

As $\text{rad.} : \text{co-f. } AC :: \tan. \angle CAB : \text{co-t. } (\angle ACD,$
call it) $m.$ (3d 114)

Or $\text{rad.} : \text{co-f. } \text{ given side} :: \tan. \text{ either given } \angle : \text{co-t. } (\text{of a fourth} =) m.$

Like $\angle$ used in this proportion, if the given side, AC , is acute;

But unlike that $\angle$, if the given side is obtuse.

Take the difference between the other $\angle$, ACB , and $\angle (ACD =) m$, call it n .

Now $\text{rad.} : \text{co-f. } CD :: \sin. (\angle ACD =) m : \text{co-f. } \angle CAB.$ (1st 116)

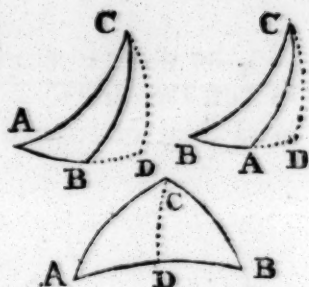
$\text{rad.} : \text{co-f. } CD :: \sin. (\angle BCD =) n : \text{co-f. } \angle ABC.$

Therefore $\sin. m : \sin. n :: \text{co-f. } \angle CAB : \text{co-f. } \angle ABC.$ (II. 155)

Or $\sin. m : \sin. n :: \text{co-f. } \angle \text{ used in first prop.} : \text{co-f. } \angle \text{ sought.}$

Like the $\angle$ used in both proportions, if m is less than the other $\angle$;

But unlike, if m is greater than the other angle.



125.

PROBLEM V.

In the oblique angled spheric triangle ABC . Plate I. Problem V.

Given the three sides AB, BC, AC ; Required the angles.

To find the angle ABC .

Let $HBKLM$ represent the quarter of a sphere, the center of which is O . Where the semicircular sections HBK, HLK , are at right angles to one another; and OB is perpendicular to HK .

Then, continuing the side BC to L , the arc HML measures $\angle ABC$. (9)

And $HQ = \frac{1}{2}$ chord HL , will be the sine of $\frac{1}{2}$ (arc $HMC = \frac{1}{2}$) $\angle ABC$.

Draw the radius OQM ; and draw LP, QN , at right angles to HK .

Then $LP = \text{fine}$, $HP = \text{versed fine}$, of $\angle ABC$; and $HN = NP$. (II. 165)

But as HQO is a right-angled triangle; OQ being perp. to HL . (II. 125)

Therefore $OH : HQ :: HQ : HN$. (II. 170)

And $OH \times HN = HQ^2$ (II. 162) = square of the sine of $\frac{1}{2} \angle ABC$.

Make $BD = BE = BC$; and $AF = AG = AC$.

Then the semicircular plane DCE , which is parallel to HLK (23), will be cut by the semicircular plane FCG , drawn at right angles to the plane HBK , in the line CI (II. 209) at right angles to DE . (II. 210)

And the arc DC , and its versed sine DI , are similar to the arc HL and its versed sine HP . (29 III. 15)

Then $\text{rad. } OH : \text{rad. } DS :: PH : ID = \left(\frac{PH}{OH} \times DS = \right) \frac{2HN}{OH} \times DS.$

Draw OR parallel to FG ; then arc $AR = (90^\circ =) \text{arc } BK$, and $RK = AB$.

Therefore $\angle DIF = (\angle KOR = \text{arc } RK =) \text{arc } AB$.

Now $DS = (SE = \text{fine arc } BE =) \text{fine arc } BC$.

And $AD = (BD - BA = BC - BA) = \text{diff. sides about } \angle \text{ sought.}$

Also $\angle DFI = \frac{1}{2} \text{ arc } (DG = AG + AD =) \overline{AC + AD}$, the sine of which is $\frac{1}{2} ID$. Schol. to art. III. 45.

And arc $FD = (AF - AD =) AC - AD$, the sine of which is $\frac{1}{2} DF$.

Now $\sin. \angle DIF : \sin. \angle DFI :: (FD : ID ::) \frac{1}{2} FD : \frac{1}{2} ID.$ (Schol. III. 45)

Or $\sin. \angle DIF : \sin. \angle DFI :: \frac{1}{2} FD : \frac{HN}{OH} \times DS.$

Therefore $\sin. \angle DIF \times DS \times \frac{HN}{OH} = \sin. \angle DFI \times \frac{1}{2} FD.$ (II. 163)

Therefore $\sin. \angle DIF \times DS \times \frac{HN}{OH} \times OH = \sin. \angle DFI \times \frac{1}{2} FD \times OH$ (II. 156)

Or $\sin. \angle DIF \times DS \times HN = \sin. \angle DFI \times \frac{1}{2} FD \times OH.$ (II. 149)

Theref. $\sin. \angle DIF \times DS : \sin. \angle DFI \times \frac{1}{2} FD :: OH : HN.$ (II. 163)

$:: OH \times OH : (HN \times OH =) \overline{HQ}^2.$ (II. 155)

Therefore $\sin. \angle DIF \times DS : \sin. \angle DFI \times \frac{1}{2} FD :: \overline{OH}^2 : \overline{HQ}^2.$

Or $\sin. AB \times \sin. BC : \sin. \frac{1}{2} AC + AD \times \sin. \frac{1}{2} AC - AD :: \text{Rad}^2 : \sin. \frac{1}{2} \angle ABC^2$
 $\sin. \frac{1}{2} AC + AD \times \sin. \frac{1}{2} AC - AD.$

Theref. $\text{sq. sin. } \frac{1}{2} \angle ABC = \frac{\sin. AB \times \sin. BC}{\sin. \frac{1}{2} AC + AD \times \sin. \frac{1}{2} AC - AD} \times \text{sq. Rad.}$ (II. 164)

Now supposing $\text{rad.} = 1$, and $L.$ to stand for logarithm.

Then $2L, \sin. \frac{1}{2} \angle ABC = L. \sin. \frac{1}{2} AC + AD + L. \sin. \frac{1}{2} AC - AD - L. \sin. AB.$
 $- L. \sin. BC.$ (I. 90, 85, 86)

And putting l for the arithmetic complement of a logarithm.

$l. \sin. AB + l. \sin. BC + L. \sin. \frac{1}{2} AC + AD + L. \sin. \frac{1}{2} AC - AD$

Then $L, \sin. \frac{1}{2} \angle ABC = \frac{\quad}{2}$

That is, having determined which angle to find,

To the arithmetic complement of $\log. \sin.$ of one containing side,

Add the arithmetic complement of $\log. \sin.$ of the other containing side,

And the $\log. \sin.$ of the $\frac{1}{2}$ sum of 3d side and difference of the containing sides,

Also the $\log. \sin.$ of the $\frac{1}{2}$ difference of 3d side and diff. of the containing sides,

Then the degrees answering to half the sum of these four logarithms, found among the sines, being doubled, will give the angle sought.

126.

PROBLEM VI.

In the oblique angled spheric triangle ABC.

Given the three angles A, B, c; Requ. the sides.

To find the side AB.

About the given angles as poles, describe arcs of great circles meeting one another, and forming the triangle FDE.

Then are the sides of FDE, the supplements of the angles A, B, c. (95)

Continue FD, FE, the supplements of the angles B, A, adjacent to the side AB required, till they meet in G.

Then in the triangle DGE, the sides GD, GE, are the measures of the angles B and A, adjacent to the side sought.

The side DE is the supplement of $\angle C$ opposite the side AB.

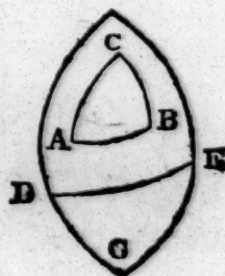
Now $\angle G (= \angle F, \text{ by } 31)$ is the supplement of AB.

Therefore the $\angle G$ being found in the triangle DGB by PROB. V. (125) will give the supplement of the side AB required.

That is, Let the given angles be taken as the sides of another triangle, observing to use the supplement of that angle opposite to the side required.

In this new triangle find (by PROB. V.) the angle opposite to that side where the supplement is used.

Then will the supplement of the angle thus found be the side required.



A TABLE containing all the cases of right angled, or Quadrantal, Spheric Triangles, with the Solutions and Determinations.

PR.	Given.	Required.	S O L U T I O N.				Determination.
			fin. Hyp.	: Rad.	: : fin. gn. leg	: fin. op. $\angle$	
127.		$\angle$ op. gn. leg					Like given leg.
128.	Hyp. and a leg	$\angle$ ad. gn. leg	Rad.	: co-t. Hyp.	: : tan. gn. leg	: co-f. adj. $\angle$	{ ac. or ob. as hyp. is like or unl. gn. leg.
129.		other leg	co-f. gn. leg	: Rad.	: : co-f. Hyp.	: co-f. req. leg	{ ac. or ob. as hyp. is like or unl. gn. leg.
130.		leg. op. gn. $\angle$	Rad.	: fin. Hyp.	: : fin. gn. $\angle$	: fin. op. leg	Like given angle.
131.	Hyp. and an angle	leg. ad. gn. $\angle$	Rad.	: tan. Hyp.	: : co-f. gn. $\angle$	: tan. adj. leg	{ ac. or ob. as hyp. is like or unl. gn. $\angle$.
132.		other angle	Rad.	: co-f. Hyp.	: : tan. gn. $\angle$	: co-t. req. $\angle$	{ ac. or ob. as hyp. is like or unl. gn. $\angle$.
133.	A leg and its op. $\angle$	Hypoth.	fin. gn. $\angle$	: fin. gn. leg	: : Rad.	: fin. Hyp.	either ac. or ob.
134.		other leg	Rad.	: co-t. gn. $\angle$	: : tan. gn. leg	: fin. req. leg	either ac. or ob.
135.		other angle	co-f. gn. leg	: co-f. gn. $\angle$	: : Rad.	: fin. req. $\angle$	either ac. or ob.
136.		other angle	Rad.	: co-f. gn. leg	: : fin. gn. $\angle$	: co-f. req. $\angle$	Like given leg.
137.	A leg and its adj. $\angle$	other leg	Rad.	: fin. gn. leg	: : tan. gn. $\angle$	: tan. req. leg	Like given angle.
138.		Hypoth.	Rad.	: co-f. gn. $\angle$	: : co-t. gn. leg	: co-t. Hyp.	{ ac. or ob. as gn. leg. like or unlike gn. $\angle$.

139.	V.	Both legs.	either angle	Rad.	: sin. either leg	: : co-t. oth. leg	: co-t. op. $\angle$	Like opposite leg.
140.			Hypoth.	Rad.	: co-f. either leg	: : co-f. oth. leg	: co-f. Hyp.	{ ac. or ob. as legs are like or unlike.
141.			either leg.	sin. eith. $\angle$	: Rad.	: : co-f. other $\angle$	: co-f. op. leg	Like opposite angle.
142.	VI.	Both angles	Hypoth.	Rad.	: co-t. either $\angle$	: : co-t. oth. $\angle$	: co-f. Hyp.	{ ac. or ob. as $\angle$ s are like or unlike.

143. In a quadrantal triangle, if the quadrantal side be called radius; the supplement of the angle opposite to that side be called hypothenuse; the other sides be called angles, and their opposite angles be called legs: Then the solution of all the cases will be as in this table; observing, that where the kind of a side or angle is determined by the hypothenuse; or the hypothenuse is to be determined; to use unlike instead of like, and like instead of unlike.

In this table, beside the contractions for sine, tangent, co-sine, co-tangent; *op.* stands for opposite; *adj.* for adjacent; *oth.* for other; *eith.* for either; *ac.* for acute; *ob.* for obtuse; *gn.* for given; *lik.* *unl.* for like, unlike; *req.* for required; *ang.* or $\angle$, for angle.



A TABLE containing all the cases of Oblique angled Spheric Triangles, with the Solutions and Determinations.

PR.	Given.	Required.	S O L U T I O N.		Determination.
			fin. one side	: fin. op. $\angle$: : fin. other side : fin. op. $\angle$	
144.		$\angle$ op. oth. fid.			either acute or obtuse.
145.	I. Two sides and an angle op. to one	$\angle$ bet. gn. fid.	Rad.	: tan. given $\angle$: : co-f. adj. side : co-t. m	$\left\{ \begin{array}{l} \text{acute or obtuse as given angle and} \\ \text{its adjacent side are like or unlike.} \\ \text{like or unlike side op. given angle} \\ \text{as that angle is acute or obtuse.} \\ \text{as given sides are like or unlike.} \end{array} \right.$
146.			co-t. fid. adj. gn. $\angle$: co-t. other side : : co-f. m. : co-f. n		
			Then req. angle is either equal to sum, or diff. of m and n.		
147.		other side	Rad.	: co-f. given $\angle$: : tan. adj. side : tan. m	$\left\{ \begin{array}{l} \text{acute or obtuse as given angle and} \\ \text{its adjacent side are like or unlike.} \\ \text{like or unlike side op. given angl.} \\ \text{as that angle is acute or obtuse.} \\ \text{as the given sides are like or unlike.} \end{array} \right.$
			co-f. S. adj. gn. $\angle$: co-f. other S. : : co-f. m : co-f. n		
			Then required side = sum or difference of m and n		
148.	II. Two angles and a side op. to one	S. op. other $\angle$	fin. one angle	: fin. opposite S. : : fin. other $\angle$: fin. op. S.	either acute or obtuse.
			Rad.	: co-f. $\angle$ adj. gn. S. : : tan. given S. : tan. m	$\left\{ \begin{array}{l} \text{like or unlike ang. adj. given side,} \\ \text{as that side is acute or obtuse.} \\ \text{either acute or obtuse.} \\ \text{as the given angles are like or un-} \\ \text{like.} \end{array} \right.$
		S. bet. gn. $\angle$ s	co-t. $\angle$ adj. gn. S. : co-t. other angle : : fin. m : fin. n		
149.		other angle	Then required side = sum or difference of m and n		$\left\{ \begin{array}{l} \text{like or unlike ang. adj. given side,} \\ \text{as that side is acute or obtuse.} \\ \text{either acute or obtuse.} \\ \text{as the given angles are like or un-} \\ \text{like.} \end{array} \right.$
			Rad.	: tan. $\angle$ adj. gn. S. : : co-f. given side : co-t. m	
			co-t. $\angle$ adj. gn. S. : co-f. other angle : : fin. m : fin. n		
150.	III. Two sides	either of other angles	Then required angle = sum or difference of m and n		$\left\{ \begin{array}{l} \text{like or unlike ang. adj. given side,} \\ \text{as that side is acute or obtuse.} \\ \text{either acute or obtuse.} \\ \text{as the given angles are like or un-} \\ \text{like.} \end{array} \right.$
			Rad.	: co-f. given angle : : tan. S. op. req. $\angle$: tan. m	$\left\{ \begin{array}{l} \text{like or unlike side op. req. angle,} \\ \text{as given angle is acute or obtuse.} \end{array} \right.$
			Take the difference between side adj. req. angle and m, call it n		
			fine n : fine m : : tan. given $\angle$: tan. req. $\angle$		$\left\{ \begin{array}{l} \text{like or unlike given angle as m is} \\ \text{less or greater than S. adj. req. \angle.} \end{array} \right.$

151.	and their included $\angle$	other side	Rad. : co-f. given angle : : tan. eith. gn. S. : tan. m Take the difference between the other side and m, call it n co-f. m : co-f. n : : co-f. S. if used : co-f. req. S.	{ like or unlike side used, as given angle is acute or obtuse. { like or unlike n, as given angle is acute or obtuse.
152.	Two angles and their included side	either of other sides	Rad. : co-f. given side : : tan. $\angle$ op. req. S. : co-t. m Take the difference between $\angle$ adj. required side and m, call it n co-f. n : co-f. m : : tan. given side : tan. req. S.	{ like or unlike angle op. req. side, as given side is acute or obtuse. { like or unlike n, as the angle opposite req. side is acute or obtuse.
153.		other angle	Rad. : co-f. given side : : tan. eith. gn. $\angle$: co-t. m Take the difference between the other angle and m, call it n fine m. : fine n : : co-f. $\angle$ if used : co-f. req. $\angle$	{ like or unlike angle used, as given side is acute or obtuse. { like or unlike angle here used as m is less or greater than other ang.
154.	Three sides	either angle	Call the sides including the angle sought e and f; the opposite side call g. Put d equal to difference between e and f; find the half sum and half difference of g and d. Then. To the Ar. Co. of Log. sine of e, add the Ar. Co. of Log. sine of f, And the Log. sine of $\frac{1}{2}$ sum of g and d; Also the Log. sine of $\frac{1}{2}$ difference of g and d. Take half the sum of these four Logarithms, which seek among the Log. sines; And the degrees and minutes answering being doubled, will give the angle sought.	
155.	Three angles	either side	Let the given angles be taken as the sides of another triangle, observing to use the supplement of that angle opposite the side required. In this new triangle, find the angle opposite to that side where the supplement is used, by the precepts in Problem V. Then will the supplement of the angle thus found be the side required.	

SECTION VIII.

The Construction and numerical Solution of the cases of right angled spheric Triangles.

156. EXAM. I. In the right-angled spheric triangle ABC.

Given the hypoth. $AC=64^{\circ} 40'$
 And one leg $BC=42^{\circ} 12'$ } Required the rest.

CONSTRUCTIONS.

1st. *To put the given leg on the primitive circle.*

Describe the primitive circle, and draw the right circle AB.

Apply the given leg ($42^{\circ} 12'$) to the primitive circle from B to c.About c, as a pole, at a distance equal to the hypotenuse ($64^{\circ} 40'$) describe (68) a small circle aa, cutting the right circle AB in A; and draw the right circle CD.

Through c, A, D, describe an oblique circle.

And ABC is the triangle sought.

2d. *To put the required leg on the primitive circle.*Describe the primitive circle, and draw the right circle CB; on which lay the given leg ($42^{\circ} 12'$) from B to c. (70)About c, as a pole (66), at a distance equal to the hypotenuse ($64^{\circ} 40'$) describe a small circle cutting the primitive in A; and draw AD.

Through A, c, D, describe an oblique circle.

(II. 72.)

Then ABC is the triangle required Whose sides and angles are measured by art. 70, 72.

COMPUTATION.

To find $\angle$ oppos. the given leg. (127)

As sin. hyp.	$=64^{\circ} 40'$	0,04391
To rad.	$=90^{\circ} 00'$	10,00000
So sin. gn. leg	$=42^{\circ} 12'$	9,82719

To sin. op. $\angle$	$=48^{\circ} 00'$	9,87110
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This angle is acute, because it is to be like the given leg, which is acute.

To find the other leg. (129)

As co-f. gn. leg	$=42^{\circ} 10'$	0,13030
To rad.	$=90^{\circ} 00'$	10,00000
So co-f. hyp.	$=64^{\circ} 40'$	9,63133

To co-f. req. leg	$=54^{\circ} 43'$	9,76163
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To find $\angle$ adj. the given leg. (128)

As rad.	$=90^{\circ} 00'$	10,00000
To co-t. hyp.	$=64^{\circ} 40'$	9,67524
So tan. gn. leg.	$=42^{\circ} 12'$	9,95748

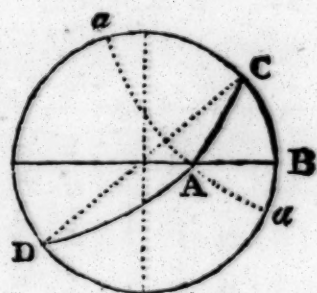
To co-f. adj. $\angle$	$=64^{\circ} 35'$	9,63272
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This angle is acute, because the hyp. and given leg are of like kinds.

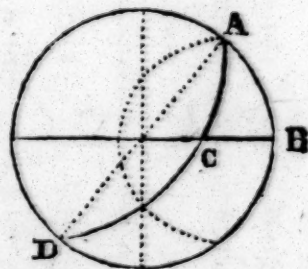
This leg is acute, because the hyp. and given leg are of like kinds.

NOTE, In these operations, and in all the following ones, although the word co-sine, or co-tangent, is used in the proportions, yet the degrees and minutes set down, are not the complements, but the real sides or angles.

157. Ex-



(II. 71)



157. EXAMPLE II. In the right-angled spheric triangle ABC.
 Given the hypoth. $AC = 64^{\circ} 40'$ } Required the rest.
 And one angle $ACB = 64^{\circ} 35'$ }

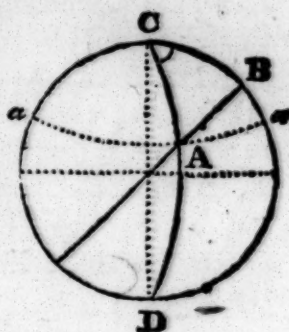
CONSTRUCTIONS.

1st. To put the leg adjacent to the given angle on the primitive circle.

Through any point c, in the primitive circle, describe (75) the oblique circle CAD, making with the primitive circle the angle BCA, equal to the given angle $64^{\circ} 35'$.

In the oblique circle CAD, take CA equal to the given hypotenuse $64^{\circ} 40'$. (70)

Through A describe the right circle AB.
 And CAB is the triangle required.



2d. To put the leg opposite the given angle on the primitive circle.

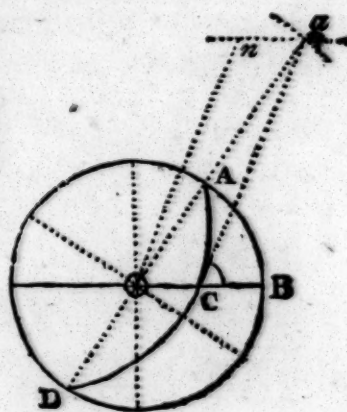
Having described the primitive circle, and drawn the right circle OB;

Describe (80) an oblique circle ACD, cutting the right circle OB in c, with the given angle $64^{\circ} 35'$, and having the part AC intercepted between the right circle OB and the primitive circle, equal to the given hypotenuse $64^{\circ} 40'$;

Then ABC is the triangle required.

The sides required are measured by art. 70.

And the required angle by art. 72.



COMPUTATION.

To find the leg opp. the giv. $\angle$ (130)

As rad. $= 90^{\circ} 00'$ 10,00000
 To fin. hyp. $= 64^{\circ} 40'$ 9,95609
 So fin. given $\angle = 64^{\circ} 35'$ 9,95579

To fin. op. leg $= 54^{\circ} 43'$ 9,91188

Like the given angle.

To find the leg adj. the giv. $\angle$ (131)

As rad. $= 90^{\circ} 00'$ 10,00000
 To tan. hyp. $= 64^{\circ} 40'$ 10,32476
 So co-f. given $\angle = 64^{\circ} 35'$ 9,63266

To tan. adj. leg $= 42^{\circ} 12'$ 9,95742

Acute, as the hypotenuse and given angle are of like kind.

To find the other angle. (132)

As rad. $= 90^{\circ} 00'$ 10,00000
 To co-f. hyp. $= 64^{\circ} 40'$ 9,63133
 So tan. given angle $= 64^{\circ} 35'$ 10,32313
 To co-t. required angle $= 48^{\circ} 00'$ 9,95446

And is acute, as the hypotenuse and given angle are of like kind.

158. EXAMPLE III. In the right angled spheric triangle ABC.
 Given one leg $CB = 42^{\circ} 12'$
 And its opp. angle $CAB = 48^{\circ} 00'$ } Required the rest.

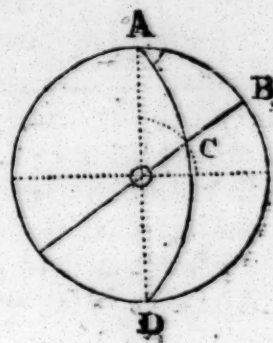
CONSTRUCTIONS.

1st. To put the required leg on the primitive circle.

Describe an oblique circle ACD (75), making with the primitive circle the angle CAB, equal to the given angle $48^{\circ} 00'$.

About the center O of the primitive circle describe (67) a small circle at the distance of the complement of the given leg $42^{\circ} 12'$, cutting ACD in C.

Draw the right circle OCB, and ACB is the triangle sought.



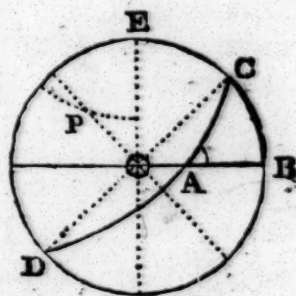
2d. To put the given leg on the primitive circle.

Draw the right circle OAB, and another OE at right angles.

Make BC equal to the given leg $42^{\circ} 12'$; draw the diameter CD, and another OP at right angles.

About E, the pole of AB, describe a small circle (68), at the distance of the given angle $48^{\circ} 00'$, cutting OP in P.

About P, as a pole (62), describe the oblique circle CAD, cutting AB in A. Then CBA is the triangle required.



The sides are measured by art. 70, and the angles by art. 72.

COMPUTATION.

To find the hypotenuse. (133)			To find the other leg. (134)		
As fin. giv. $\angle = 48^{\circ} 00'$	0,12893		As rad. $= 90^{\circ} 00'$	10,00000	
To fin. giv. leg $= 42 12$	9,82719		To co-t. giv. $\angle = 48 00$	9,95444	
So rad. $= 90 00$	10,00000		So tan. giv. leg $= 42 12$	9,95748	
To fin. hyp. $= 64 40\frac{1}{2}$	9,95612		To fin. req. leg $= 54 44$	9,91192	
And is either acute or obtuse.			And is either acute or obtuse.		

To find the other angle. (135)

As co-f. given leg $= 42^{\circ} 12'$	0,13030	
To co-f. given $\angle = 48 00$	9,82551	
So rad. $= 90 00$	10,00000	
To fin. required $\angle = 64 35$	9,95581	
And is either acute or obtuse.		

159. EXAMPLE IV. In the right angled spheric triangle ABC ,
 Given a leg $AB=54^{\circ} 43'$ } Required the rest.
 And its adj. angle $CAB=48^{\circ} 00'$

CONSTRUCTIONS.

1st. To put the given leg on the primitive circle.

Having described the primitive, and right circle

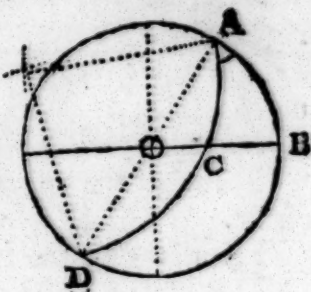
OB;

Make BA equal to the given leg $54^{\circ} 43'$

Draw the diameter AD.

Through A describe the oblique circle ACD (75) making with the primitive the given angle BAC $48^{\circ} 00'$, cutting OB in C.

Then is ACB the triangle required.



2d. To put the required leg on the primitive.

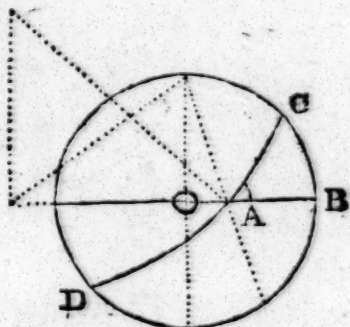
In the right circle OB, take (71) AB, equal to the given leg $54^{\circ} 43'$.

Through the point A, describe (76) the oblique circle CAD, making with AB the angle BAC, equal to the given angle $48^{\circ} 00'$, cutting the primitive circle in C.

Then is ABC the triangle sought.

The sides required are measured by art. 70.

And the required angle by art. 72.



COMPUTATION.

To find the other angle. (136)			To find the other leg. (137)		
As rad.	$=90^{\circ} 00'$	10,00000	As rad.	$=90^{\circ} 00'$	10,00000
To co-f. giv. leg	$=54^{\circ} 43'$	9,76164	To fin. giv. leg	$=54^{\circ} 43'$	9,91185
So fin. given $\angle$	$=48^{\circ} 00'$	9,87107	So tan. giv. $\angle$	$=48^{\circ} 00'$	10,04556
To co-f. req. $\angle$	$=64^{\circ} 35'$	9,63271	To tan. req. leg	$=42^{\circ} 42'$	9,95741
And is like the given angle.			And is like the given leg.		

To find the hypotenuse. (138).

As rad.	—	$=90^{\circ} 00'$	10,00000
To co-f. given $\angle$	—	$=48^{\circ} 00'$	9,82551
So co-t. given leg	—	$=54^{\circ} 43'$	9,84979
To co-t. hypoth.	—	$=64^{\circ} 40'$	9,67530

And is acute, as the given leg and angle are of a like kind.

160. EXAMPLE V. In the right angled spheric triangle ABC.
 Given one leg $BA = 54^\circ 43'$
 And the other leg $BC = 42^\circ 12'$ } Required the rest.

CONSTRUCTION.

To put either leg on the primitive circle.

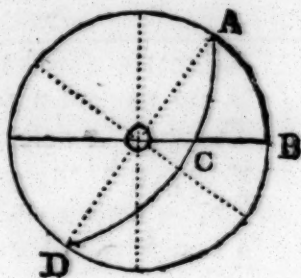
Describe the primitive circle, and draw the right circle OB.

Then, let the given legs $54^\circ 43'$, and $42^\circ 12'$, be applied, one from B to A, and the other from B to C (74); and draw the diameter AD.

Through the points A, C, D, describe an oblique circle. (II. 72)

Then is ABC the triangle required.

The angles A and C may be measured by art. 72.
 And the hypotenuse AC by art. 70.



COMPUTATION.

To find the angle A. (139)

As radius	—	= $90^\circ 00'$	10,00000
To sin. of leg AB		= $54^\circ 43'$	9,91185
So co-t. other leg BC		= $42^\circ 12'$	10,04251
To co-t. op. angle A		= $48^\circ 00'$	9,95436

And is acute, as the opposite leg CB is acute.

To find the angle C. (139)

As radius	—	= $90^\circ 00'$	10,00000
To sin. of leg CB		= $42^\circ 12'$	9,82719
So co-t. other leg AB		= $54^\circ 43'$	9,84979
To co-t. op. angle C		= $64^\circ 35'$	9,67698

And is acute, because the opposite leg AB is acute.

To find the hypotenuse AC. (140)

As radius	—	= $90^\circ 00'$	10,00000
To co-f. either leg AB		= $54^\circ 43'$	9,76164
So co-f. other leg CB		= $42^\circ 12'$	9,86970
To co-f. hypoth. AC		= $64^\circ 40'$	9,63134

And is acute, as the legs are of the same kind.

161. EXAMPLE VI. In the right angled spheric triangle ABC.
 Given one angle $A = 48^\circ 00'$
 And the other angle $c = 64^\circ 35'$ } Required the rest.

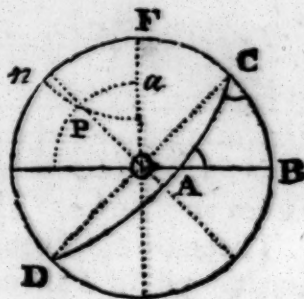
CONSTRUCTION.

To put either leg, as CB, on the primitive circle.

Having described the primitive circle, and drawn the right circle OB;

Then (81) describe the oblique great circle CAD, cutting the primitive circle in the given angle c, and the right circle OB in the given angle A.

The sides are to be measured by art. 70.



COMPUTATION.

To find the leg CB. (141)

As fin. $\angle$ adj. req. leg	$c = 64^\circ 35'$	0,04421
To radius	$= 90^\circ 00'$	10,00000
So co-f. other angle	$A = 48^\circ 00'$	9,82551
To co-f. of its op. leg	$CB = 42^\circ 12'$	9,86972

And is acute, because the opposite angle is acute.

To find the leg AB. (141)

As fin. $\angle$ adj. req. leg	$A = 48^\circ 00'$	0,12893
To radius	$= 90^\circ 00'$	10,00000
So co-f. other angle	$c = 64^\circ 35'$	9,63266
To co-f. of its op. leg	$AB = 54^\circ 43'$	9,76159

And is acute, because the opposite $\angle$ is acute.

To find the hypotenuse AC. (142)

As radius	$= 90^\circ 00'$	10,00000
To co-t. either angle as	$A = 48^\circ 00'$	9,95444
So co-t. other angle as	$c = 64^\circ 35'$	9,67687
To co-f. hypoth.	$AC = 64^\circ 40'$	9,63131

And is acute, because the angles are both acute, or like.

162. EXAMPLE VII. In the quadrantal triangle ABC.

Given the quadrantal side $AC = 90^\circ 00'$
 an adjacent angle $A = 42^\circ 12'$
 And the opposite angle $B = 64^\circ 40'$ } Required the rest.

CONSTRUCTION.

To put the quadrantal side on the primitive circle.

Having described the primitive circle, and drawn the diameters AD, BC, at right angles;

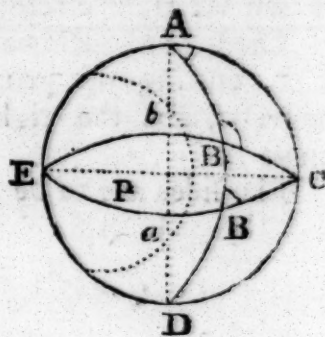
Describe the oblique circle ABD, making with AC an angle of $42^\circ 12'$. (75)

Through c describe a great circle CBE, cutting the circle ABD in an $\angle$ of $64^\circ 40'$. (74)

Then is ABC the triangle sought.

The angle c is to be measured by art. 72.

And the sides AB, CB, are measured by art. 70.



COMPUTATION.

Imagine the given triangle ABC to be changed into a right angled triangle, where the supplement of the angle B is to represent the hypothenuse, and the angle A to be one of the legs.

Then will the solution fall under art. 127, 128, 129, in the table; and the numerical computations will be the same as in Example I. Observing that the angles there found are, in this example, the measures of the sides AB, CB; and the side AB in that example stands for the angle c in this.

Now in determining the value of the parts of this triangle, as they arise in the computation, the words like and unlike are to be changed one for the other, where the hypothenuse is concerned in the determination: Thus the leg AB is taken acute, because the supplement of the angle opposite to the quadrantal side, which is here used as the hypothenuse, is unlike the other given angle; and its opposite angle c is to be acute for the same reason: But the kind of the side BC being known by the kind of its opposite angle A, it must be taken acute, as the opposite angle is acute.

In the construction there arises two triangles, either of which will answer the conditions in the example: For the small circle described about P, the pole of the oblique circle ABD, cuts the diameter AD in the points, a, b; and either of these points may be taken for the pole of the oblique circle wanting to complete the triangle.

Now if a be taken for the pole, then in the triangle ABC, the measure of the things sought, will be equal to those arising from the computation: But the angle B is the supplement of what was given.

And if b is taken for the pole; then the triangle ABC will arise from the construction; wherein the angles A and B are respectively equal to what is propounded: But then the side AB, and the angle c, will both be obtuse.

SECTION

SECTION IX.

The Construction and numerical Solution of the cases of oblique angled spheric Triangles.

163. EXAMPLE I. In the oblique angled spheric triangle ABC.

Given the side $AB = 114^\circ 30'$
 the side $BC = 56^\circ 40'$ } Required the rest.
 And an angle opposite to one side, $BCA = 125^\circ 20'$

CONSTRUCTION.

To put the given side, adjacent to the known angle, on the primitive circle.

Describe the primitive circle, and draw the diameter BD.

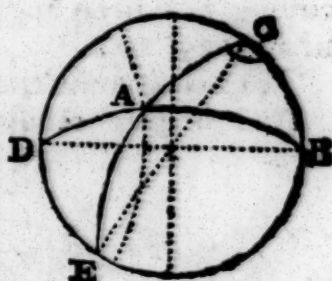
Make BC equal to the side adjacent to the given angle $= 56^\circ 40'$ (70)

Describe the great circle CAE, making the angle BCA equal to the given one, $= 125^\circ 20'$. (75)

Through B describe a great circle BAD, cutting AE in A, at the distance of AB, the other given side from B, $= 114^\circ 30'$. (68)

Then ABC is the triangle sought.

And the parts required are measured by art. 70, 72.



COMPUTATION.

To find the angle A, opposite to the other given side. (144)

As sin. one side $AB = 114^\circ 30'$	0,04098	} Which may be either acute or obtuse from the things given: But the construction shews it to be acute.
To sin. op. $\angle C = 125^\circ 20'$	9,91158	
So sin. oth. side $CB = 56^\circ 40'$	9,92194	
To sin. op. $\angle A = 48^\circ 30'$	9,87450	

To find the angle B between the given sides. (145)

As rad.	$= 90^\circ 00'$	10,00000	As co-t. S. ad. g. $\angle BC = 56^\circ 40'$	0,18197
To tan. giv. $\angle C = 125^\circ 20'$	10,14941		To co-t. oth. side $AB = 114^\circ 30'$	9,65870
So co-f. adj. side $BC = 56^\circ 40'$	9,73997		So co-f. m	$= 127^\circ 47'$
				9,78723
To co-t. m	$= 127^\circ 47'$	9,88938	To co-f. n	$= 64^\circ 53'$
				9,62790

And is obtuse, as the given angle and its given adjacent side are unlike.

Which is acute, being unlike side opposite given $\angle$, that $\angle$ being obtuse.

Then as the given sides are unlike, the diff. of m and n, or $62^\circ 54' = \angle B$.

To find the other side AC. (146)

As rad.	$= 90^\circ 00'$	10,00000	As co-f. S. ad. g. $\angle BC = 56^\circ 40'$	0,26002
To co-f. giv. $\angle C = 125^\circ 20'$	9,76218		To co-f. oth. side $AB = 114^\circ 30'$	9,61773
So tan. adj. side $BC = 56^\circ 40'$	10,18197		So co-f. m	$= 138^\circ 40'$
				9,87557
To tan. m	$= 138^\circ 40'$	9,94415	To co-f. n	$= 55^\circ 29'$
				9,75332

And is obtuse, as $\angle C$ and CB are unl.

And is acute, being unl AB as above.

Then as BC and BA , are unlike the diff. of m and n, or $83^\circ 11' = AC$.

164. Ex.

164. EXAMPLE II. In the oblique angled spheric triangle ABC.

Given the angle

$$BAC = 48^{\circ} 30'$$

the angle

$$BCA = 125^{\circ} 20'$$

And the side opposite to one angle,

$$AB = 114^{\circ} 30'$$

} Required the rest.

CONSTRUCTION.

To put the given side AB on the primitive circle.

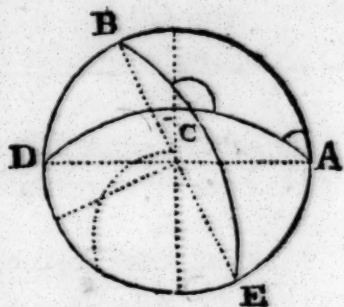
Describe the primitive circle; draw the diameter DA; and through A describe the great circle ACD, making the given angle $BAC = 48^{\circ} 30'$. (75)

Make the arc AB equal to the given side $= 114^{\circ} 30'$ (70); and draw the diameter BE.

Through B, describe the great circle BCE, cutting ACD in an angle equal to the given angle $BCA = 125^{\circ} 20'$. (78)

Then is ACB the triangle sought.

And the parts required are to be measured by art. 70, 72.



COMPUTATION.

To find the side opposite the other given angle. (147)

$$\text{As fin. one } \angle C = 125^{\circ} 20' \quad 0,08842$$

$$\text{To fin. op. side } AB = 114^{\circ} 30' \quad 9,95902$$

$$\text{So fin. other } \angle A = 48^{\circ} 30' \quad 9,87446$$

$$\text{To fin. op. side } BC = 56^{\circ} 40' \quad 9,92190$$

Which may either be acute or obtuse from what is given. But the construction shews it to be acute.

To find the side AC between the given angles. (148)

$$\text{As rad.} \quad = 90^{\circ} 00' \quad 10,00000$$

$$\text{To co-f. } \angle \text{ad.g. } S.A = 48^{\circ} 30' \quad 9,82126$$

$$\text{So tan. gn. } S. AB = 114^{\circ} 30' \quad 10,34130$$

$$\text{To tan. } M \quad = 124^{\circ} 31' \quad 10,16256$$

$$\text{As co-t. } \angle \text{ad.g. } S.A = 48^{\circ} 30' \quad 0,05319$$

$$\text{To co-t. other } \angle C = 125^{\circ} 20' \quad 9,85059$$

$$\text{So sine } M \quad = 124^{\circ} 31' \quad 9,91591$$

$$\text{To sine } N \quad = 41^{\circ} 19' \quad 9,81969$$

And is obtuse, being unlike $\angle A$, as AB is greater than 90° .

Which may be either acute or obtuse; either $41^{\circ} 20'$ or $138^{\circ} 40'$.

Then as the given angles are unlike, the difference of M and N , or $83^{\circ} 12'$, is the side AC . Or the sum of $138^{\circ} 41'$, and $124^{\circ} 31'$, lessened by 180° , leaves $83^{\circ} 12'$.

To find the other angle ABC. (149)

$$\text{As rad.} \quad = 90^{\circ} 00' \quad 10,00000$$

$$\text{To tan. } \angle \text{ad.g. } S.A = 48^{\circ} 30' \quad 10,05319$$

$$\text{So co-f. gn. } S. AB = 114^{\circ} 30' \quad 9,61773$$

$$\text{To co-t. } m \quad = 115^{\circ} 07' \quad 9,67092$$

$$\text{As co-f. } \angle \text{ad.g. } S.A = 48^{\circ} 30' \quad 0,17874$$

$$\text{To co-f. other } \angle C = 125^{\circ} 20' \quad 9,76218$$

$$\text{So sine } m \quad = 115^{\circ} 07' \quad 9,95686$$

$$\text{To sine } n \quad = 52^{\circ} 13' \quad 9,89778$$

And is obtuse, being unlike $\angle A$, as its adj. side AB is greater than 90° .

Which may be either acute or obtuse, viz. $52^{\circ} 13'$, or $127^{\circ} 47'$.

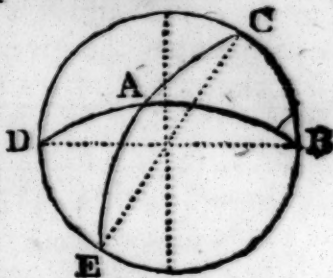
Then as the given angles are unlike, the difference of m and n , or $62^{\circ} 54'$, is the angle B required. Or the sum of $115^{\circ} 07'$, and $127^{\circ} 47'$, lessened by 180° , leaves $62^{\circ} 54'$.

165. EXAMPLE III. In the oblique angled spheric triangle ABC.
 Given the side $AB = 114^\circ 30'$
 the side $BC = 56^\circ 40'$ } Required the rest.
 And the contained angle $ABC = 62^\circ 54'$

CONSTRUCTION.

To put either of the given sides, as BC, on the primitive circle.

Describe the primitive circle; draw the diameter BD; and through B describe a great circle BAD, making the given angle $ABC = 62^\circ 54'$.



On the circles BCD, BAD, take the arcs BC, BA, respectively equal to the given sides, viz. $BC = 56^\circ 40'$, and $BA = 114^\circ 30'$.

Draw the diameter CE, and through C, A, E, describe the great circle CAE; then ABC is the triangle sought.

The required parts of ABC are measured by art. 70, 72.

COMPUTATION.

To find the angle C. (150).

As rad.	$= 90^\circ 00'$	10,00000	} Obtuse, being like side op. req. $\angle$, the given angle being acute. Take the difference between M and BC, and it is $78^\circ 21'$; call it N.
To co-f. given $\angle B = 62^\circ 54'$		9,65853	
So t. S.op.re. $\angle AB = 114^\circ 30'$		10,34130	
To tan. M	$= 135^\circ 01'$	9,99983	} And is obtuse, being unlike the given angle, because M is greater than BC, the side adjacent to the required angle.
As fine N	$= 78^\circ 21'$	0,00904	
To fine M	$= 135^\circ 01'$	9,84936	
So tan. given $\angle B = 62^\circ 54'$		10,29096	
To tan. req. $\angle C = 125^\circ 20'$		10,14936	

To find the angle A. (150)

As rad.	$= 90^\circ 00'$	10,00000	} Acute, being like side op. req. $\angle$, the given angle being acute. Take the difference between M and BA, and it is $79^\circ 48'$; call it N.
To co-f. given $\angle B = 62^\circ 54'$		9,65853	
So t. S.op.re. $\angle BC = 56^\circ 40'$		10,18197	
To tan. of M	$= 34^\circ 42'$	9,84050	} And is acute, being like the given angle, as M is less than AB, the side adjacent to the required angle.
As fine N	$= 79^\circ 48'$	0,00692	
To fine M	$= 34^\circ 42'$	9,75533	
So tan. given $\angle B = 62^\circ 54'$		10,29096	
To tan. req. $\angle A = 48^\circ 30'$		10,05321	

To find the other side AC. (151).

As rad.	$= 90^\circ 00'$	10,00000	As co-f. M	$= 135^\circ 01'$	0,15039
To co-f. given $\angle B = 62^\circ 54'$		9,65853	To co-f. N	$= 78^\circ 21'$	9,30521
So tan. eith. S. AB = 114 30		10,34130	So co-f. S. used AB = 114 30		9,61773
To tan. M	$= 135^\circ 01'$	9,99983	To co-f. S. req. AC = 83 12		9,07333

Obtuse, being like AB, the side used,
because the given angle is acute.

The diff. of M and BC, or $78^\circ 21' = N$.

And is acute, being like N, because
the given angle is acute.

166. EXAMPLE IV. In the oblique angled spheric triangle ABC.

Given the angle $BCA = 125^\circ 20'$
 the angle $BAC = 48^\circ 30'$ } Required the rest.
 And the included side $AC = 83^\circ 12'$

CONSTRUCTION.

To put the given side on the primitive circle.

Describe the primitive circle; draw the diameter AD; and through A describe the great circle ABD, making the given $\angle BAC = 48^\circ 30'$ (75)

Make AC equal to the given side $= 83^\circ 12'$ (70)

Draw the diameter CE, and through c describe the great circle CBE, making the given angle $BCA = 125^\circ 20'$ (75), cutting ABD in B.

Then is ABC the triangle sought.

And the parts required are measured by art. 70, 72.

COMPUTATION.

To find the side AB. (152)

As rad.	$= 90^\circ 00'$	10,00000	} Obtuse, being like $\angle$ op. side req. the given side being acute. Take the diff. between m and $\angle$ A, and it is $50^\circ 59'$; call it n.
To co-f. gn. side AC	$= 83^\circ 12'$	9,07337	
To tan. $\angle$ op. r. S. c	$= 125^\circ 20'$	10,14941	
To co-t. m	$= 99^\circ 29'$	9,22278	} And is obtuse, being unlike n, because the angle opposite to the side required is obtuse.
As co-f. n	$= 50^\circ 59'$	6,20097	
To co-f. m	$= 99^\circ 29'$	9,21615	
Sotan. gn. side AC	$= 83^\circ 12'$	10,92357	
Totan. req. fid. AB	$= 114^\circ 30'$	10,34139	

To find the side BC. (152)

As rad.	$= 90^\circ 00'$	10,00000	} Acute, being like $\angle$ op. side required, the given side being acute. Take the diff. between m and $\angle$ c, and it is $42^\circ 57\frac{1}{2}'$; call it n.
To co-f. gn. fid. AC	$= 83^\circ 12'$	9,07337	
Sotan. $\angle$ op. r. S. A	$= 48^\circ 30'$	10,05319	
To co-t. of m	$= 82^\circ 22\frac{1}{2}'$	9,12656	} And is acute, being like n, because the angle A opposite to BC, the side required, is acute.
As co-f. n.	$= 42^\circ 57\frac{1}{2}'$	0,13558	
To co-f. m	$= 82^\circ 22\frac{1}{2}'$	9,12283	
Sotan. gn. fid. AC	$= 83^\circ 12'$	10,92357	
Totan. req. fid. BC	$= 56^\circ 40'$	10,18198	

To find the other angle B. (153)

As rad.	$= 90^\circ 00'$	10,00000	As fine m	$= 99^\circ 29'$	0,00598
To co-f. gn. fid. AC	$= 83^\circ 12'$	9,07337	To fine n	$= 50^\circ 59'$	9,89040
Sotan. either $\angle$ c	$= 125^\circ 20'$	10,14941	So co-f. $\angle$ used, c	$= 125^\circ 20'$	9,76218
To co-t. m	$= 99^\circ 29'$	9,22278	To co-f. req. $\angle$ B	$= 62^\circ 54'$	9,65856

Obtuse, being like $\angle$ c here used, because the given side is acute.

Take difference of m and $\angle$ A, viz. $42^\circ 57'$; and call it n.

And is acute, being unlike the angle c here used, as m is greater than the other angle A.

167. EXAMPLE V. In the oblique angled spheric triangle ABC.

Given the side AB = $114^{\circ} 30'$
 the side AC = $83^{\circ} 13'$
 the side BC = $56^{\circ} 40'$ } Required the reft.

CONSTRUCTION.

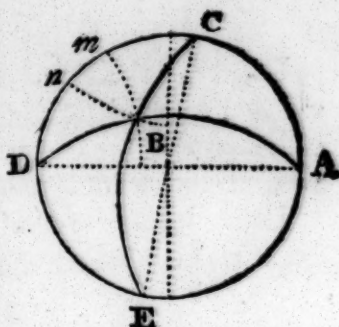
To put either side, as AC, on the primitive circle.

Describe the primitive circle, and from any point in the circumference, as A, set off one of the given sides, as AC, = $83^{\circ} 13'$ (70); and draw the diameters AD, CE.

About C, as a pole, and at a distance equal to the given side BC, = $56^{\circ} 40'$, describe a small circle *nB*. (68)

About A, as a pole, and at a distance equal to the given side AB (when AB is less than 90°) describe another small circle *mB* (68), cutting the former in B: But when the side, as AB, = $114^{\circ} 30'$, is greater than 90° ; then about D, the opposite pole to A, describe a small circle with the supplement of AB, as *mB*, cutting the former small circle *nB* in B.

Thro' the points A, B, D, and C, B, E, describe the great circles ABD, CBE. Then is ABC the triangle sought, and the angles are measured by art. 72.



COMPUTATION.

To find the angle C. (154.)

Here AC=E= $83^{\circ} 13'$	Ar. Co. sine E = $83^{\circ} 13'$	0,00305
CB=F= $56^{\circ} 40'$	Ar. Co. sine F = $56^{\circ} 40'$	0,07806
E-F=D= $26^{\circ} 33'$	Sine $\frac{1}{2}$ sum = $70^{\circ} 31\frac{1}{2}'$	9,97441
AB=G= $114^{\circ} 30'$	Sine $\frac{1}{2}$ diff. = $43^{\circ} 58\frac{1}{2}'$	9,84158
$\text{E} + \text{D} = 141^{\circ} 03'$	Sum of the four Log. -	19,89710
$\text{E} - \text{D} = 87^{\circ} 57'$	$\frac{1}{2}$ sum is sin. of $62^{\circ} 39\frac{1}{2}'$	9,94855
$70^{\circ} 31\frac{1}{2}' = \frac{1}{2}$ sum	Which doubled gives $125^{\circ} 19' = \angle C$.	
$43^{\circ} 58\frac{1}{2}' = \frac{1}{2}$ diff.		

To find the angle A. (154)

Here AB=E= $114^{\circ} 30'$	Ar. Co. sine E = $114^{\circ} 30'$	0,04098
AC=F= $83^{\circ} 13'$	Ar. Co. sine F = $83^{\circ} 13'$	0,00305
E-F=D= $31^{\circ} 17'$	Sine $\frac{1}{2}$ sum = $43^{\circ} 58\frac{1}{2}'$	9,84158
BC=G= $56^{\circ} 40'$	Sine $\frac{1}{2}$ diff. = $12^{\circ} 41\frac{1}{2}'$	2,34184
$\text{E} + \text{D} = 87^{\circ} 57'$	Sum of four Log. -	19,22745
$\text{E} - \text{D} = 25^{\circ} 23'$	$\frac{1}{2}$ sum is sin. of $24^{\circ} 15\frac{1}{2}'$	9,61372
$43^{\circ} 58\frac{1}{2}' = \frac{1}{2}$ sum	Which doubled gives $48^{\circ} 31' = \angle A$.	
$12^{\circ} 41\frac{1}{2}' = \frac{1}{2}$ diff.		

To find the angle B. (154)

Here AB=E= $114^{\circ} 30'$	Ar. Co. sine E = $114^{\circ} 30'$	0,04098
BC=F= $56^{\circ} 40'$	Ar. Co. sine F = $56^{\circ} 40'$	0,07806
E-F=D= $57^{\circ} 50'$	Sine $\frac{1}{2}$ sum = $70^{\circ} 31\frac{1}{2}'$	9,97441
AC=G= $83^{\circ} 13'$	Sine $\frac{1}{2}$ diff. = $12^{\circ} 41\frac{1}{2}'$	9,34184
$\text{E} + \text{D} = 141^{\circ} 03'$	Sum of four Log. -	19,43529
$\text{E} - \text{D} = 25^{\circ} 23'$	$\frac{1}{2}$ sum is sin. of $31^{\circ} 28'$	9,71764
$70^{\circ} 31\frac{1}{2}' = \frac{1}{2}$ sum	Which doubled gives $62^{\circ} 56' = \angle B$.	
$12^{\circ} 41\frac{1}{2}' = \frac{1}{2}$ diff.		

168. EXAMPLE VI. In the oblique angled spheric triangle ABC.

Given the angle $A = 48^\circ 31'$
 the angle $B = 62^\circ 52'$ } Required the rest.
 the angle $C = 125^\circ 20'$

CONSTRUCTION.

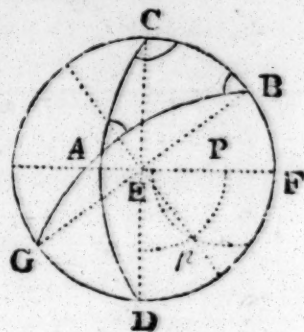
To put either two angles, as C and B , at the primitive.

Describe the primitive circle, draw the diameters CD and EF at right angles to one another; and thro' c describe a great circle CAD , making the angle BCA equal to the given angle $C = 125^\circ 20'$. (75)

Describe a great circle BAG , cutting the given great circles CFD , CAD , in the given angles $B = 62^\circ 52'$, and $A = 48^\circ 31'$. (81)

Then is ABC the triangle sought.

Where the sides are measured by art. 70.



COMPUTATION.

To find the side AB . (155)

Here $\angle B = E = 62^\circ 52'$

$\angle A = F = 48^\circ 31'$

$E - F = D = 14^\circ 21'$

Sup. $\angle C = G = 54^\circ 40'$

$G + D = 69^\circ 01'$

$C - D = 40^\circ 19'$

$34^\circ 30\frac{1}{2}' = \frac{1}{2}\text{sum}$

$20^\circ 09\frac{1}{2}' = \frac{1}{2}\text{diff.}$

Ar. Co. sine $E = 62^\circ 52' \quad 0.05064$

Ar. Co. sine $F = 48^\circ 31' \quad 0.12543$

Sine $\frac{1}{2}\text{sum} = 34^\circ 30\frac{1}{2}' \quad 9.75322$

Sine $\frac{1}{2}\text{diff.} = 20^\circ 09\frac{1}{2}' \quad 9.53733$

Sum of the four Log. 19.46662

$\frac{1}{2}\text{sum}$ is sin. of $32^\circ 45\frac{1}{2}' \quad 9.73331$

The sup. of its double is $114^\circ 29' = AB$.

To find the side AC . (155)

Here $\angle C = E = 125^\circ 20'$

$\angle A = F = 48^\circ 31'$

$E - F = D = 76^\circ 49'$

Sup. $\angle B = G = 117^\circ 08'$

$G + D = 193^\circ 57'$

$G - D = 40^\circ 19'$

$96^\circ 58\frac{1}{2}' = \frac{1}{2}\text{sum}$

$20^\circ 09\frac{1}{2}' = \frac{1}{2}\text{diff.}$

Ar. Co. sine $E = 125^\circ 20' \quad 0.08842$

Ar. Co. sine $F = 48^\circ 31' \quad 0.12543$

Sine $\frac{1}{2}\text{sum} = 96^\circ 58\frac{1}{2}' \quad 9.99677$

Sine $\frac{1}{2}\text{diff.} = 20^\circ 09\frac{1}{2}' \quad 9.53733$

Sum of four Log. 19.74795

$\frac{1}{2}\text{sum}$ is sin. of $48^\circ 25\frac{1}{2}' \quad 9.87397$

The sup. of its double is $83^\circ 09' = AC$.

To find the side BC . (155)

Here $\angle C = E = 125^\circ 20'$

$\angle B = F = 62^\circ 52'$

$E - F = D = 62^\circ 28'$

Sup. $\angle A = G = 131^\circ 29'$

$G + D = 193^\circ 57'$

$G - D = 69^\circ 01'$

$96^\circ 58\frac{1}{2}' = \frac{1}{2}\text{sum}$

$34^\circ 30\frac{1}{2}' = \frac{1}{2}\text{diff.}$

Ar. Co. sine $E = 125^\circ 20' \quad 0.08842$

Ar. Co. sine $F = 62^\circ 52' \quad 0.05064$

Sine $\frac{1}{2}\text{sum} = 96^\circ 58\frac{1}{2}' \quad 9.99677$

Sine $\frac{1}{2}\text{diff.} = 34^\circ 30\frac{1}{2}' \quad 9.75322$

Sum of four Log. 19.88905

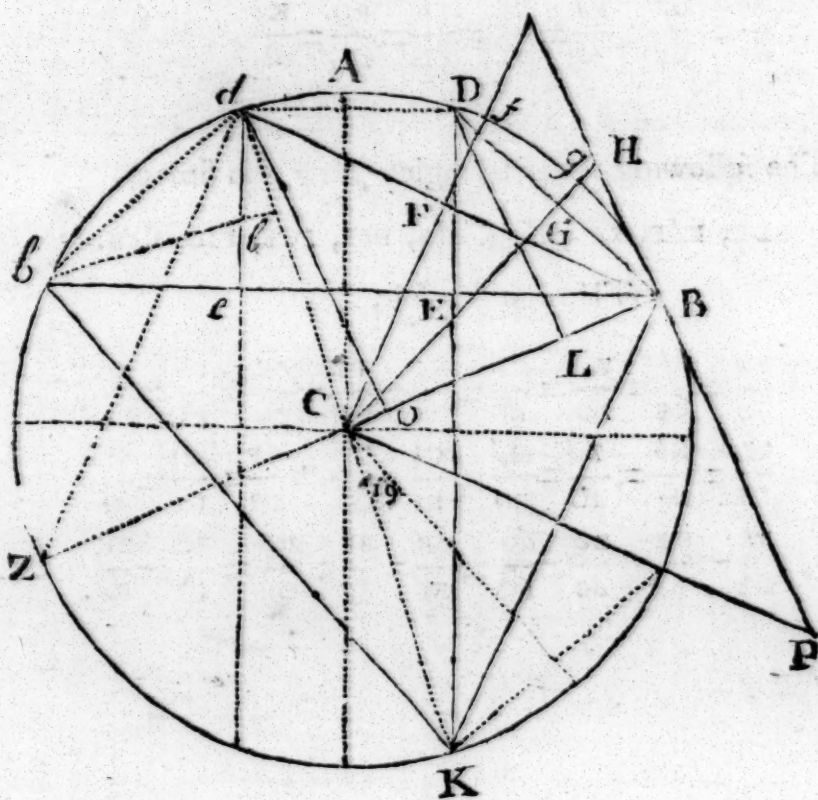
$\frac{1}{2}\text{sum}$ is the sin. of $61^\circ 39' \quad 9.94452$

The sup. of its double is $56^\circ 42' = BC$.

SECTION

SECTION X.

169. The principles already delivered have been shewn sufficient for deriving methods for the solution of all the cases in spherical Trigonometry: yet as there are many other useful and curious particulars which appertain to the subject, it was thought proper to add some of them for the entertainment of speculative readers. The chief of these relations cannot, perhaps, be better investigated, than by imitating the method of the late William Jones, Esq. who published in the year 1747, in the Philosophical Transactions. N^o 483, some properties of Goniometrical lines; which properties are mostly derived from a general figure which Mr. Jones improved from one communicated to him by the great Dr. Halley. See *Synopsis Palmariorum Matheseos*, p. 245.



Let AB, AD ; or Ab, Ad , be any two arcs, each less than 90 degrees.
 Be and BE , or be and be , be the sum and difference of their right sines.
 KE and DE , the sum and difference of their co-sines.

The arcs Bd, BD ; or bD, bd ; express the sum and difference of the arcs AB, AD . do, DL , are sines of the arcs Bd, BD , the sum and difference of arcs AB, AD : BO, BL , the versed sines of that sum and difference.
 $ZO, ZL = Kl$, the versed sines of the supplements of their sum and diff.

Let the arcs Bf , Bg , be the half sum and half diff. of the arcs AB , AD :
 BF , BG , the fines
 CF , CG , the co-fines
 BI , BH , the tangents
 CI , CH , the secants
 Bd , BD , twice the fines
 KB , Kb , twice the co-fines
 PB , PC , the co-tangent and the co-secant of the half sum of the arcs AB , AD .

} of the half sum and half diff. of AB and AD .

Now the following set of triangles being similar,

viz. CBG , Bde , KBE , Kbl , DBL , CHB , BHG , Kdb .

$$\begin{aligned} \text{Then } \frac{CB}{CG} &= \frac{Bd}{Be} = \frac{KB}{KE} = \frac{Kb}{Kl} = \frac{BD}{DL} = \frac{CH}{CB} = \frac{BH}{BG} = \frac{Kd}{Kb} \\ \frac{CB}{BG} &= \frac{Bd}{de} = \frac{KB}{BE} = \frac{Kb}{bl} = \frac{BD}{BL} = \frac{CH}{BH} = \frac{BH}{HG} = \frac{Kd}{db} \\ \frac{CG}{BG} &= \frac{Be}{de} = \frac{KE}{BE} = \frac{Kl}{bl} = \frac{DL}{BL} = \frac{CB}{BH} = \frac{BG}{HG} = \frac{Kb}{db}. \end{aligned}$$

The following set of triangles being also similar.

viz. CBF , BDE , KbE , zdo , dbo , CIB , BIF , PCB , PIC , KdB .

There will result,

$$\begin{aligned} \frac{CB}{CF} &= \frac{BD}{DE} = \frac{Kb}{Eb} = \frac{zd}{do} = \frac{Bd}{Bo} = \frac{CI}{BI} = \frac{BI}{IF} = \frac{PC}{CB} = \frac{PI}{CI} = \frac{Kd}{Bd} \\ \frac{CB}{BF} &= \frac{BD}{DE} = \frac{Kb}{Eb} = \frac{zd}{do} = \frac{Bd}{Bo} = \frac{CI}{BI} = \frac{BI}{IF} = \frac{CP}{CB} = \frac{PI}{CI} = \frac{dK}{Bd} \\ \frac{CF}{BF} &= \frac{BE}{DE} = \frac{KE}{Eb} = \frac{zo}{do} = \frac{do}{Bo} = \frac{CB}{BI} = \frac{BF}{IF} = \frac{PB}{CB} = \frac{PC}{CI} = \frac{BK}{Bd}. \end{aligned}$$

170. Now the several values of the radius CB being collected, are placed in the annexed table; where the letters s, t, f, v , stand for the sine, tangent, secant, versed sine; and the letters s', t', f', v' , the co-sine, co-tangent, co-secant, of the arcs a ; or of the arcs $\frac{1}{2}A+a, \frac{1}{2}A-a$; and v' , the versed sine of the supplement.

Goniometrical Properties.

(171) $\frac{s, A+s, a}{s, A+s, a} \times t, \frac{1}{2}A+a.$	(172) $\frac{s', A+s', a}{s, A-s, a} \times t, \frac{1}{2}A-a.$	(173) $\frac{2s', \frac{1}{2}A-a}{s, A+s, a} \times s, \frac{1}{2}A+a.$	(174) $\frac{2s', \frac{1}{2}A-a}{s', A+s', a} \times s', \frac{1}{2}A+a.$
(175) $\frac{A+s, a}{A+s, a} \times t', \frac{1}{2}A+a.$	(176) $\frac{s, A-s, a}{s', A+s', a} \times t', \frac{1}{2}A-a.$	(177) $\frac{s, A+s, a}{2s', \frac{1}{2}A-a} \times f, \frac{1}{2}A+a.$	(178) $\frac{s', A+s', a}{2s, \frac{1}{2}A-a} \times f, \frac{1}{2}A+a.$
(179) $\frac{A-s, a}{s, A} \times t, \frac{1}{2}A+a.$	(180) $\frac{s, A+s, a}{s', a-s', A} \times t, \frac{1}{2}A-a.$	(181) $\frac{2s', \frac{1}{2}A-a}{s', a-s', A} \times s, \frac{1}{2}A+a.$	(182) $\frac{2s', \frac{1}{2}A-a}{s, A-s, a} \times s', \frac{1}{2}A+a.$
(183) $\frac{s, a-s', A}{A-s, a} \times t', \frac{1}{2}A+a.$	(184) $\frac{s', a-s', A}{s, a+s, A} \times t', \frac{1}{2}A-a.$	(185) $\frac{s', a-s', A}{2s, \frac{1}{2}A-a} \times f, \frac{1}{2}A-a.$	(186) $\frac{s, A-s, a}{2s', \frac{1}{2}A-a} \times f, \frac{1}{2}A+a.$
(187) $\frac{s, \frac{1}{2}A+a}{s, \frac{1}{2}A+a} \times t, \frac{1}{2}A+a.$	(188) $\frac{s', \frac{1}{2}A-a}{s, \frac{1}{2}A-a} \times t, \frac{1}{2}A-a.$	(189) $\frac{2s', \frac{1}{2}A+a}{s, A+a} \times s, \frac{1}{2}A+a.$	(190) $\frac{2s', \frac{1}{2}A-a}{s, A-a} \times s, \frac{1}{2}A-a.$
(191) $\frac{v, A+a}{s, A+a} \times t, \frac{1}{2}A+a.$	(192) $\frac{v, A-a}{s, A-a} \times t, \frac{1}{2}A-a.$	(193) $\frac{2s, \frac{1}{2}A+a}{v, A+a} \times s, \frac{1}{2}A+a.$	(194) $\frac{2s, \frac{1}{2}A-a}{v, A-a} \times s, \frac{1}{2}A-a.$
(195) $\frac{A+a}{s, A+a} \times t, \frac{1}{2}A+a.$	(196) $\frac{s, A-a}{v, A-a} \times t, \frac{1}{2}A-a.$	(197) $\frac{2s', \frac{1}{2}A+a}{v, \frac{1}{2}A+a} \times s', \frac{1}{2}A+a.$	(198) $\frac{2s', \frac{1}{2}A-a}{v, A-a} \times s', \frac{1}{2}A-a.$
(199) $\frac{s, \frac{1}{2}A+a}{s, \frac{1}{2}A+a} \times f, \frac{1}{2}A+a.$	(200) $\frac{s, \frac{1}{2}A-a}{t, \frac{1}{2}A-a} \times f, \frac{1}{2}A-a.$	(201) $\frac{s, \frac{1}{2}A+a}{t, \frac{1}{2}A+a} \times f, \frac{1}{2}A+a.$	(202) $\frac{s', \frac{1}{2}A+a}{t', \frac{1}{2}A+a} \times f, \frac{1}{2}A+a.$
(203) $\frac{s, \frac{1}{2}A+a}{r} \times t, \frac{1}{2}A+a.$	(204) $\frac{s', \frac{1}{2}A-a}{r} \times f, \frac{1}{2}A-a.$	(205) $\frac{s, \frac{1}{2}A+a}{r} \times f, \frac{1}{2}A+a.$	(206) $\frac{s', \frac{1}{2}A+a}{r} \times f, \frac{1}{2}A+a.$
(207) $\frac{s, \frac{1}{2}A+a}{s, \frac{1}{2}A+a} \times t, \frac{1}{2}A+a.$	(208) $\frac{s, \frac{1}{2}A-a}{f-s, \frac{1}{2}A-a} \times t, \frac{1}{2}A-a.$	(209) $\frac{s, \frac{1}{2}A+a}{f, \frac{1}{2}A+a} \times t+t', \frac{1}{2}A+a.$	(210) $\frac{s', \frac{1}{2}A+a}{f, \frac{1}{2}A+a} \times t+t', \frac{1}{2}A+a.$

From the preceding table, a very great number of properties are readily deduced; some of which are here annexed, as examples of its use; where analogies are, in general, expressed by equal ratios.

$$211. \text{ The } \frac{\text{sum of the sines of two arcs}}{\text{diff. of the sines of those arcs}} = \frac{\text{tan. of half the sum of those arcs}}{\text{tan. of half the diff. of the arcs}} \quad (171, 172)$$

$$212. \text{ The } \frac{\text{sum of the co-sin. of two arcs}}{\text{diff. of the co-sin. of those arcs}} = \frac{\text{co-tan. of half the sum of the arcs}}{\text{tan. of half the diff. of the arcs}} \quad (175, 180)$$

$$213. \text{ The } \frac{\text{fine of the sum of two arcs}}{\text{fine of the diff. of those arcs}} = \frac{\text{sum of the tan. of those arcs}}{\text{diff. of the tan. of those arcs}}$$

$$\text{For } \frac{s, A + s, a}{s, A - s, a} = \frac{t, \frac{1}{2}A + a}{t, \frac{1}{2}A - a} \quad (211.) \quad \text{And } \frac{s, A + s, A + s, A - s, a}{s, A + s, a - s, A + s, A} = \frac{t, \frac{1}{2}A + a + t, \frac{1}{2}A - a}{t, \frac{1}{2}A + a - t, \frac{1}{2}A - a}$$

by composition.

$$\text{Then } \frac{t, \frac{1}{2}A + a + t, \frac{1}{2}A - a}{t, \frac{1}{2}A + a - t, \frac{1}{2}A - a} = \left(\frac{s, A + s, A}{s, a + s, a} \right) \quad (\text{III. 47, '48}) = \frac{2s, A}{2s, a} = \frac{s, A}{s, a}$$

Here the arcs A, a , are the sum and diff. of the arcs $\frac{1}{2}A + a, \frac{1}{2}A - a$.

$$214. \text{ The } \frac{\text{co-f. of the sum of two arcs}}{\text{co-f. of the diff. of the arcs}} = \frac{\text{diff. of tan. of one and co-t. of other}}{\text{sum of tan. of one and co-t. of other}}$$

taking the tan. of the same arc.

$$\text{For } \frac{t, \frac{1}{2}A + a}{t, \frac{1}{2}A - a} = \frac{s, A + s, a}{s, a - s, A} \quad (212.) \quad \text{And } \frac{t, \frac{1}{2}A + a - t, \frac{1}{2}A - a}{t, \frac{1}{2}A + a + t, \frac{1}{2}A - a} = \frac{s, A + s, a - s, a + s, A}{s, A + s, a + s, a - s, A}$$

$$\text{Then } \frac{t, \frac{1}{2}A + a - t, \frac{1}{2}A - a}{t, \frac{1}{2}A + a + t, \frac{1}{2}A - a} = \left(\frac{2s, A}{2s, a} \right) = \frac{s, A}{s, a}$$

Here the arcs A, a , are the sum and diff. of the arcs $\frac{1}{2}A + a, \frac{1}{2}A - a$.

215. The fine of the sum of two arcs, into radius; is equal to the sum of the products, of the fine of the greater by the co-sine of the less, and the fine of the less by the co-sine of the greater. And,

The fine of the difference of two arcs, into radius; is equal to the difference of the products, of the fine of the greater by the co-sine of the less, and the fine of the less by the co-sine of the greater.

$$\text{For } \left\{ \begin{array}{l} R \times \frac{1}{2}s, A + \frac{1}{2}s, a = s, \frac{1}{2}A + a \times s, \frac{1}{2}A - a \quad (173). \\ R \times \frac{1}{2}s, A - \frac{1}{2}s, a = s, \frac{1}{2}A - a \times s, \frac{1}{2}A + a \quad (182). \end{array} \right\} \text{ Here } \frac{1}{2}A + a \text{ and } \frac{1}{2}A - a$$

are the arcs.

$$\text{Hence } \left\{ \begin{array}{l} R \times s, A \text{ (the sum)} = s, \frac{1}{2}A + a \times s, \frac{1}{2}A - a + s, \frac{1}{2}A - a \times s, \frac{1}{2}A + a \\ R \times s, a \text{ (the diff.)} = s, \frac{1}{2}A + a \times s, \frac{1}{2}A - a - s, \frac{1}{2}A - a \times s, \frac{1}{2}A + a \end{array} \right.$$

216. The

216. The co-sine of the sum of two arcs, into radius; is equal to the difference, between the product of the co-sines, and product of the sines, of those arcs.

The co-sine of the difference of two arcs, into radius; is equal to the sum, of the product of the co-sines, and product of the sines, of those arcs.

For $\left\{ \begin{array}{l} R \times \frac{1}{2}s, A + \frac{1}{2}s, a = s, \frac{1}{2}A + a \times s, \frac{1}{2}A - a \text{ (174).} \\ R \times \frac{1}{2}s, a - \frac{1}{2}s, A = s, \frac{1}{2}A + a \times s, \frac{1}{2}A - a \text{ (181).} \end{array} \right\} \frac{1}{2}A + a \text{ and } \frac{1}{2}A - a \text{ being}$
the arcs.

Then $\left\{ \begin{array}{l} R \times s, A \text{ (the sum)} = s, \frac{1}{2}A + a \times s, \frac{1}{2}A - a - s, \frac{1}{2}A + a \times s, \frac{1}{2}A - a. \\ R \times s, a \text{ (the diff.)} = s, \frac{1}{2}A + a \times s, \frac{1}{2}A - a + s, \frac{1}{2}A + a \times s, \frac{1}{2}A - a. \end{array} \right.$

217. Radius, less the co-sine of an arc $\frac{\text{Square the tan. of half that arc}}{\text{Square of the radius.}}$
Radius, more the co-sine of an arc

$$\text{For } \left\{ \begin{array}{l} R - s, A + a = (v, A + a) = \frac{t, \frac{1}{2}A + a}{R} \times s, A + a. \\ R + s, A + a = (v, A + a) = \frac{R}{t, \frac{1}{2}A + a} \times A + a. \end{array} \right. \quad \begin{array}{l} (195) \\ (191) \end{array}$$

$$\text{Then } \frac{R - s, A + a}{R + s, A + a} = \frac{t, \frac{1}{2}A + a}{R}$$

218. The $\frac{\text{sum of the fine \& co-fine of an arc}}{\text{diff. of the fine \& co-fine of that arc}} = \frac{\text{radius}}{\text{tan. of diff. of that arc \& } 45^\circ}.$

The $\frac{\text{sum of rad. and tan. of an arc}}{\text{diff. of rad. and tan. of that arc}} = \frac{\text{radius}}{\text{tan. of diff. of that arc \& } 45^\circ}.$

For if $A + a = 90^\circ$; then $\frac{1}{2}A = 45^\circ - \frac{1}{2}a$; and $\frac{1}{2}a = 45^\circ - \frac{1}{2}A$.

Also $s, a = s, A$; $s, a = s, A$; and $s, A = t, A \times s, A \div R$. (III. 33)

$$\text{Then } \frac{s, A + s, a}{s, a - s, A} = \frac{t, \frac{1}{2}A + a}{t, \frac{1}{2}A - a}. \quad (212)$$

$$\text{Or } \frac{s, A + s, A}{s, A - s, A} = \left(\frac{t, 45^\circ - \frac{1}{2}a + \frac{1}{2}a}{t, A - 45^\circ + \frac{1}{2}A} \right) = \frac{R}{t, A \infty 45^\circ}.$$

$$\text{Again, } \frac{s, A + s, A}{s, A - s, A} = \left(\text{III. 33} \right) \frac{t, A \times s, A \div R + s, A}{t, A \times s, A \div R - s, A} = \frac{t, A \times s, A + s, A \times R}{t, A \times s, A - s, A \times R}.$$

$$\text{Then } \left(\frac{s, A + s, A}{s, A - s, A} = \frac{t, A + R \times s, A}{t, A - R \times s, A} \right) \frac{R + t, A}{R \infty t, A} = \frac{R}{t, A \infty 45^\circ}.$$

* This mark ∞ shews the difference of the values it stands between.

219. The difference of the co-fines of two arcs, is equal to the difference of the versed fines of those arcs.

220. The product of the fines of two arcs, is equal to the product of half the radius into the difference of the co-fines, of the sum and difference of those arcs.

$$\text{That is, } s, \frac{1}{2}A+a \times s, \frac{1}{2}A-a = \frac{1}{2}R \times s, a-s, A=s, \frac{1}{2}A+a - \frac{1}{2}A-a = s, \frac{1}{2}A+a + \frac{1}{2}A-a. \quad (181)$$

$$\text{Or } s, z \times s, x = \frac{1}{2}R \times s, z+x-s, z-x. \quad \text{Putting } z = \frac{1}{2}A+a; x = \frac{1}{2}A-a.$$

221. The product of the fines of two arcs, is equal to the product of half the radius into the difference between the versed fines, of the sum and difference of those arcs.

$$\text{That is, } s, z \times s, x = (\frac{1}{2}R \times s, z+x-s, z-x) (220) = v, z+x - v, z-x \times \frac{1}{2}R. \quad (219)$$

$$222. \text{ Half the radius} = \frac{\text{square of the fine of an arc}}{\text{versed line of twice that arc}} \quad (193)$$

$$= \frac{\text{square of the co-fine of an arc}}{\text{sup-versed fine of twice that arc}} \quad (197)$$

$$= \frac{\text{product of the fines of two arcs}}{\text{diff. of ver. fines of the sum and diff. of those arcs,}} \quad (221)$$

$$223. \text{ The sq. of rad.} = \frac{\text{prod. of the squares, of the fine and co-t. of an arc}}{\text{square of the co-fine of that arc}},$$

$$= \frac{\text{prod. of the squares of the co-fine & tan. of an arc}}{\text{square of the fine of that arc}},$$

$$\text{For } R = \frac{s, A \times t, A}{s, A} = \frac{s, A \times t, A}{s, A} (187). \text{ Then } RR \frac{ss, A \times tt, A}{ss, A} = \frac{s, s, A \times tt, A}{ss, A}.$$

224. The product of radius, and the co-fine of an arc, is equal to the difference of the squares, of the fine and co-fine of half that arc.

$$\text{For } \frac{s, s, \frac{1}{2}A+a}{s, \frac{1}{2}A+a} = \left(\frac{v, A+a}{v, \frac{1}{2}A+a} (193, 197) = \right) \frac{R \times s, A+a}{R-s, A+a}. \quad \text{Put } z = A+a.$$

$$\text{Then } \frac{R + s, z - R + s, z}{R + s, z + R - s, z} = \frac{s, s, \frac{1}{2}z - ss, \frac{1}{2}z}{s, s, \frac{1}{2}z + ss, \frac{1}{2}z}. \quad \text{By composition and division.}$$

$$\frac{2s, z}{2R} = \frac{s, s, \frac{1}{2}z - ss, \frac{1}{2}z}{RR} \quad (\text{II. III.}) \quad \text{And } R \times s, z = s, s, \frac{1}{2}z - ss, \frac{1}{2}z.$$

$$= s, \frac{1}{2}z + s, \frac{1}{2}z \times s, \frac{1}{2}z - s, \frac{1}{2}z. \quad (\text{II. 119}).$$

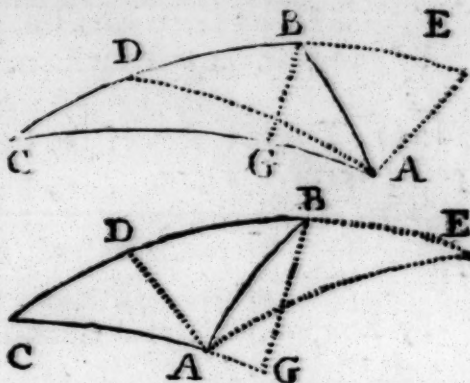
In

In any spheric triangle ABC , if in the side CB produced, be taken BE, BD , each equal to BA , and BG be drawn at right angles to CA .

Then $CE = BC + BA$ is the sum
 $CD = BC - BA$ is the diff. } of the legs including the angle at B .

CG and AG are the segments of the base, or side opposite to the angle B . $\angle A$ and $\angle C$ are called base angles. $\angle CBG = a$, $\angle ABG = c$ are the vertical angles.

Now a very great number of relations may be formed between the sides and angles; some of which are here enumerated.



225. The sines of the legs, are as the sines of the opposite base angles.

That is, $s, BC : s, BA :: s, A : s, C$. (110)

Hence $\frac{\text{sum of the sines of the legs}}{\text{diff. of the sines of the legs}} = \frac{\text{sum of the sines of the base angles}}{\text{diff. of the sines of the base angles}}$ by composition.

226. The co-sines of the base angles, are as the sines of the vertical angles.

That is, $s, C : s, A :: s, a : s, c$. (3d of 122, and 2d of 124)

Hence $\frac{\text{sum of co-sines of base angles}}{\text{diff. of co-sines of base angles}} = \frac{\text{sum of the sines of vertical angles}}{\text{diff. of sines of vertical angles}}$ by composition and division of ratios.

227. The co-sines of the legs, are as the co-sines of the adjacent segments of the base.

That is, $s, BC : s, BA :: s, CG : s, AG$. (3d of 121, and 2d of 123)

Hence $\frac{\text{sum of co-sines of the legs}}{\text{diff. of co-sines of the legs}} = \frac{\text{sum of co-sines of base segments}}{\text{diff. of co-sines of base segments}}$ by composition and division of ratios.

228. The co-tangents of the legs, are as the co-sines of the adjacent vertical angles.

That is, $t, BC : t, BA :: s, a : s, c$. (2d of 121, and 2d of 124)

Hence $\frac{\text{sum of co-t. of the legs}}{\text{diff. of co-t. of the legs}} = \frac{\text{sum of co-sines of vert. angles}}{\text{diff. of co-sines of vert. angles}}$ by composition and division of ratios.

229. The tangents of the legs, are as the co-f. of the adjacent vertical angles reciprocally.

Hence $\frac{\text{sum of tan. of the legs}}{\text{diff. of tan. of the legs}} = \frac{\text{sum of co-f. of vert. angles}}{\text{diff. of co-f. of vert. angles}}$ by comp. &c.

230. The

230. The fines of the base segments, are as the tangents of the adjacent base angles reciprocally.

That is, $s, CG : s, AG :: (t, c : t, A ::) t, A : t, c$. (2d of 122, and 1st of 123)

Hence $\frac{\text{sum of fines of base segments}}{\text{diff. of fines of base segments}} = \frac{\text{sum of tan. of base angles}}{\text{diff. of tan. of base angles}}$
by composition and division.

231. The tangents of the base segments, are as the tangents of the opposite vertical angles.

That is, $t, CG : t, AG :: t, a : t, c$. (108)

Hence $\frac{\text{sum of tan. of base segments}}{\text{diff. of tan. of base segments}} = \frac{\text{sum of tan. of vert. angles}}{\text{diff. of tan. of vert. angles}}$
by composition and division of ratios.

232. The $\frac{\text{tan. of half the sum of the legs}}{\text{tan. of half the diff. of the legs}} = \frac{\text{tan. of half the sum of the base ang.}}{\text{tan. of half the diff. of the base ang.}}$

$$\text{For } \frac{s, BC + s, BA}{s, BC - s, BA} = \frac{s, A + s, C}{s, A - s, C} \quad (225) = \frac{t, \frac{1}{2}BC + BA}{t, \frac{1}{2}BC - BA} = \frac{t, \frac{1}{2}A + C}{t, \frac{1}{2}A - C} \quad (211)$$

233. The $\frac{\text{tan. of } \frac{1}{2} \text{ sum of base segments}}{\text{tan. of } \frac{1}{2} \text{ sum of the legs}} = \frac{\text{tan. of } \frac{1}{2} \text{ diff. of the legs}}{\text{tan. of } \frac{1}{2} \text{ diff. of the base segments.}}$

$$\text{For } \frac{s, BA + s, BC}{s, BA - s, BC} = \frac{s, GA + s, GC}{s, GA - s, GC} \quad (227) = \frac{t, \frac{1}{2}BC + BA}{t, \frac{1}{2}BC - BA} = \frac{t, \frac{1}{2}CG + GA}{t, \frac{1}{2}CG - GA} \quad (212)$$

$$\text{Then } \frac{t, \frac{1}{2}BC - BA}{G - GA} = \left(\frac{t, \frac{1}{2}BC + BA}{t, \frac{1}{2}CG + GA} \right) \text{ (II. 145) } = \frac{t, \frac{1}{2}CG + GA}{t, \frac{1}{2}CB + BA} \quad (\text{III. 37})$$

234. The $\frac{\text{fine of sum of legs}}{\text{fine of diff. of legs}} = \frac{\text{co-tan. of } \frac{1}{2} \text{ sum of vert. angles}}{\text{tan. of } \frac{1}{2} \text{ diff. of vert. angles}}$
 $\frac{\text{co-tan. } \frac{1}{2} \text{ diff. of vert. angles}}{\text{tan. of } \frac{1}{2} \text{ sum of vert. angles}}$

$$\text{For } \frac{s, BC + BA}{s, BC - BA} = \left(\frac{t, BC + t, BA}{t, BC - t, BA} \right) \quad (213) = \frac{s, c + s, a}{s, c - s, a} \quad (226) = \frac{t, \frac{1}{2}a + c}{t, \frac{1}{2}a - c} = \frac{t, \frac{1}{2}a - c}{t, \frac{1}{2}a + c} \quad (212)$$

235. The $\frac{\text{co-t. of } \frac{1}{2} \text{ sum of vert. angles}}{\text{tan. of } \frac{1}{2} \text{ sum of the base angles}} = \frac{\text{tan. of } \frac{1}{2} \text{ diff. of base angles}}{\text{tan. of } \frac{1}{2} \text{ diff. of vert. angles.}}$

$$\text{For } \frac{t, \frac{1}{2}A + C}{t, \frac{1}{2}A - C} = \left(\frac{s, C + s, A}{s, C - s, A} \right) \quad (212) = \frac{s, a + s, c}{s, a - s, c} \quad (226) = \frac{t, \frac{1}{2}a + c}{t, \frac{1}{2}a - c} \quad (211)$$

$$\text{Hence } \frac{t, \frac{1}{2}A - C}{t, \frac{1}{2}a - c} = \left(\frac{t, \frac{1}{2}A + C}{t, \frac{1}{2}a + c} \right) \text{ (II. 145) } = \frac{t, \frac{1}{2}a + c}{t, \frac{1}{2}A + C} \quad (\text{III. 37})$$

236. The

236. The $\frac{\text{fine of sum of the legs}}{\text{fine of diff. of the legs}} = \frac{\text{square of co-t. of } \frac{1}{2} \text{ sum of vert. angles}}{\tan. \frac{1}{2} \text{ sum, into tan. } \frac{1}{2} \text{ diff. of base } \angle^s}$

$$\text{For } \frac{t, \frac{1}{2}A+C \times t, \frac{1}{2}A-C}{t, \frac{1}{2}a+c} = t, \frac{1}{2}a-c. \quad (235)$$

$$\text{Then } s, \overline{BC+BA} : s, \overline{BC-BA} :: t, \frac{1}{2}a+c : \frac{t, \frac{1}{2}A+C \times t, \frac{1}{2}A-C}{t, \frac{1}{2}a+c}. \quad (234)$$

$$:: t, \frac{1}{2}a+c : t, \frac{1}{2}A+C \times t, \frac{1}{2}A-C. \quad (\text{II. 151})$$

237. The $\frac{\text{fine of } \frac{1}{2} \text{ the sum of legs}}{\text{fine of } \frac{1}{2} \text{ the diff. of legs}} = \frac{\text{co-tan. of } \frac{1}{2} \text{ the sum of vert. angles}}{\tan. \text{ of } \frac{1}{2} \text{ the diff. of the base angles}}$

$$\text{For } \frac{t, \frac{1}{2}a+c}{t, \frac{1}{2}A+C \times t, \frac{1}{2}A-C} = \frac{s, \overline{BC+BA}}{s, \overline{BC-BA}} (236). \text{ And } \frac{t, \frac{1}{2}A+C}{t, \frac{1}{2}A-C} = \frac{t, \frac{1}{2}BC+BA}{t, \frac{1}{2}BC-BA}. \quad (232)$$

$$\text{Then } \frac{t, \frac{1}{2}a+c \times t, \frac{1}{2}A+C}{t, \frac{1}{2}A+C \times t, \frac{1}{2}A-C \times t, \frac{1}{2}A-C} = \frac{s, \overline{BC+BA} \times t, \frac{1}{2}BC+BA}{s, \overline{BC-BA} \times t, \frac{1}{2}BC-BA}. \quad (\text{II. 156})$$

$$\text{And } \frac{t, \frac{1}{2}a+c}{t, \frac{1}{2}A-C} = \frac{2ss, \frac{1}{2}BC+BA}{2ss, \frac{1}{2}BC-BA} (195, 193). \text{ Then } \frac{s, \frac{1}{2}BC+BA}{s, \frac{1}{2}BC-BA} = \frac{t, \frac{1}{2}a+c}{t, \frac{1}{2}A-C}.$$

238. The $\frac{\text{co-f. of } \frac{1}{2} \text{ sum of the legs}}{\text{co-f. of } \frac{1}{2} \text{ diff. of the legs}} = \frac{\text{co-t. of } \frac{1}{2} \text{ the sum of vertical angles}}{\tan. \text{ of } \frac{1}{2} \text{ the sum of the base angles}}$

$$\text{For } \frac{t, \frac{1}{2}a+c}{t, \frac{1}{2}A+C \times t, \frac{1}{2}A-C} = \frac{s, \overline{BC+BA}}{s, \overline{BC-BA}} (236). \text{ And } \frac{t, \frac{1}{2}A-C}{t, \frac{1}{2}A+C} = \frac{t, \frac{1}{2}BC+BA}{t, \frac{1}{2}BC-BA}. \quad (232)$$

$$\text{Then } \frac{t, \frac{1}{2}a+c \times t, \frac{1}{2}A-C}{t, \frac{1}{2}A+C \times t, \frac{1}{2}A-C \times t, \frac{1}{2}A+C} = \frac{s, \overline{BC+BA} \times t, \frac{1}{2}BC+BA}{s, \overline{BC-BA} \times t, \frac{1}{2}BC-BA}. \quad (\text{II. 156})$$

$$\text{And } \frac{t, \frac{1}{2}a+c}{t, \frac{1}{2}A+C} = \frac{2ss, \frac{1}{2}BC+BA}{2ss, \frac{1}{2}BC-BA} (191, 197). \text{ Then } \frac{s, \frac{1}{2}BC+BA}{s, \frac{1}{2}BC-BA} = \frac{t, \frac{1}{2}a+c}{t, \frac{1}{2}A+C}.$$

The two last propositions solve the problem where two sides, and the included angle, of a spheric triangle, are given to find the other angles.

Or where two angles and the included side are given, to find the other sides, using the word angles for legs; the given side for sum of vertical angles; the other side for base angles.

In art. 237, 238, the conclusions were gained from this principle, namely, that the sides of proportional squares, are in the same proportion as those squares.

239. The co-sine of an angle, is to radius;
 As the radius into co-s. of the opposite side, less the product of
 the co-sines of the including sides,
 To the product of the sines of the including sides.

$$\text{For } s, CG = (s, AC - AG) = s, AC \times s, AG + s, AC \times s, AG \div R. \quad (216)$$

$$\text{And } (s, CG) = \frac{s, BC \times s, AG}{s, AB} \quad (227) = s, AC \times s, AG + s, AC \times s, AG \div R. \quad (\text{II. 46})$$

$$\text{Therefore } s, BC \times s, AG = \frac{s, AB}{R} \times s, AC \times s, AG + \frac{s, AB}{R} \times s, AC \times s, AG.$$

$$\text{Therefore } \frac{R \times s, BC - s, AB \times s, AC}{R} \times s, AG = \frac{s, AB \times s, AC}{R} \times s, AG.$$

$$\text{Then } \frac{R \times s, BC - s, AB \times s, AC}{s, AB \times s, CA} = \left(\frac{s, AG}{s, AG} (\text{II. 163}) \right) \frac{t, AG}{R}$$

$$\text{But } t, AG = \frac{s, A}{R} \times t, AB \quad (\text{I 31}). \text{ And } \frac{t, AG}{R} = \left(\frac{s, A}{RR} \times t, AB = \right) \frac{s, A}{RR} \times \frac{s, AB \times R}{s, AB}. \quad (187)$$

$$\text{Then } \frac{s, A}{R} \times \frac{s, AB}{s, AB} = \frac{R \times s, BC - s, AB \times s, AC}{s, AB \times s, AC}. \text{ And } \frac{s, A}{R} = \frac{R \times s, BC - s, AC \times s, AB}{s, AC \times s, AB}.$$

$$240. \text{ Hence } R \times s, BC = \frac{s, A \times s, AC \times s, AB}{R} + s, AC \times s, AB.$$

241. The versed side of an angle, is to the square of radius:
 As the diff. of the versed sines of op. side, and diff. of including sides,
 To the product of the sines of the sides including that angle.

$$\text{For } (239) \frac{R \times s, BC - s, AC \times s, AB}{s, AB \times s, AC} = \left(\frac{s, A}{R} \right) \frac{R - v, A}{R}.$$

$$\text{Therefore } RR \times s, BC - R \times s, AC \times s, AB = R \times s, AC \times s, AB - s, AC \times s, AB \times v, A.$$

$$\text{Therefore } RR \times s, BC + s, AC \times s, AB \times v, A = s, AC \times s, AB + s, AC \times s, AB \times R. \\ = s, AC - AB \times RR. \quad (216)$$

$$\text{Then } s, AC \times s, AB \times v, A = (RR \times s, CD - RR \times s, BC) = s, CB - s, BC \times RR.$$

$$\text{And } \frac{v, A}{RR} = \left(\frac{s, CD - s, BC}{s, AC \times s, AB} \right) \frac{v, CB - v, CD}{s, AC \times s, AB}. \quad (219)$$

242. Hence

242. Hence $\frac{v, A}{2R} = \frac{v, BC - v, CD}{v, CE - v, CD}$. Or $\frac{1}{2} v, A = \frac{v, BC - v, CD}{v, CE - v, CD}$, when $R = 1$.

For $(s, AC \times s, AB =) \frac{1}{2} R \times \overline{v, CE - v, CD} \quad (222) = \overline{v, CB - v, CD} \times \frac{RR}{v, A} \quad (241)$

Then $\frac{v, CB - v, CD}{v, CE - v, CD} = \left(\frac{\frac{1}{2} R \times v, A}{RR} = \right) \frac{v, A}{2R}$.

243. The verfed sine of the sup. of an angle, is to the square of radius;
As the diff. of the verfed sines of the opposite side, and sum of the
including sides,
To the product of the sines of the sides including that angle.

For (239) $\frac{R \times s, BC - s, AC \times s, AB}{s, AC \times s, AB} = \left(\frac{s, A}{R} = \right) \frac{v, H - R}{R}$.

Therefore $RR \times s, BC - R \times s, AC \times s, AB = s, AC \times s, AB \times v, A - R \times s, AC \times s, AB$.

Therefore $RR \times s, BC - s, AC \times s, AB \times v, A = R \times s, AC \times s, AB - s, AC \times s, AB$.
 $= (RR \times s, AC + AB =) RR \times s, CE. \quad (216)$

Then $RR \times s, BC - RR \times s, CE = s, AC \times s, AB \times v, A$.

And $\frac{v, A}{RR} = \left(\frac{s, BC - s, CE}{s, AC \times s, AB} = \right) \frac{v, CE - v, BC}{s, AC \times s, AB}$.

244. Hence $\frac{v, A}{2R} = \frac{v, CE - v, CB}{v, CE - v, CD}$. Or $\frac{1}{2} v, A = \frac{v, CE - v, CB}{v, CE - v, CD}$, when $R = 1$.

For $(s, AC \times s, AB =) \frac{1}{2} R \times \overline{v, CE - v, CD} \quad (222) = \overline{v, CE - v, CB} \times \frac{RR}{v, A} \quad (243)$

Then $\frac{v, CE - v, CB}{v, CE - v, CD} = \left(\frac{\frac{1}{2} R \times v, A}{RR} = \right) \frac{v, A}{2R}$.

245. The square of the sine of half an angle, is to the square of the radius;
As $\frac{1}{2}$ radius into the diff. of the verfed sines of the side opposite,
and diff. of the sides including that angle,
To the product of the sines of the sides including that angle.

For $(v, A =) \frac{ss, \frac{1}{2} A}{\frac{1}{2} R} \quad (222) = \frac{v, CB - v, CD}{s, AC \times s, AB} \times RR. \quad (241)$

Then $\frac{ss, \frac{1}{2} A}{RR} = \frac{v, CB - v, CD}{s, AC \times s, AB} \times \frac{1}{2} R = \frac{s, CD - s, CB}{s, AC \times s, AB} \times \frac{1}{2} R. \quad (219)$

246. Hence

$$\begin{aligned}
 246. \text{ Hence } \frac{s, \frac{1}{2}A}{RR} &= \frac{s, \frac{1}{2}CB+CD \times s, \frac{1}{2}CB-CD}{s, AC \times s, AB} & (181) \\
 &= \frac{s, \frac{1}{2}CB+AC-AB \times s, \frac{1}{2}CB-AC+AB}{s, AC \times s, AB}; \text{ because } CD=AC-AB \\
 &= \frac{s, H-AC \times s, H-AB}{s, AC \times s, AB}. \text{ Putting } 2H=AC+AB+BC.
 \end{aligned}$$

247. The square of the co-sine of half an angle, is to the squ. of radius ;
 As $\frac{1}{2}$ radius into the diff. of versed lines of the side opposite, and
 sum of the included sides,
 To the product of the fines of the sides including that angle.

$$\text{For } (v, A) = \frac{s, \frac{1}{2}A}{\frac{1}{2}R} \quad (222) = \frac{v, CE - v, CB}{s, AC \times s, AB} \times RR. \quad (243)$$

$$\text{Then } \frac{s, \frac{1}{2}A}{RR} = \frac{v, CE - v, CB}{s, AC \times s, AB} \times \frac{1}{2}R = \frac{s, CB - s, CE}{s, AC \times s, AB} \times \frac{1}{2}R. \quad (219)$$

$$\begin{aligned}
 248. \text{ Hence } \frac{s, \frac{1}{2}A}{RR} &= \frac{s, \frac{1}{2}CE+CB \times s, \frac{1}{2}CE-CB}{s, AC \times s, AB} & (177) \\
 &= \frac{s, \frac{1}{2}AC+AB+CB \times s, \frac{1}{2}AC+AB-CB}{s, AC \times s, AB}. \text{ For } CE=AC+AB. \\
 &= \frac{s, H \times s, H-CB}{s, AC \times s, AB}. \text{ Putting } 2H=AC+AB+BC.
 \end{aligned}$$

249. The square of the tan. of half an angle, is to the square of radius ;
 As the diff. of ver. fines of the side op. and diff. of including sides,
 To the diff. of ver. fines of the side op. and sum of including sides.

$$\text{For } \frac{v, A}{v, A} = \frac{s, \frac{1}{2}A}{s, \frac{1}{2}A} \quad (222) = \frac{tt, \frac{1}{2}A}{RR} \quad (223) = \frac{v, CE - v, CD}{v, CE - v, CB}. \quad (241, 243)$$

$$\begin{aligned}
 250. \text{ Hence } \frac{tt, \frac{1}{2}A}{RR} &= \frac{s, \frac{1}{2}CB+CD \times s, \frac{1}{2}CB-CD}{s, \frac{1}{2}CE+CB \times s, \frac{1}{2}CE-CB} & (219, 181) \\
 &= \frac{s, \frac{1}{2}CB+AC-AB \times s, \frac{1}{2}CB-AC+AB}{s, \frac{1}{2}CB+AC+AB \times s, \frac{1}{2}AC+AC-CB} \\
 &= \frac{s, H-AB \times s, H-AC}{s, H \times s, H-BC}.
 \end{aligned}$$

From the articles in the three last pages may be deduced many rules
 for solving the problem, where the three sides are given to find an angle ;
 and thence, from the three angles given to find a side.

251. When

251. When two sides and the included angle are given to find the third side: Or when the three sides are given to find an angle; for these particular cases there have been given compendiums by Sir Jonas Moore, in his Mathematics, Vol. II. page 383: Also by Nicholas Facio Duillier, in a small tract of his, which is very scarce; and by the learned Dr. Pemberton, in the Philosophical Transactions for the year 1760: The principle employed by each of them is the same as in Art. 245; which will be here illustrated on account of its utility in some astronomical subjects.

In the triangle ABC, where $CD = AC \propto AB$.

Given AB, AC, $\angle A$; required BC.

$$\text{Now } \frac{s, \frac{1}{2} \angle A}{R^3} = \frac{\frac{1}{2} s, CB \propto \frac{1}{2} s, CD}{s, AC \times s, AB} \quad (245)$$

$$\text{And } \frac{s, \frac{1}{2} \angle A \times s, AC \times s, AB}{R^3} = \frac{1}{2} s, CB \propto \frac{1}{2} s, CD = N$$

Then $2N - s, CD = s, CB$.

Hence the following practical Rules.

252. I. To twice the log. sine of half the given angle,

Add the log. sines of the two containing sides;

The sum, abating three radii in the index, leaves the log. sine of an arc.

From twice the nat. sine of that arc; take the nat. co-sine of the diff. of the given sides,

Leaves the nat. co-sine of the third side, or of its supplement.

253. II. But the side required may be found without the use of natural sines. Thus

To twice the log. sine of half the given angle,

Add the log. sines of the two containing sides;

From half the sum of these logs. subtract the log. sine of half the diff. of the sides,

And the remainder is the log. tangent of an arc;

The log. sine of which arc, subtracted from the said half sum of logs,

Leaves the log. sine of half the required side.

Take the Example used in the six cases of oblique spheric triangles.

Where $AC = 83^\circ 11'$, $AB = 56^\circ 40'$; $\angle A = 125^\circ 20'$

Required BC, which was there found to be $114^\circ 30'$.

Given $\angle = 125^\circ 20'$

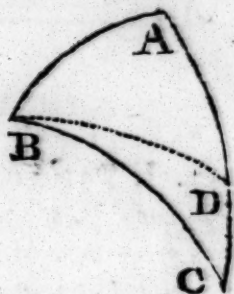
Its half	=	62	40	—	—	Log. sine	{ 9.94858
One side	=	83	11	—	—	Log. sine	{ 9.94858
Other side	=	56	40	—	—	Log. sine	{ 9.99692
							9.92194
Are found	=	40	53 $\frac{2}{3}$	its nat. sine	65467 (256)		9.81602

Diff. sides = 26 30 the nat. co-sine

65 30 the nat. co-sine 41454 (257)

The side required $114^\circ 30'$

The



The said Example wrought by the second Rule is as follows;

Given $\angle = 125^\circ 20'$

Its half $62^\circ 40'$ log. sine $\begin{cases} 9,94858 \\ 9,94858 \end{cases}$

One side $= 83^\circ 11'$ log. sine $9,99692$

Other side $= 56^\circ 40'$ log. sine $9,92194$

Sum logs. $= 39,81602$

Half sum $19,90801$ half sum log. $19,90801$

$\frac{1}{2}$ diff. sides $= 13^\circ 15\frac{1}{2}'$ $9,36048$

Arc $= 74^\circ 10'$ log. tan. $10,54753$ Log. sine $74^\circ 10'$ $9,98320$

Log. sine of $57^\circ 15'$ $9,92481$

The required side $= 114^\circ 30'$

When the three sides are given to find an angle;

254. I. To the nat. co-s. of the side opposite the required angle, add the nat. co-s. of the diff. of the sides about that angle; half the sum is the nat. sine of an arc. To the log. sine of that arc, add the arith. comps. of the log. sines of the sides about the required angle, and also the radius.

The half of this sum is the log. sine of half the angle sought.

Or without using the natural sines.

255. II. To the log. sine of half the diff. of the sides about the angle, add the arith. comp. of the log. sine of half the base; the sum is the log. sine of an arc. To the log. co-sine of this arc, add the log. sine of half the base; reject radius from the sum, and to the double of what will then remain add the arith. comps. of the log. sines of the contained sides.

Half the sum is the log. sine of half the angle.

EXAM. Let the three sides be $BC = 114^\circ 30'$, $AC = 80^\circ 11'$, $AB = 56^\circ 40'$. Required the angle A.

By I. Rule

Base $BC = 114^\circ 30'$

Its suppl. $63^\circ 30'$ nat. co-sine $= 41469$ (256)

Diff. sides $26^\circ 31'$ nat. co-s. $= 89480$

Sum 130949

Rad. $10,00000$

Half sum is the nat. sine. (217) 65474 arc $40^\circ 54'$ log. sine $9,81607$

Ar. co. log. sine $83^\circ 11'$ $0,00308$

Ar. co. log. sine $56^\circ 40'$ $0,07806$

$19,89721$

Half the angle sought $62^\circ 40'$ log. sine $9,94860$

By II. Rule.

$\frac{1}{2} DC = 13^\circ 15\frac{1}{2}'$ Log. sine $9,36048$

$\frac{1}{2} BC = 57^\circ 15'$ Ar. Co. L. fin. $0,07518$ log. sine $9,92482$

Log. sine $15^\circ 49\frac{1}{2}'$ an arc $9,43566$ log. co-s. $15^\circ 49\frac{1}{2}'$ $9,98322$

$9,90804$

2

$19,81608$

Ar. Co. Log. sine AC $=$ $=$ $=$ $0,00308$

Ar. Co. Log. sine AB $=$ $=$ $=$ $0,07806$

Sum $=$ $=$ $=$ $19,89722$

Half sum is Log. sine of $62^\circ 40'$

$9,94861$

Angle sought is $125^\circ 20' = \angle A.$

X

As

As the natural fines of arcs are not contained in this work, and are on some occasions necessary, it will be proper to shew how they may be found from the Logarithmic tables contained herein.

256. *First. An arc being given, to find its natural sine to five places of figures.*

Rule. Take out the Log. sine of the arc, rejecting the Index;
Seek these figures among the logarithms of numbers;
The corresponding number is the natural sine of the given arc;
which is to be reckoned as a decimal fraction of the radius, or unity:

Prefixing the decimal comma (,) if the index of the log. sine was 9;
But if the index was 8; 7; or 6; prefix ,0; ,00; or ,000; by which means the left hand digit of the natural sine will stand in the place of the firsts, seconds, thirds, or fourths. (l. 18)

Ex. I. *Required the natural sine and co-sine of $4^{\circ} 22'$?*

Log. sines	sine 8,88161	Co-sine 9,99874
Num. or nat. sines	0,07614	0,99710

Ex. II. *Required the natural sine and co-sine of $28^{\circ} 35'$?*

Log. sines	sine 9,67982	Co-sine 9,94355
Num. or nat. sines	0,47844	0,87812

If a given log. sine is found in the table of logs. of numbers, its natural number consisting of four places is seen at sight; and its right hand place is 0 when the index of the log. sine was 9.

But if a given log. sine is not found to every figure in the tables of log. numbers, its 5th, or right-hand place, is thus found.

Take the diff. between the log. num. next greater and less, than the given log. sine; and also the diff. between the given log. sine and its next less log. numb.

Then, As first diff. is to 2d diff. so is 10 to the digit for the right hand place.

Thus to $4^{\circ} 22'$, the nat. sine is 0,07614; and co-sine is 0,99710.

But to $28^{\circ} 35'$, the log. sine and co-s. does not appear exactly among the log. numb.

And the above-mentioned two differences, for sine, are 9 and 3; for co-s. are 5 and 1.

Then $9 : 3 :: 10 : 3$, the 5th place. And $5 : 1 :: 10 : 2$, the 5th place.

257. *On the contrary, A natural sine being given, its corresponding arc may be thus found.*

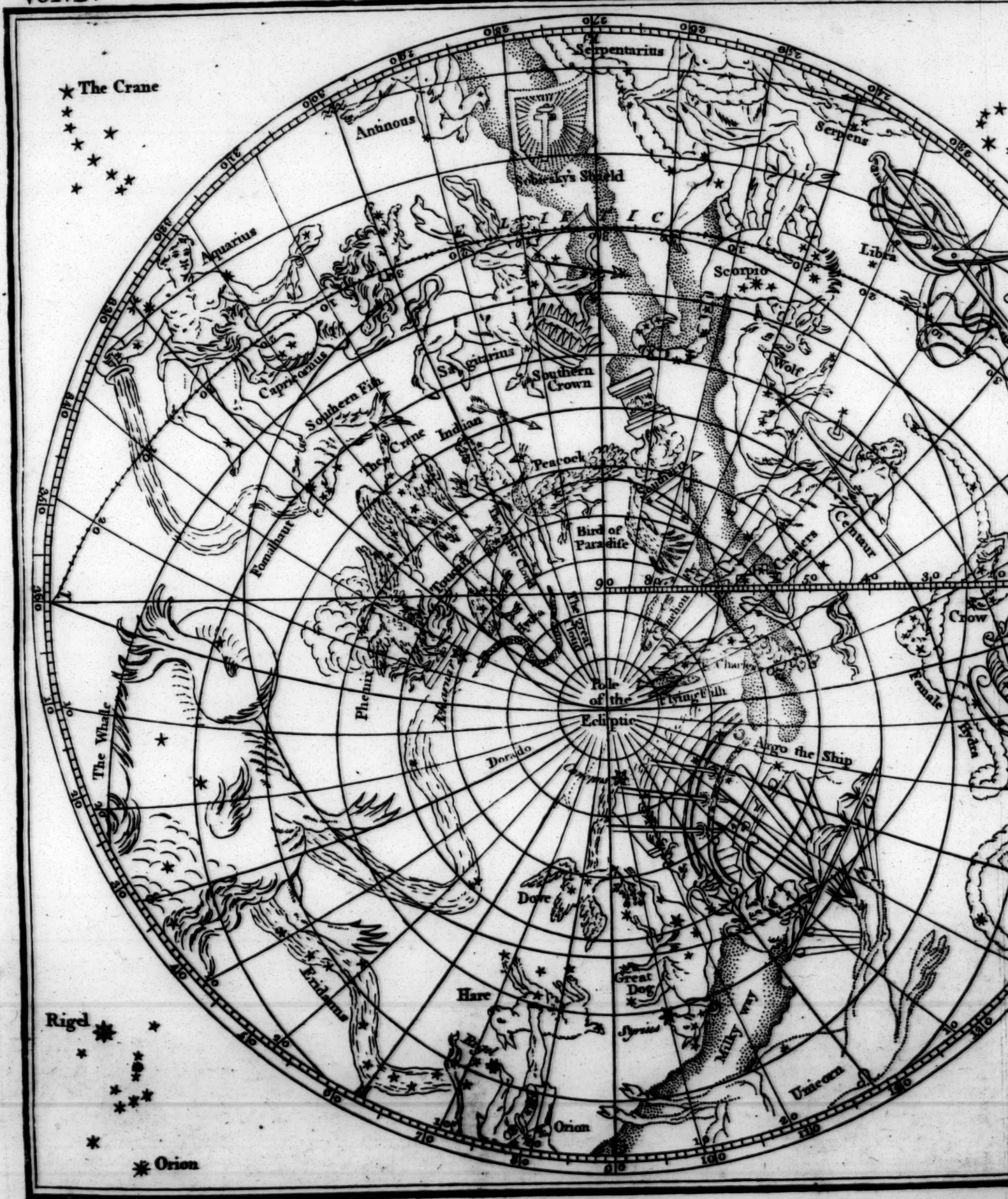
In the tables of num. and logs. enter with the natural sine as a num. and take out its log.

Seek this log. in the table of log. sines, and the corresponding degrees and minutes shew the arc required.

Prefixing the index 9, 8, 7, 6; according as the left hand digit stood in the place of firsts, seconds, thirds, or fourths.

What has been said of the nat. and log. sines of arcs, is also applicable to the nat. and log. tangents of arcs.

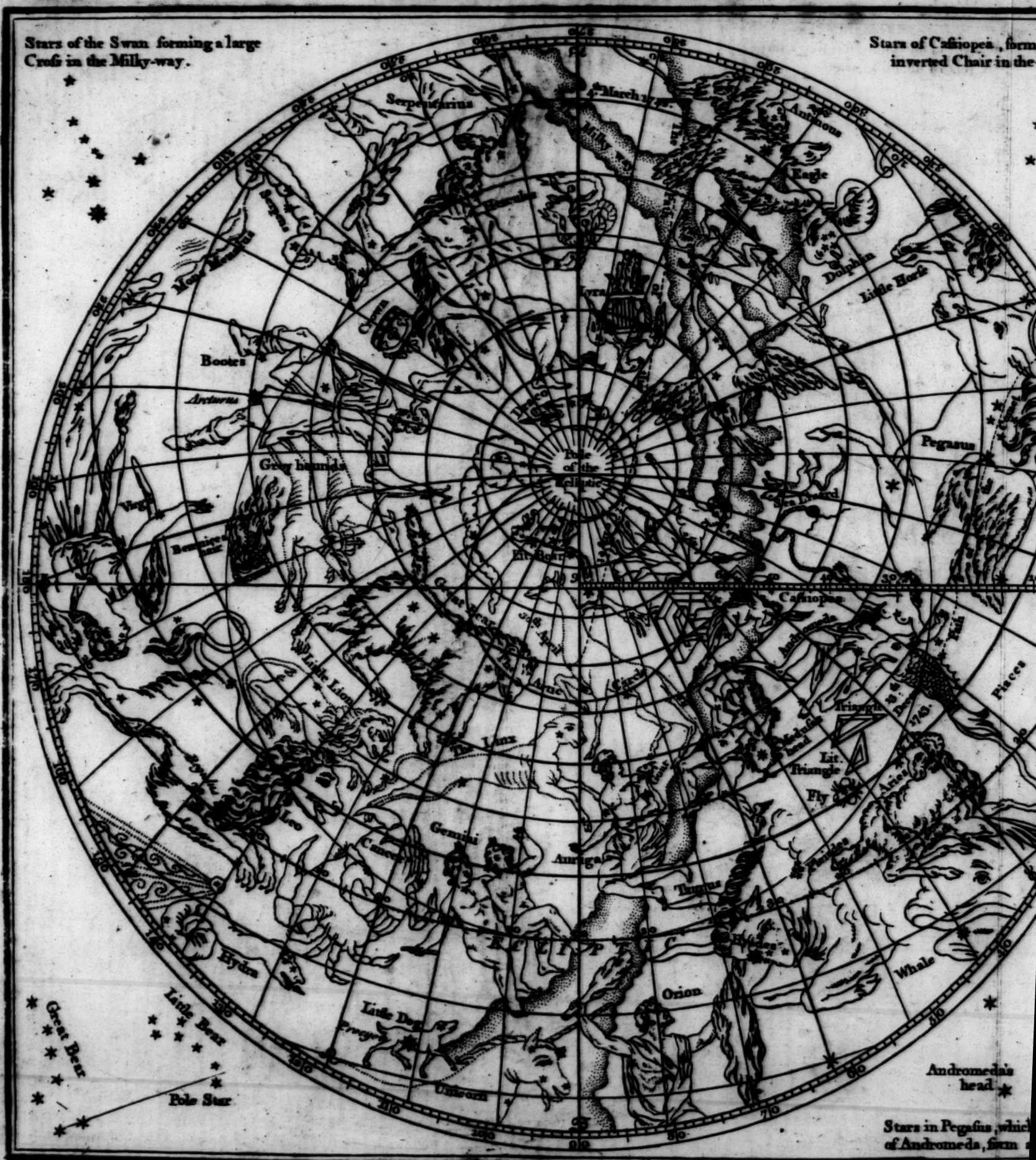
END OF BOOK IV.





Stars of the Swan forming a large
Cross in the Milky-way.

Stars of Cassiopea, form
inverted Chair in the



Pl. II.

ing a kind of
Milky-way.



which with the head
form a Quadrilateral.



THE
ELEMENTS
OF
NAVIGATION.

BOOK V.
OF ASTRONOMY.

SECTION I.
Of Solar Astronomy.

1. **A**STRONOMY is a science which treats of the motions and distances of the heavenly bodies, and of the appearances thence arising.

There have been great variety of opinions among the philosophers of preceding ages concerning the situation of the great bodies in the universe, or of the positions of the bodies which appear in the heavens: But the notion now embraced by the most judicious Astronomers is, that the universe is composed of an infinite number of systems, or worlds; in every system there are certain bodies moving in free space, and revolving at different distances around a SUN, placed in, or near, the center of the system; and that these *suns* and other *bodies* are the stars which are seen in the heavens.

2. The SOLAR SYSTEM, so called by Astronomers, is that in which our Earth is placed; and in which the Sun is supposed to be fixed in or near the center, with several bodies similar to our Earth revolving round him at different distances. This hypothesis, which is confirmed by all the observations hitherto made, is called the COPERNICAN SYSTEM,

from *Nicolas Copernicus*, a Polish Philosopher, who about the year 1500 revived this notion from the oblivion it had been buried in for many ages.

Stars are distinguished into two kinds, namely, *fixed* and *wandering*.

3. The **FIXED STARS** are the suns, in the centers of their systems, shining by their own light; and are observed to preserve always the same situation in respect to one another.

4. The fixed stars appear of various sizes, which is, undoubtedly, occasioned by their different distances; these sizes are usually distinguished into six or seven classes, called **Magnitudes**: The largest, or brightest, are said to be of the **FIRST MAGNITUDE**; those in the next class or degree of brightness, are said to be of the **SECOND MAGNITUDE**; and so on to the least, or those just discernable to the naked eye. But besides these, there is scattered throughout the heavens a great number of other stars, called **TELESCOPIC STARS**, because they cannot be seen except through a telescope. And, indeed, it is to the assistance of that most admirable instrument, that a great part of the modern Astronomy owes its rise; which will transmit with honour to the latest posterity the name of **GALILEO**, among whose many inventions the telescope is ranked.

5. Although the visible fixed stars are probably at very unequal distances from the center of the *solar system*, yet Astronomers, for their ease in computation, consider them as equally distant from our Sun, forming the surface of a sphere which incloses our system, and is called the **CELESTIAL SPHERE**. This supposition, with regard to the Solar System, may be strictly admitted, considering the immense distance even of the nearest fixed stars from the center of the system.

6. A **CONSTELLATION** is a number of stars lying in the neighbourhood of one another on the surface of the *celestial sphere*, which Astronomers, for the sake of remembering them with greater ease, supposed to be circumscribed by the outlines of some *animal*, or other *figure*: by this means the motions of the wandering stars are more readily described and compared.

The number of these constellations is 80, each containing several stars of different magnitudes. The number of stars of each magnitude, and also the constellation in which they are ranged, are contained in the following table; where it may be observed, that the constellations are distinguished under three heads; namely, in the *zodiac*, and in the northern, and southern hemispheres.

7. CONSTELLATIONS IN THE ZODIAC.

Names.	Marks.	Number.	Magnitudes.						Names.	Marks.	Number.	Magnitudes.					
			I	II	III	IV	V	VI				I	II	III	IV	V	VI
Northern.									Southern.								
Aries.	♈	46	0	1	1	3	5	36	Libra.	♎	33	0	2	1	8	3	19
Taurus.	♉	109	1	1	3	9	24	71	Scorpio.	♏	44	1	3	6	14	5	15
Gemini.	♊	94	1	2	4	8	13	66	Sagittarius.	♐	48	0	1	5	9	11	22
Cancer.	♋	75	0	0	0	6	8	61	Capricornus.	♑	58	0	0	2	5	9	42
Leo.	♌	91	2	2	6	13	11	57	Aquarius.	♒	93	0	0	4	7	28	54
Virgo.	♍	93	1	0	5	11	24	52	Pisces.	♓	110	0	0	1	7	28	74

8. NORTHERN

8.

NORTHERN CONSTELLATIONS.

Names.	Numb.	Magnitudes.						Names.	Numb.	Magnitudes.					
		I	II	III	IV	V	VI			I	II	III	IV	V	VI
Little Bear.	12	0	2	1	4	3	2	Camelopardalus.	23	0	0	0	5	7	11
Great Bear.	105	0	5	5	16	30	49	Serpent.	50	0	1	7	6	5	31
Dragon.	49	0	1	7	8	10	23	Serpentarius.	67	0	1	6	12	17	31
Cepheus.	40	0	0	3	7	10	20	Sobieski's Shield.	8	0	0	0	2	3	3
Greyhounds.	24	0	0	0	1	7	16	Eagle.	29	1	0	5	1	4	18
Bootes.	53	1	0	7	10	12	23	Antinous.	34	0	0	5	2	7	20
Mons Mæmalus.	11	0	0	0	1	0	10	Dolphin.	18	0	0	6	0	2	10
Berenice's Hair.	24	0	0	0	6	8	10	Colt.	12	0	0	0	4	1	7
Charles's Heart.	3	0	1	0	0	0	2	Arrow.	13	0	0	0	4	1	8
Northern Crown.	11	0	1	0	6	3	1	Andromeda.	66	0	3	2	10	16	35
Hercules.	92	0	0	12	12	28	40	Perseus.	67	1	1	5	10	14	36
Cerberus.	9	0	0	0	3	1	5	Pegasus.	81	0	3	4	9	11	54
Harp.	24	1	0	3	2	8	10	Auriga.	46	1	1	1	9	9	25
Swan.	73	0	1	5	15	20	32	Lynx.	55	0	0	1	8	21	25
Fox.	29	0	0	0	6	11	12	Little Lion.	20	0	0	1	6	5	8
Goose.	10	0	0	0	0	2	8	Great Triangle.	10	0	0	0	3	1	6
Lizard.	12	0	0	0	3	5	4	Little Triangle.	8	0	0	0	0	0	5
Cassiopea.	52	0	0	5	7	7	33	Musca.	6	0	0	1	2	2	1

9.

SOUTHERN CONSTELLATIONS.

Names.	Numb.	Magnitudes.						Names.	Numb.	Magnitudes.					
		I	II	III	IV	V	VI			I	II	III	IV	V	VI
Whale.	80	0	2	8	13	10	47	Peacock.	14	0	1	3	5	4	1
Eridanus.	72	1	0	10	24	19	18	Southern Crown.	12	0	0	0	1	3	8
Phoenix.	13	0	1	5	5	0	2	Crane.	14	0	2	1	2	9	0
American Goose.	9	0	0	4	2	3	0	Southern Fish.	15	1	0	2	9	2	1
Orion.	93	2	4	3	19	15	50	Hare.	25	0	0	4	9	4	8
Monoceros.	32	0	0	1	10	10	11	Noah's Dove.	10	0	2	0	1	6	0
Little Dog.	14	1	0	1	0	2	10	Charles's Oak.	13	0	1	2	6	4	0
Hydra.	53	0	1	3	14	13	22	Ship Argo.	48	1	6	11	13	14	3
Sextans Uraniz.	4	0	0	0	0	0	4	Great Dog.	29	1	5	1	4	10	8
Cup.	17	0	0	0	8	2	1	Bee.	4	0	0	0	2	2	0
Crow.	8	0	0	2	2	2	2	Swallow.	11	0	0	0	4	3	4
Centaur.	36	2	6	6	14	8	0	Indus.	12	0	0	0	4	6	2
Wolf.	36	0	0	3	6	18	9	Camelion.	10	0	0	0	0	9	1
Altar.	9	0	0	1	6	1	1	Flying Fish.	7	0	0	0	0	6	1
Southern Triangle.	5	0	1	2	0	2	0	Sword Fish.	7	0	0	2	2	1	2

Constellations in the zodiac 12, contain
 Northern constellations 36, contain
 Southern constellations 32, contain

Number of stars in the 80 constellations

Stars	I	II	III	IV	V	VI
894	6	12	38	100	160	569
243	5	21	92	200	291	634
706	9	32	75	185	188	217
2843	20	65	205	485	648	1420

As these stars are found not to alter their situation in respect to one another, they serve Astronomers as fixed points, by which the motions of other

other bodies may be compared ; and therefore their relative positions have been sought after with great care for many ages, and catalogues of their places have from time to time been published by those, who have been at the pains to make the observations. Among these catalogues, the most copious, and, as generally esteemed, the best, is that called the *Historia Cælestis* of our countryman FLAMSTEED.

10. The positions of the stars being obtained, their relative places may be delineated on a sphere or plane ; and thus are the maps or charts of the heavens made, and the constellations drawn inclosing their respective stars. There are two maps, usually called Celestial hemispheres, which are prefixed to this book ; by the help of which a person may readily become acquainted with the positions and names of many of the principal fixed stars, thus :

On a clear night, let these prints be laid so as to correspond to the north and south parts of the heavens ; then the observer looking on the stars, and then on the hemispheres, will with a little practice know some of the stars in the heavens, the like positions and names of which he has observed on the prints.

11. The WANDERING STARS are those bodies within our system, or celestial sphere, which revolve round the Sun ; they appear luminous or bright, only by reflecting the light they receive from the Sun ; and are of three kinds, namely, *primary planets*, *secondary planets*, and *comets*.

12. The PRIMARY PLANETS are those bodies, which in revolving round the Sun respect him only as the center of their courses ; the motions of which are regularly performed in tracks, or paths, that are found by observations to be nearly circular and concentric to one another.

13. A SECONDARY PLANET, commonly called a SATELLITE or MOON, is a body, which, while it is carried round the Sun, does also revolve round a primary planet, which it respects as its center.

14. COMETS, vulgarly called *blazing stars*, are bodies which also revolve round the Sun ; probably in as regular order as the planets, but in much longer periods of time. From what is already known of them, they are many more in number than all the planets, and their tracks or courses pass among the paths of the planets in a great variety of directions.

15. The ORBIT of a planet or comet is that track or path along which it moves.

There are six *primary planets* ; and reckoned in order from the Sun, their names and marks are, MERCURY ☿, VENUS ♀, the EARTH ♂ or ⊕, MARS ♂, JUPITER ♃, SATURN ♄.

Mars, *Jupiter*, and *Saturn*, are called SUPERIOR PLANETS, as their orbits include that of the *Earth* ; but *Venus* and *Mercury*, the orbits of which are contained within the *Earth's*, are called INFERIOR PLANETS.

16. It has been discovered by the help of telescopes, that there are ten *secondary planets* ; the *Earth* being attended by one, called the Moon, *Jupiter* by four, and *Saturn* by five.

Saturn is also observed to have a kind of circle, called his RING, which surrounds the planet at some distance from his surface : and *Jupiter* has certain appearances, which seem like zones or girdles round him ; and these are called JUPITER'S BELTS.

Every

Every primary planet is supposed to have two motions, namely, *annual* and *diurnal*.

17. The **ANNUAL MOTION** of a planet is that whereby the planet is carried in its orbit round the Sun; which in every one is found by observation to be from west to east.

This motion is discovered by the planets changing their places in the celestial sphere; upon the surface of which they appear to move among the fixed stars; and in certain times to return to the same stars from which they were seen to depart; and so on continually.

18. The **DIURNAL MOTION** of a planet is that by which it turns or spins about its axis, and is also from west to east.

This motion is discovered by the spots that are seen by telescopes on the surfaces of the planets. The spots appear first on the eastern margin, or side of the planet, and gradually move from thence across it, till they disappear on the western side, or *limb*; after a certain time they appear again on the eastern side, and so on.

19. Each planet is observed always to pass through the constellations *Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, Pisces*; and it also appears, that every one has a track peculiar to itself; hence the paths of the six planets form among the stars a kind of road, which is called the **ZODIAC**, the middle path of which, called the **ECLIPTIC**, is the orbit described by the Earth, with which the orbits of the other planets are compared.

As the ecliptic runs through twelve constellations, it is supposed to be divided into twelve equal parts, of 30 degrees each, called signs, having the same names with the twelve constellations which they run through.

20. The **EQUINOCTIAL POINTS** are those two points of the Ecliptic, opposite to one another, through which the Earth passes in its annual motion, when the length of the day and night is equal in all parts on the Earth. One of these points, called the **VERNAL EQUINOX**, answers nearly to the 20th of March; and the other, called the **AUTUMNAL EQUINOX**, nearly to the 22d of September.

21. The *Plane of the Ecliptic* is supposed to divide the celestial sphere into two equal parts, called the *northern* and *southern celestial hemispheres*; and any body in either of these hemispheres is said to have north or south latitude, according to the hemisphere it is in: So that the **LATITUDE** of a celestial object is its nearest distance from the ecliptic, taken on the sphere.

The Planes of the other five orbits are observed to lie partly in the northern, and partly in the southern hemisphere; so that every one cuts the ecliptic in two opposite points called **NODES**. One called the **ASCENDING NODE**, marked thus, ♈, is that through which the planet passes when it moves out of the southern into the northern hemisphere; and the other called the **DESCENDING NODE**, marked thus, ♎, is that through which the planet must pass in going out of the northern into the southern hemisphere.

The right line joining the two Nodes of any planet, is called the **LINE OF THE NODES**.

22. The names of most of the constellations were given by the ancient Astronomers, who reckoned that star in Aries, now marked γ, (according to

to *Bayer's* maps) to be the first point in the ecliptic, this star being next the Sun when he entered the *Vernal Equinox*; and at that time each constellation was in the sign by which it was called. But observations shew, that the point marked in the heavens by the *Vernal Equinox* has been constantly going backward by a small quantity every year, from which cause the stars appear to have advanced forward as much; so that the constellation *Aries* is now removed almost into the sign *Taurus*, the said first star γ having got almost 30 degrees forward from the equinox; which difference is called the PRECESSION OF THE EQUINOXES, and the yearly alteration is about 50 seconds of a degree, or about a degree in 72 years.

23. It was said in art. 12, that the planets revolved round the Sun in orbits nearly circular and concentric: for the several phenomena arising from their motions shew they are not strictly so; and the only curve they can move in, to reconcile all the various appearances, is found to be an Ellipsis: So that the orbits of the primary planets and comets are Ellipses of different curvatures, having one common focus, in which the Sun is fixed. But every secondary planet respects the primary planet round which it revolves, as the focus of its elliptic motion. For as no other suppositions can solve all the appearances that are observed in the motions of the planets, and as these agree with the strictest physical and mathematical reasoning, therefore they are now received as elementary principles.

24. The line of the nodes of every planet passes through the Sun: For as the motion of every planet is in a plane passing through the Sun, consequently the intersections of these planes, that is, the lines of the nodes, must also pass through the Sun.

25. All the planets, in their revolutions, are sometimes nearer to, sometimes farther from the Sun: This is the consequence of the Sun not being placed in the center of each orbit, and of their being ellipses.

26. The APHELION, or SUPERIOR APSIS, is that point of the orbit where the planet is farthest from the Sun. The PERIHELION, or INFERIOR APSIS, is that point where it is nearest to the Sun: And the transverse diameter of the orbit, or the line joining the two apses, is called the LINE OF THE APSES, or ASPIDES.

27. The planets move faster as they approach the Sun, or come nearer to the perihelion, and slower as they recede from the Sun, or come nearer to the aphelion. This is not only a consequence from the nature of the planets motions about the Sun, but is confirmed by all good observations.

28. If a right line drawn from the Sun through any planet, usually called the *Radius Vector*, is supposed to revolve round the Sun with the planet, then this line will describe or pass through every part of the plane of the orbit; so that the *Radius Vector* may be said to describe the area of the orbit.

29. There are two chief laws observed in the Solar System, which regulate the motions of all the planets; namely,

- I. The planets describe equal areas in equal times: That is, in equal portions of time the *Radius Vector* describes equal areas, or portions of the space contained within the planet's orbit.

II. The

- II. *The squares of the periodical times of the planets are as the cubes of their mean distances from the Sun*: That is, as the square of the time which a planet A takes to revolve in its orbit, is to the square of the time taken by any other planet B to run through its orbit; so is the cube of the mean distance of A from the Sun, to the cube of the mean distance of B from the Sun.

30. The MEAN DISTANCE of a planet from the Sun is its distance from him, when the planet is at either extremity of the conjugate diameter; and it is equal to half the transverse diameter.

31. The foregoing laws are the two famous laws of KEPLER, a great Astronomer, who flourished in Germany about the beginning of the 17th century, and who deduced them from a multitude of observations: But the first who demonstrated these laws was the incomparable Sir ISAAC NEWTON.

By the second law, the relative distances of the planets from the Sun are known; and were the real distance of any one known, the absolute distances of all the others would be obtained by it.

32. Every thing already said of the planets is found in a great measure to be applicable also to the comets, as well from the observations that have been made of them, as from the physical and mathematical considerations of their motions.

33. Were the motions of the planets to be observed from the Sun, each of them would be ever seen to move the same way, though with different velocities; those nearer to the Sun running their courses through the Zodiac in less time than those at greater distances: And hence it would happen, that some of them overtaking the others would in passing by them appear to be sometimes above, sometimes below, and sometimes as if they touched one another, according to the parts of the orbits in which those planets happened to be with respect to their nodes.

34. When two planets are seen together in the same sign equally advanced, they are said to be in CONJUNCTION: But when they are in direct opposite parts of the Zodiac, they are said to be in OPPOSITION.

35. As the planes of the orbits are inclined to one another, therefore when two planets happen to be in conjunction at the time they come near a node of one of them, they would be seen from the Sun apparently to touch one another; and the farthest of those planets from the Sun would see the nearest moving over the face of the Sun like a black spot, being then directly between the Sun and the remoter planet; so the planet Venus was observed from the Earth in the transits of the years 1761 and 1769. Also, should an opposition of two planets happen near a node of one of them, the sun, being then directly between them, would hide the light of one from the other. These obscurations, or interceptions of the light of the planets one from the other, are called ECLIPSES.

36. The place that any planet appears to occupy in the celestial sphere, when seen by an observer supposed to be in the Sun, is called its *Heliocentric place*: And indeed all celestial appearances, as seen from the Sun, are called *Heliocentric phenomena*.

37. The

37. The following table exhibits at once some of the most material conclusions that have been deduced from the observations hitherto made, the mean distance of the Earth from the Sun being reckoned at 1000.

TABLE OF THE SOLAR SYSTEM.

Planets Names.	Marks.	Mean dist. compared to that of the Earth.	Eccentricity.	Inclination of the orbits to the eclipt.	Times of the periodical revolutions.	Diurnal re- volutions.	True diameter.	N ^o of Moons.
Mercury.	☿	387	81 60 52'	3 m. or	87 ^d 23 ^h	uncertain	0,32	0
Venus.	♀	724	53 23	8 m. or	224 17	23 ^h 0 ^m 0 ^s	0,87	0
Earth.	♁	1000	17 0 00	1 y. or	365 6	23 56 4	1,00	1
Mars.	♂	1524	141 1 52	2 y. or	686 23	24 40 0	0,73	0
Jupiter.	♃	5201	250 1 20	12 y. or	4332 12	9 56 0	7,70	4
Saturn.	♄	9538	543 2 20	30 y. or	10759 8	uncertain	4,19	5

38. By all the observations made on the secondary planets, it appears,

1st. That the satellites revolve round their superior planets from west to east, in curved-lined orbits like ellipses, the primary planet being in the focus, and one of the orbit's diameters directed toward the Sun.

2d. That the planets of the orbits of the satellites are inclined to the plane of the orbit of their respective planet.

3d. That, like the primary planets, they describe equal areas in equal times; and the squares of the times of their revolutions are as the cubes of the mean distances from their primary planet.

In every revolution of a moon round its primary planet, there must be two conjunctions betwixt the planet, moon, and Sun: namely, once, when the moon is in that part of its orbit nearest to the Sun; and once, when in that part of its orbit farthest from the Sun: And an eclipse may happen at either conjunction, according as the moon's nodes happen to be posited at those times. For the plane of a moon's orbit is inclined to that of its primary, and so makes two nodes: And whenever the Sun, planet, and moon happen to be at the same time in the line of the nodes, there must be an eclipse; which would occur at every conjunction, but for the inclination of the orbit.

One of the conjunctions, namely, that made by the moon's going beyond the primary, from the Sun, is called the SUPERIOR CONJUNCTION; and the other, by the satellite on the side of the planet next the Sun, is called the INTERIOR CONJUNCTION.

The following table shews the time taken by each satellite in its revolution, and also its mean distance from the primary in semidiameters of it.

39.	I	II	III	IV	V
	d h m	d h m	d h m	d h m	d h m
Saturn's satell.	1 21 18 $\frac{1}{2}$	2 17 41 $\frac{1}{3}$	3 12 25 $\frac{1}{3}$	15 22 41 $\frac{1}{4}$	79 7 48
Dist. from Jup.	8 $\frac{7}{8}$ f. diam.	11 $\frac{1}{4}$ f. diam.	15 f. diam.	36 f. diam.	108 f. diam.
Jupiter's satell.	1 18 28 $\frac{1}{3}$	3 13 18 $\frac{5}{6}$	7 3 59 $\frac{2}{3}$	16 18 5 $\frac{1}{10}$	
Dist. from Jup.	5 $\frac{2}{3}$ f. diam.	9 f. diam.	14 $\frac{2}{3}$ f. diam.	25 $\frac{1}{3}$ f. diam.	
The Moon.	29 12 44 $\frac{1}{20}$	and is dist. from the Earth 60 $\frac{1}{5}$ semidiameters.			

30.

Of the Figure and Light of the Planets.

That the Sun and planets are spherical bodies is evident from all the observations that have been made of them; and that the Earth is of like figure is not only deduced by analogy, but sufficiently confirmed by observation*. Now although Astronomers generally say that the planets are spherical, yet they do not mean a Geometrical sphere, but a figure called an oblate spheroid, which is something like the figure that a flexible sphere would be formed into by gently pressing it at its poles. Observations have determined this in Jupiter, and it is known that the Earth is of this figure both from observations and actual mensuration.

That the planets must have this oblate figure, is evident from this consideration; that as they are of matter, and violently whirled on their axes, all the parts would endeavour to fly off, like water from a trundled mop; those in the equator moving swiftest, have the greatest tendency to depart: And although the parts are retained in the sphere by the superior force of gravity, yet the equatorial diameters will be somewhat increased, and the polar lessened.

41. The planets are all opaque or dark bodies, and consequently shine only by the light they receive from the Sun: This is known by observing, that those bodies are not visible, when they are in such parts of their orbits as are between the Sun and Earth, or partly so. Now, as all the planets sometimes appear with a strong light, therefore the rays they receive from the Sun must convey to them a degree of warmth proportional in some measure to their distance; which proportion is reciprocally as the squares of the distances; and this must be readily inferred from the heat which the inhabitants of the Earth receive from the Sun.

42. As a planet revolves on its axis, every part of its surface will be turned toward the Sun, and so enjoy its light and heat.

* Observations shew, that in eclipses of the moon the darkened part is bounded by a circular curve; and consequently the body, which casts the shade, or obstructs the light, must be bounded by a like curve: but as these obscurations are caused by different parts of the Earth, consequently its surface must be limited by a circular figure, that is, it must be globular.

SECTION II.

Of Terrestrial Astronomy.

43. TERRESTRIAL ASTRONOMY is that which considers the motions of the celestial bodies as seen from the Earth, which is also in motion.

The motions described in the preceding section were such, as would appear to an observer viewing the heavens from the Sun: But were he placed in one of the planets, suppose the Earth, and there observed the motions of the rest, the Sun, and other planets, would appear to him to revolve round the Earth as a center; but the Sun would be the only one that moved uniformly the same way: For the other planets would seem to move sometimes from west to east, and then to stand still; then they would seem to run from east to west; and after standing some time, they would again move from west to east, and so on continually.

44. The place in the celestial sphere that any planet appears in, when seen from the center of the Earth, is called its **GEOCENTRIC PLACE**.

45. The **DIRECT MOTION** of a planet is that by which it appears to move from west to east, and this motion is said to be *according to the order of the signs*, or in *consequentia*. When the planet appears to stand still, it is said to be **STATIONARY**; and when its motion is apparently from east to west, it is then called **RETROGRADE**, or has a motion in *antecedentia*, or contrary to the order of the signs. These different appearances follow partly in consequence of the observer being himself in motion while he is viewing the motions of the planets, and partly because he is not in the center of the motions which he observes.

46. *The Phenomena of the Inferior Planets.* See Fig. 1. Plate IV.

Let ABC represent an arc of the celestial sphere; EOP the Earth's orbit; LNIG the orbit of an inferior planet, as of *Venus*; and s the Sun: Let the Earth at first be supposed to stand still in its orbit at E: Now it is evident that the Sun will appear at the point B, and the planet always within the arc AC. Whilst the planet moves in its orbit from I through Q to N, it will seem to move from B to A in *consequentia*: But passing from N to L, it will seem to an eye at E to return back from A to B, or be *retrograde*. While the planet is at, or near, the point N, and moving as it were in a right line toward the Earth, it will for some time seem to stand still near A, and it is then said to be *stationary*.

47. When the planet is in that part of its orbit N or G, which is contiguous to the tangent EA or EC, it will then appear at A or C, its greatest distance from the Sun, and is said to have then the greatest **ELONGATION**: This elongation is measured by the angle SEG. The more distant a planet is from the Sun, the greater will its angle of elongation

tion be ; that of Venus is about 48 degrees, and that of Mercury about 28 degrees.

In the space of a revolution, the two inferior planets will, with respect to the Earth, undergo two conjunctions ; one when it is beyond the Sun at *I*, the other when it is at *L* between the Sun and the Earth ; the former is called the *superior*, and the latter the *inferior conjunction*.

48. Whilst Venus goes from her superior conjunction, she appears in the arc *BA* always to the eastward of the Sun, and therefore sets some time after the Sun, and is called the *evening star*. But during the time she is going from her inferior to her superior conjunction, she will be seen somewhere in the arc *BC* to the westward of the Sun, and so will set before him in the evening, and rise before him in the morning ; hence she is called the *morning star*.

Hitherto the planet only has been supposed to move while the Earth stood still ; but when both move, the foregoing phenomena will be much the same, only the planet will be more direct in the farthest part of the orbit, and less retrograde in the nearest ; the former arising from the sum of their motions, and the latter from the difference.

49. *Of the Phenomena of the Superior Planets.*

The *direct*, *stationary*, and *retrograde* appearances of the superior planets, are explained much after the same manner as those of the inferior ones, but with these differences.

1st. The retrograde motions of the superior planets happen oftener the slower their motions are, as the retrogradations of the inferior planets happen oftener the swifter their angular motions are : Because the retrograde motions of the superior planets depend upon the motions of the Earth, but those of the inferior on their own angular motions. A superior planet is retrograde once in each revolution of the Earth ; an inferior one, in every one of its own revolutions.

2. The superior planets do not always accompany the Sun as the inferior do, but are often in opposition to him ; which necessarily follows from the orbit of the Earth being included in the orbits of the superior planets.

50. *Of the apparent Motion of the Sun.*

As a spectator in the Sun would see the Earth revolve through the signs in the ecliptic ; so to a spectator in the Earth the Sun apparently revolves the same way, but is always in the opposite point of the ecliptic : (For it is well known to every one, especially to those who use the sea, that fixed objects appear to change their place by the motion of the observer :) So that the heliocentric place of the Earth, and the geocentric place of the Sun, are always in direct opposite points of the ecliptic.

Now although it is the motion of the Earth that really causes a great variety in the apparent motions of the other planets, yet as the motion of the Sun being known gives that of the Earth, therefore Astronomers speak

of the motion of the sun ; and in their computations use the quantities of those motions as if they were real.

51. Beside the various appearances that arise from the annual motion of the Earth, there are many resulting from its diurnal motion : For that the Earth has a daily motion round its axis must necessarily be inferred from the most strict reasoning on the motions of the planets : and the notion, that bodies so immensely distant as the stars are, really revolve round the earth in 24 hours, is now treated as a great absurdity by every one who has rightly considered these things : However, as the motions are apparent, and the speaking of them as real is customary and no way affects the conclusions ; therefore Astronomers treat of those motions as they appear.

52. Any sphere revolving as on an axis, must have two points on its surface at the extremities of its axis, that do not revolve at all ; these points, with respect to the Earth, are called its *poles*.

53. By the Earth's rotation on its axis from west to east in a day, the surface of the celestial sphere appears to move from east to west in the same time ; and all the celestial objects appear to describe circles in the heavens, which are greater or less according as they are farther from, or nearer to, the apparent centers of those motions : For there are two points in the heavens which are apparently fixed, and the nearer any stars are to these points, the slower are their motions. These points are called the **CELESTIAL POLES** ; the right line joining them is called the **AXIS OF THE SPHERE**, and passes through the poles of the Earth ; the circle in the heavens, equally distant from the poles of the celestial sphere, is called the **EQUINOCTIAL** ; the corresponding circle on the Earth is called the **EQUATOR**, which is equally distant from both the poles of the Earth.

54. As the Sun's rays falling on any sphere enlighten one half of its surface ; therefore one half of the Earth is always illuminated at once, and consequently the enlightened part is bounded by a great circle, which may be called the **TERMINATOR**, from its property of terminating, or bounding, the verges of light and darkness. Now, by the rotation of the Earth on its axis once in 24 hours, there will be a constant succession of light on all parts of its surface as they are turned toward the Sun, and of darkness in those parts as they move out of his rays ; and hence arise the vicissitudes of **DAY** and **NIGHT**.

55. If the plane of the equator coincided with the plane of the ecliptic, and the axis of the Earth stood perpendicular to it, the terminator would always pass through the poles of the Earth, and there would be a constant equality of day and night in every part of its surface, except at the two poles, where there would be constant day. But the contrary of this is known to every one, and observations shew, that the Earth's axis is inclined to the plane of the ecliptic in an angle of about $66\frac{1}{2}$ degrees ; therefore the poles of the ecliptic and equator are about $23\frac{1}{2}$ degrees distant from one another ; consequently the ecliptic and equinoctial, which in the heavens intersect one another in the opposite points of Aries and Libra, make at those intersections angles of about $23\frac{1}{2}$ degrees (IV. 33) : This angle is called the **OBLIQUITY OF THE ECLIPTIC**.

The axis of the Earth being thus inclined to the plane of the ecliptic, and moving parallel to itself in all points of the **ANNUAL ORBIT**, or ecliptic,

ecliptic, is the occasion of the inequality of days and nights, and of the different seasons of the year; which two phænomena are explained as follows:

56. It must be observed, that the Sun will appear to be vertical to that part of the Earth, which is cut by a straight line joining the centers of the Sun and Earth.

57. Now when the Earth is at ψ , Fig. 3. Plate IV. the Sun appearing then in $\odot$ will be vertical to that point of the terrestrial ecliptic, it lying in the right line joining the centers of the Sun and Earth. And this point being in the Earth's northern hemisphere, all those who live there will enjoy summer, or the hottest time of the year, the solar rays falling more copiously, and more perpendicularly, upon their hemisphere at that time.

58. At the same time the inhabitants of the southern hemisphere will have winter, the rays of the sun falling more obliquely, and in less quantity, on them, and consequently affording them less heat.

59. Again, the inhabitants of the northern hemisphere will have their days longer than their nights, in proportion as they are more distant from the equator; while those who live under the equator will have an equal share of day and night all the year round. For in this position the terminator, which is always at right angles to the plane of the ecliptic, will pass $23\frac{1}{2}$ degrees beyond the north pole, and consequently will cut all the circles parallel to the equator which it meets with into two unequal parts: those that are in north latitude will have the greater portions of those parallels in the enlightened hemisphere: but the terminator being a great circle, will cut the equator into two equal parts; therefore half the equator is always illuminated.

60. Hence it necessarily follows, that those who live under the equator will have their days and nights equal: those who live within the limits of $23\frac{1}{2}$ degrees round the north pole, will have no night; and the inhabitants between this limit and the northern neighbourhood of the equator will have their nights shorter than their days. In the mean time those who live in the southern hemisphere will have their nights longer than their days, in proportion as they approach nearer to the south pole; and the regions contained within the limits of $23\frac{1}{2}$ degrees round the south pole will have no day.

61. Suppose the Earth now to move in its orbit from ψ through the signs ϵ , κ to γ , the Sun will seem to run through the signs $\odot$, φ to $\cap$; and this will be the place of the Sun in autumn.

While the Earth is in γ , the days and nights will be equal in both hemispheres, and the season is a medium between summer and winter: For at that time the Sun will appear vertical to the equator, because a right line joining the centers of the Sun and Earth will then cut the surface of the Earth in the equator: so that the terminator, the plane of which is always at right angles to the said line, will pass through the poles; consequently, all the Earth will then have an equal share of day and night. And because the rays of the Sun then fall perpendicularly upon the axis of the Earth, it will then follow, that they must fall with an equal obliquity, and with equal number, upon either hemisphere; therefore they must enjoy an equal degree of heat and cold.

Now

Now suppose the Earth to move from γ to $\ominus$, the Sun will seem to move from $\sphericalangle$ to ν , where it will be in its nearest approach to the south pole; and at this time of the year it will be winter in the northern hemisphere. For to this hemisphere the like phenomena will now happen, which did before to the southern, when the Earth was in ν ; and by a parity of reason, when the Earth has got as far as $\sphericalangle$, and the Sun is apparently in γ , the northern hemisphere will enjoy spring, and the southern will have autumn.

62. The four points of the ecliptic, in which the Earth has been considered in summer, autumn, winter, and spring, are called the four **CARDINAL POINTS**; ν and $\ominus$ are called **SOLSTITIAL POINTS**; $\sphericalangle$ and γ **EQUINOCTIAL POINTS**.

63. The first point of Cancer is called the **SUMMER SOLSTICE**; because when the Sun enters it, which is about the 21st of June, he has then got to the greatest extent northward, and being about to return toward the equator, he seems for a day or two to be at a stand. And for the same reason, the first point of Capricorn, which the Sun enters about the 21st of December, is called the **WINTER SOLSTICE**, with respect to the northern hemisphere.

64. The first points of Aries and Libra are called the **VERNAL** and **AUTUMNAL EQUINOCTIAL POINTS**, from the equality of days and nights all over the surface of the Earth, when the Sun enters those points.

65. *Of the Rising and Setting of the Stars.*

There is only one half of the celestial sphere visible at one time to any observer on the surface of the Earth, the other half being hid by the Earth itself. Now the apparent plane on which the observer stands, seems to be extended to the heavens, and there marks out a circle that divides the visible from the invisible hemisphere; this circle is called the **HORIZON**, above which all the celestial motions are seen. When this horizon is a great circle of the celestial sphere, it is called the **RATIONAL HORIZON**: but when by the particular situation of the observer, he sees more or less than half the celestial sphere, then the circle bounding his view is called the **SENSIBLE HORIZON**.

The horizon is one of the most useful circles in Astronomy; for to this circle, which is the only apparent one, almost all the celestial motions are referred. It is the common termination of day and night; it marks out the times of the rising and setting of the Sun and stars, and many other particulars, of which hereafter.

66. *Of Parallaxes.*

The **PARALLAX** of any object is the difference between the places that object is referred to in the celestial sphere, when seen at the same time from two different places within that sphere: Or, it is the angle under which any two places in the inferior orbits are seen from a superior planet, or even from the fixed stars: But the parallaxes which are most used by Astronomers are those which arise from seeing the object from the cen-
ters

ters of the Earth and Sun; from the surface and center of the Earth; and from all three compounded.

67. The difference between the heliocentric and geocentric place of a planet, is called the *parallax of the annual orbit* (namely, that of the Earth); that is, the angle at any planet, subtended by the distance between the Sun and Earth, is called the parallax of the Earth's, or annual orbit.

68. The difference in the two longitudes is called the parallax of longitude; and that of the two latitudes is called the parallax of latitude.

69. In the SYZIGIES, that is, in the *oppositions* or *conjunctions*, the Sun and planet being equally advanced in the same sign, or in like places in opposite signs, the parallax of latitude is then greatest.

70. And when the planet is in its QUADRATURES, that is, when it is 90 degrees distant from the Sun, the parallax of longitude is then the greatest.

71. To explain the parallaxes which respect the Earth only. Fig. 2. Plate IV.

Let HSW represent the Earth, where T is the center; OR part of the Moon's orbit, Prg part of a planet's orbit, and zAA part of a great circle in the celestial sphere. Now to a spectator at s upon the surface of the Earth, let the Moon appear in G, that is in the sensible horizon of s, and it will be referred to A; but if viewed from the center T, it will be referred to the point D, which is its true place.

The arc AD will be the Moon's parallax; the angle sGT the parallactic angle: Or the parallax is expressed by the angle under which the semidiameter Ts of the Earth is seen from the Moon.

If the parallax is considered with respect to different planets, it will be greater or less as those objects are more or less distant from the Earth. Thus the parallax AD of G is greater than the parallax Ad of g. If it is considered with respect to the same planet, it is evident that the horizontal parallax (or the parallax when the object is in the horizon) is greatest of all; and diminishes gradually as the body rises above the horizon, until it comes to the zenith, where the parallax vanishes, or becomes equal to nothing. Thus AD and Ad, the horizontal parallaxes of G and g, are greater than AB and ab, the parallaxes of R and r; and the objects o or p, seen from s or T, appear in the same place z, or the zenith.

72. By knowing the parallax of any celestial object, its distance from the center of the Earth may be easily obtained by Trigonometry. Thus, if the distance of G from T is sought; in the triangle STG, the side ST being known, and the angle sGT determined by observation, the side TG is thence known.

The parallax of the Moon may be determined by two persons observing her from different stations at the same time, she being vertical to the one, and horizontal to the other: and it is generally concluded to be about 57 minutes of a degree; consequently her mean distance TG is about 60 semidiameters of the Earth, or 60 times Ts.

But the parallax most wanted is that of the Sun, by which his absolute distance from the Earth would be known; and thence the abso-

lute distances of all the other planets would be obtained from their relative distances found by the second *Keplerian Law*.

Before the year 1761 some Astronomers reckoned the Sun's parallax at $12\frac{1}{2}$ seconds, others at 10; these different parallaxes gave very different distance between the Sun and the Earth; the former making the distances near 8270 diameters of the Earth, and the latter 10313 diameters.

But in the years 1761 and 1769 the planet Venus passed between the Earth and the Sun, and was seen like a black spot moving over the face of the Sun. These phenomena (which had not happened in more than 100 years before) were observed by many Astronomers from different parts of the Earth, and the result of their observations make the Sun's mean parallax about $8\frac{2}{3}$ seconds, and hence the mean distance between the Sun and Earth comes very nearly to 11900 diameters of the Earth: And from what was shewn many years ago by the excellent Dr. Halley, if these observations were made with the accuracy he supposed, the distance between the Sun and the Earth might be obtained to less than a 500th part of the whole distance.

73.

Of the Measure of the Earth.

The relative distances of the planets are discovered by the 2d *Keplerian Law*, and their relative magnitudes are gathered from the angles which they appear under (when viewed with very accurate instruments) compounded with their distances. Now as these distances and magnitudes can by means of the parallaxes be compared with the diameter of the Earth, consequently this diameter being accurately known would serve as a measure with which the magnitudes and distances of all the other planets might be compared.

To find the measure of the Earth is a problem of such importance in Astronomy, that it has been attempted by some of the most considerable men in almost all the preceding ages. But its solution was not brought to any degree of accuracy till the year 1635, when it was very nearly ascertained by our countryman RICHARD NORWOOD, an eminent mathematician at that time. The principle he proceeded upon was this, that as 360 degrees were contained in every great circle, both of the celestial sphere and of the Earth, and as these circles are considered as concentric to the center of the Earth; therefore, were the measure of a degree known on a great circle of the Earth, corresponding to a degree of a great circle of the heavens, then, by analogy, the whole circumference of a great circle of the Earth would be known in that measure, and consequently its diameter would be obtained. (II. 197)

NORWOOD solved this problem in the following manner: He chose two distant places which were known to lie nearly north and south one of the other, as London and York; and by a method like that of Traverse sailing (explained in Book VII.) he found their difference of latitude, or, the distance between the parallels of latitude passing through those places; or, which is the same thing, the length of that arc of the terrestrial meridian. He also with a good instrument found the distance between

between the zeniths of those places, and consequently he thence knew the quantity of the celestial arc answering to the measured terrestrial one. Then saying, As that celestial arc is to a great circle of the celestial sphere, or 360 degrees; so is the arc of the terrestrial great circle measured in feet, to the circumference of a great circle of the Earth in feet measure.

And thus he found that about $69\frac{1}{2}$ English miles answered to one degree; hence the circumference of the Earth appears to be 25020 miles, and its diameter about 8000 miles.

By the same kind of reasoning, the distances found in art. 72. from the parallaxes, were obtained.

$$\begin{array}{l}
 \text{For } 12\frac{1}{2}'' : 360^\circ :: 1 \text{ semi-diam.} : 103680 \\
 10'' : 360^\circ :: 1 \text{ semi-diam.} : 129600 \\
 8\frac{2}{3}'' : 360^\circ :: 1 \text{ semi-diam.} : 149538
 \end{array}
 \left. \vphantom{\begin{array}{l} 12\frac{1}{2}'' : 360^\circ \\ 10'' : 360^\circ \\ 8\frac{2}{3}'' : 360^\circ \end{array}} \right\} \text{the } \left\{ \begin{array}{l} \text{Circumference of the} \\ \text{Earth's orbit in semi-} \\ \text{diameters of the Earth.} \end{array} \right.$$

$$\begin{array}{l}
 \text{And } 6,283185 : 103680 :: 1 : 16539,5 \\
 6,283185 : 129600 :: 1 : 20626,4 \\
 6,283185 : 149538 :: 1 : 23799,8
 \end{array}
 \left. \vphantom{\begin{array}{l} 6,283185 : 103680 \\ 6,283185 : 129600 \\ 6,283185 : 149538 \end{array}} \right\} \text{the } \left\{ \begin{array}{l} \text{Mean dist. of the Earth} \\ \text{from the Sun, in semi-d.} \\ \text{of the Earth. (II. 197)} \end{array} \right.$$

$$\begin{array}{l}
 \text{Then } 16539,5 \times 4000 = 66158000 \\
 20626,4 \times 4000 = 82505600 \\
 23799,8 \times 4000 = 95199200
 \end{array}
 \left. \vphantom{\begin{array}{l} 16539,5 \times 4000 \\ 20626,4 \times 4000 \\ 23799,8 \times 4000 \end{array}} \right\} \text{the } \left\{ \begin{array}{l} \text{Mean distance of the} \\ \text{Earth from the Sun, in} \\ \text{miles.} \end{array} \right.$$

74.

Of the Moon.

The Moon revolves in her orbit from west to east round the Earth, and is carried perpetually with it through the annual orbit round the Sun, making in the space of one year 13 *periodical*, and 12 *synodical revolutions*.

75. A PERIODICAL MONTH, or REVOLUTION, is the time the Moon takes up in revolving from one point of her orbit to the same point again, and consists of 27 d. 7 h. 43 m.

76. A SYNODICAL MONTH, or REVOLUTION, is the time the Moon spends in passing from one conjunction with the Sun to another, which is 29 d. 12 h. 44 m.; being 2 d. 5 h. 1 m. longer than the *Periodical Month*. For whilst the Moon is passing from her former conjunction with the Sun round to it again; the Earth has proceeded forward in its annual course, as it were leaving the Moon behind it; so that, in order to complete her next conjunction with the Sun, she must not only come round to her former point again, but also go beyond it.

77. Besides this monthly motion of the Moon round the Earth, she has also a motion round her axis, which is performed exactly in the same time with her periodical revolution: Hence it comes to pass, that the same face of the Moon is always turned toward the Earth, her diurnal motion turning just as much of her face to us, as her periodical motion turns it from us.

78. Though the same side of the Moon is ever turned toward us, yet it is not always visible, but seems daily to put on different appearances,

called PHASES : For the Moon being an opaque body like the rest of the planets, borrows its light from the Sun, having always one hemisphere enlightened by the solar rays.

When the enlightened hemisphere is wholly turned from the Earth, as at her change or time of new-moon, the planet then being betwixt us and the Sun, the Moon's whole enlightened face, or *disk*, must needs be invisible to the Earth. When she passes from this state, and turns some little portion of the illuminated half to us, she must appear horned, the CUSPS or points being turned from the Sun towards the east. When the Moon is in her quadratures, or at 90 degrees from the Sun, then half the illuminated face becomes visible : She afterward continues to shew more than half the enlightened disk, until she comes in opposition to the Sun or time of full-moon, when the whole of the illuminated orb is presented to us ; from whence receding, she must put on the like phases as before, but in an inverse order, the cusps being now turned toward the west.

79.

Of Solar and Lunar Eclipses.

Eclipses of the Sun and Moon can only happen about the times of the conjunctions and oppositions : those of the Sun fall out at the conjunctions, when the Moon intercepts the light of the Sun from the Earth ; and those of the Moon occur in the oppositions, when the Earth getting between the Sun and Moon, the latter loses her light during the time of that interposition.

The cause why there is not an eclipse in every syzgie is the inclination of the plane of the Moon's orbit to that of the ecliptic, which is about $5^{\circ} 18'$: for it is certain, that unless the Sun, Earth, and Moon, are all in the plane of the ecliptic, or nearly so, the shadows of the Earth and Moon can never fall on one another, but must be directed either above or below. Now they can never be in the same plane, and in one right line, except when the Moon is in her nodes, the nodes and Sun's center being in the same right line.

80. The solar and lunar eclipses do not happen every year in the same places of the zodiac, but in succeeding years they fall in places gradually removed backward, or toward the antecedent signs : For since the nodes are found to go continually backward, the eclipses must also observe the same order.

81. Eclipses of the Moon are either *total* or *partial* : the total happen when the node falls in or near the center of the shadow : and the partial, when the node happens to be on either side the center, within or without the shadow. Now the longer the duration of a partial eclipse is, so much the greater is that part of the Moon which enters into the shadow of the Earth.

82. Hence it is usual to conceive the Moon's diameter as divided into 12 parts, called DIGITS, by which the greatness of partial eclipses is measured, they being said to be of so many digits as they are parts covered by the Earth's shadow : Thus if 5 of the 12 parts are covered, it is called an eclipse of 5 digits.

83. As the planet Mars is never eclipsed by the Earth, it is plain the shadow of the latter does not reach so far as the orbit of the former, but

but tapers to a point at a less distance ; and consequently the Earth's shadow must be a cone, the vertex of which is extended beyond the orbit of the Moon. It naturally follows from hence, that the Sun is a much larger body than the Earth ; it is indeed, in diameter, above 100 times that of the Earth.

84. If a person was placed just at the vertex, or point of this shadow, he would see nothing of the Sun but a small rim of light round his disk ; and the farther the observer was removed from the vertex, the larger would the rim of light appear, and consequently the fewer rays would be intercepted by the opaque body, till at last it would appear only as a spot in the Sun ; in like manner as the planets Venus and Mercury appear when they are seen to pass over the Sun's disk.

85. What has hitherto been said of the shadow of the Earth includes that of the atmosphere surrounding the Earth : for in lunar eclipses the shadow of the atmosphere is to be considered. And hence it is that the Moon is visible in eclipses, the shadow cast by the atmosphere being not near so dark as that cast by the Earth.

86. The Moon always enters the western side of the shadow with her eastern limb, and quits it with her western limb ; and in her approach to and recess from the shadow, she must pass through a *PENUMBRA*, or imperfect shade, which is caused by the Earth itself.

87. In the same manner, in which it has been shewn that the Moon must come into the shadow of the atmosphere, when she is at full and at or near a node, it may also be shewn, that her shadow must fall upon the Earth at the time of new Moon, provided she is in or near a node : But the penumbra of the Moon's shadow is much more sensible in solar eclipses, than that accompanying the shadow of the Earth in lunar ones.

88. It is observed, that to determinate parts of the Earth solar eclipses are not seen so oft as lunar ones ; which is owing to the shadow of the Moon being less than that of the Earth : For the Earth's shadow often covers all the Moon ; but that of the Moon cannot cover all the Earth ; and as it sometimes falls on one part, sometimes on another, it causes solar eclipses, in general, to be more frequent than lunar ones ; yet to any determinate place on the Earth there are more eclipses of the Moon visible than of the Sun.

What has hitherto been said, may suffice to give beginners a general idea of the motions of the bodies in the solar system, and of some of the phenomena thence arising ; those who desire to be farther acquainted with particulars, may find them fully treated of in M. de la Caille's *Elements of Astronomy*, published in English a few years since * ; and also in the works of Gregory, Keil, and others.

* Translated by the Author of these Elements.

SECTION III.

The Astronomy of the Sphere.

DEFINITIONS and PRINCIPLES.

89. By the Astronomy of the sphere is meant the finding, from proper things given, the measure of certain arcs and angles formed on the surfaces of the celestial and terrestrial spheres, by the apparent motions of the bodies which are seen in the heavens.

The surfaces of those spheres are supposed to be concentric to the center of the Earth, and to have correspondent circles described on both spheres.

90. GREAT CIRCLES are those which divide either sphere into two equal parts.

LESSER CIRCLES, those which divide the sphere into unequal parts.

The POLES of a circle are the points on the sphere equally distant from that circle.

An AXIS is a right line supposed to connect the poles.

The CELESTIAL AXIS is that right line about which the heavens seem to revolve.

The NORTH and SOUTH POLES of the world are those two points where the axis cuts the celestial sphere.

91. The EQUINOCTIAL or EQUATOR, is the great circle of the sphere equally distant from the poles of the world.

92. MERIDIANS, or HOUR CIRCLES, or CIRCLES of RIGHT ASCENSION, or CIRCLES of TERRESTRIAL LONGITUDE, are great circles perpendicular to the equator, and passing through the poles of the world.

93. The ECLIPTIC is a great circle inclined to the equator in an angle of about $23\frac{1}{2}^{\circ}$, and cutting it in two points diametrically opposite.

The ecliptic is supposed to be divided into 12 equal parts, called SIGNS, beginning from one of its intersections with the equator; each sign containing 30 degrees, named and noted thus:

<i>Aries</i>	<i>Taurus</i>	<i>Gemini</i>	<i>Cancer</i>	<i>Leo</i>	<i>Virgo</i>
♈	♉	♊	♋	♌	♍
<i>Libra</i>	<i>Scorpio</i>	<i>Sagittarius</i>	<i>Capricornus</i>	<i>Aquarius</i>	<i>Pisces</i>
♎	♏	♐	♑	♒	♓

The first six are called *northern*, and the latter six *southern* signs.

94. The CARDINAL POINTS of the ecliptic are the four first points of the signs ♈, ♋, ♎, ♑; those of ♈ and ♎ are called EQUINOCTIAL POINTS, and those of ♋ and ♑ are called SOLSTITIAL POINTS.

95. The EQUINOCTIAL COLURE is a meridian passing through the equinoctial points; and the SOLSTITIAL COLURE is another meridian passing through the solstitial points. The colures cut one another at right angles in the poles of the world.

96. CIR-

96. CIRCLES OF CELESTIAL LONGITUDE are great circles perpendicular to the ecliptic.

97. The LATITUDE of any point in the heavens is an arc of a circle of longitude intercepted between that point and the ecliptic, and is called north or south latitude, as the point is on the north or south side of the ecliptic.

98. PARALLELS OF CELESTIAL LATITUDE are small circles parallel to the ecliptic.

99. The LONGITUDE of any object in the heavens is an arc of the ecliptic intercepted between the first point of Aries, and a circle of longitude passing through that point.

100. The RIGHT ASCENSION of any object is an arc of the equator, contained between the first point of Aries and a meridian passing through the center of the object: Or, it is the angle formed by the equinoctial colure, and the meridian passing over that object.

101. The DECLINATION of any object is an arc of a meridian contained between the center of the object and the equinoctial: If the object be on the north side of the equinoctial, it is said to have *north declination*; but if on the south side, it has *south declination*.

102. The OBLIQUITY OF THE ECLIPTIC is the angle made by the intersection of the equator and ecliptic, and is measured by the Sun's greatest declination; which, according to modern observations, is about $23^{\circ} 28'$.

103. PARALLELS OF DECLINATION are small circles parallel to the equinoctial. The TROPIC OF CANCER is a parallel of declination at $23^{\circ} 28'$ distant from the equinoctial in the northern hemisphere; and the TROPIC OF CAPRICORN is the parallel of declination as far distant in the southern hemisphere.

104. The ARCTIC POLAR CIRCLE is a parallel of declination at $23^{\circ} 28'$ distant from the north pole; and the ANTARCTIC POLAR CIRCLE is the parallel of declination as far distant from the south pole.

105. The ZENITH is the point of the heavens directly over a place; and the NADIR is the point directly underneath.

106. The HORIZON is that great circle of the sphere which is equally distant from the *zenith* and *nadir* of any place, and divides the sphere into the upper and lower hemispheres.

107. The RISING of a celestial object is when its center appears in the eastern part of the horizon; and its SETTING is when its center disappears in the western quarter of the horizon.

108. AZIMUTH, or VERTICAL CIRCLES, are great circles perpendicular to the horizon, passing through its poles, which are the *zenith* and *nadir*.

109. The PRIME VERTICAL is that vertical circle which passes through the east and west points of the horizon. The MERIDIAN OF THE PLACE is a vertical circle passing through the north and south points of the horizon.

110. The meridian of the place is called the *twelve o'clock hour circle*. The hour circle at right angles to the meridian is called the *six o'clock hour circle*.

111. The AZIMUTH of any celestial object is an angle at the *zenith* formed by the meridian of the place, and a vertical circle passing through that

that object when it is above or below the horizon: And it is measured by the arc of the horizon intercepted between those vertical circles.

112. The **AMPLITUDE** of any object in the heavens is an arc of the horizon contained between the eastern point of it, and the center of the object at its rising, or between the western point of it and the center of the object at its setting; or it may be taken as an angle at the zenith, included between the meridian of a place and a vertical circle passing through the object at its rising or setting.

113. The **ALTITUDE** of any object in the heavens is an arc of a vertical circle intercepted between the center of that object and the horizon.

114. The **ZENITH DISTANCE** of any object is an arc of a vertical circle contained between the center of that object and the zenith.

The altitude and zenith distance are complements one of the other.

115. The **MERIDIAN ALTITUDE**, or **MERIDIAN ZENITH DISTANCE**, is the altitude or zenith distance when the object is on the meridian of the place.

116. The **CULMINATING** of any celestial object, is the time it *transits*, or comes to the Meridian. And the **MEDIUM COELI**, or **MID-HEAVEN**, to any place, is that degree of the ecliptic, or part of the heavens, over the meridian of that place, at any time. Or the **MID-HEAVEN** is the distance of the meridian from the first point of Aries, reckoned on the equinoctial.

117. The **NONAGESIMAL DEGREE** is the 90th degree of the Ecliptic, reckoned from its intersection with the eastern point of the horizon, at any given time.

— Consequently the altitude or height of the nonagesimal degree above the horizon is equal to the distance of the poles of the ecliptic and horizon; and is the measure of the angle which the ecliptic makes with the horizon.

118. **ALMICANTHERS**, or **PARALLELS OF ALTITUDE**, are small circles parallel to the horizon.

119. A **PARALLEL SPHERE** is that position of the sphere in which the circles apparently described by the diurnal rotation, are parallel to the horizon; which can happen only at the poles.

120. A **RIGHT SPHERE** is that in which the diurnal motions are at right angles to the horizon: Thus it appears in all places under the equator.

121. An **OBLIQUE SPHERE** has all the diurnal motions oblique to the horizon: And thus the motions appear to all parts of the Earth, except under the poles and equator.

122. **DIURNAL ARCS** are those parts of the parallels of declination of celestial objects which are apparently described between the times of the rising and setting of those objects: And **NOCTURNAL ARCS** are the parts of those parallels apparently described from the time of setting to the time of rising.

123. **SEMI-DIURNAL** and **SEMI-NOCTURNAL ARCS**, or the halves of diurnal and nocturnal arcs, are the parts of the parallels intercepted between the meridian and the horizon. The corresponding part of the equator answering to the semi-diurnal arc, gives the times between noon and the rising or setting; and the equatorial part answering to the semi-nocturnal

nocturnal arc, shews the time between midnight and the time of setting or rising.

124. The **OBLIQUE ASCENSION** of any object in the heavens, is an arc of the equinoctial intercepted between the first point of Aries and the eastern part of the horizon when that object is rising; and the **OBLIQUE DESCENSION** is an arc of the equinoctial intercepted between the first point of Aries and the western part of the horizon at its setting.

125. The **ASCENSIONAL DIFFERENCE** belonging to any celestial object is an arc of the equinoctial intercepted between the horizon and the hour-circle which the object is on when it rises or sets; or it is the difference between the right and oblique ascension of that object. In the Sun, it is the time that he rises or sets before or after the hour of six.

126. The **LATITUDE** of any place on the Earth is an arc of a terrestrial meridian contained between that place and the equator; or it is an arc of a celestial meridian intercepted between the zenith of the place and the equinoctial; being north or south, according to the side of the equator it is on.

127. The **LONGITUDE** of any place on the Earth is an arc of the equator contained between the meridian of that place and the meridian which is chosen for the first, where the reckoning of longitude begins: Or, it is the angle at the pole formed by the first meridian and that of the place.

128. **REFRACTION**, in an astronomical sense, is the difference between the true and apparent altitudes of celestial objects; they appearing more elevated above the horizon than they really are, on account of the density of the Earth's atmosphere, or air and vapours surrounding it.

129. The **PARALLAX OF A CELESTIAL OBJECT** is the distance between its place in the heavens, as seen from the center of the Earth, and its place when seen from the surface of it.

130. The **TWILIGHT** is that medium between light and darkness, which happens in the morning before sun-rise, and in the evening after sun-set.

This is occasioned by the atmosphere's refracting the solar rays upon any place, although the Sun is below the horizon of that place, and by observation it is found to begin and end when the Sun is about 18° below the horizon.

131. The **CREPUSCULUM** is a small circle parallel to the horizon at 18° below it, where the twilight begins and ends.

For under the pole, or in the latitude of 90 degrees, the pole is in the zenith, or is 90 degrees above the horizon; so that, in this case, the horizon coincides with the equinoctial.

And as many degrees as the observer goes from the pole toward the equator, so many degrees does his horizon go below the equator on one side, and approach the pole on the other side.

Therefore the pole approaches the horizon just as much as the zenith approaches the equator; that is, the height of the pole above the horizon, is equal to the distance of the zenith from the equinoctial, which is equivalent to the distance of the observer from the equator, or is equal to the latitude.

132. ASTRONOMICAL TABLES in general contain numbers shewing either the measure of the distances of the heavenly bodies from certain limits which are used to represent remarkable *times* and *places*; or, the times when those bodies had, or will have, given positions relative to those limits.

Some of the chief astronomical tables are,
Solar and lunar tables for finding the places of those luminaries at given times.

Tables for finding the places of the other planets.

Stellar tables for finding the places of the stars.

Tables shewing the Sun's place, declination, and right ascension for given times.

Tables of refractions for correcting observations on altitudes.

Tables of the equation of time; or the difference between the times shewn by a sun-dial and a well-regulated clock, &c.

The astronomical tables chiefly wanted in this work are placed at the end of this book; and are preceded by an account of their construction and use.

133. As the Earth makes one revolution on its axis in a common day of 24 hours; therefore every point of the equator will describe the circle of 360 degrees in 24 hours; and consequently, if 360 degrees give 24 hours, any other number of degrees will give its proportional hours: And if 24 hours give 360 degrees, any other number of hours will give its proportional number of degrees.

And hence are derived methods for converting arcs of circles into measures of time, and measures of time into arcs of circles.

To reduce degrees, minutes, &c. to time.

Multiply by 24, and divide by 360; or multiply by 4, and divide by 60:
Or, Divide the given degrees by 15 for hours; multiply the remainder by 4 for minutes, adding to the product 1 minute for every 15' of a degree; the overplus minutes of a degree, multiplied by 4, give seconds of time, &c.

Or thus: Let the quotient of the given degrees by 60 stand for the first name; the remaining degrees for the second name; and the other given names in order following: Then this number multiplied by 4, will give the hours, minutes, seconds, &c. in order.

To reduce time into degrees.

Multiply the given hours by 15 gives degrees, to which add 1° for every 4 minutes of time; for every overplus minute reckon 15' of a degree; and for every second of time take 15'' of a degree.

Or thus: Divide the time by 4, carrying by sixties, the quotient will be in order, sixties of degrees, degrees, minutes, seconds, &c. Then sixties of degrees and degrees being reduced, will give the degrees, &c. required.

EXAM. I. Reduce $69^{\circ} 20', 45''$,
to its corresponding time.

$$\begin{array}{r} 15) 69^{\circ} 20' 45'' \quad (4^h 37^m 23^s \\ \hline 9 \times 4 + 1 = 37 \\ \hline 5 \times 4 + 3 = 23 \end{array}$$

EXAM. II. Reduce $4^h, 37^m, 23^s$,
to its corresponding degrees.

$$\begin{array}{r} 4^h 37^m 23^s \\ 15 \\ \hline 60^{\circ} \quad 0' \quad 0'' \quad \text{for 4 hours.} \\ 9 \quad 15 \quad 0 \quad \text{for 37 minutes.} \\ \quad 5 \quad 45 \quad \text{for 23 seconds.} \\ \hline 69^{\circ} 20' 45'' \end{array}$$

EXAM. III. Reduce $237^{\circ}, 44', 37''$,
to its equivalent time.

$$\begin{array}{r} 60) 237^{\circ} \\ \hline 3 \quad 57 \quad 44 \quad 37 \\ \hline \quad \quad \quad 4 \end{array}$$

Answer $15^h 50^m 58^s 28^t$

EXAM. IV. Reduce $15^h, 50^m, 58^s, 28^t$,
to its equivalent degrees.

$$\begin{array}{r} 4) 15^h 50^m 58^s 28^t \\ \hline 3^{\circ} 57' 44'' 37'' \\ \hline \text{Or } 237^{\circ} 44' 37'' \text{ Answer.} \end{array}$$

As the sun is constantly changing his place, the tables of his right ascension shew for every day at noon (when he comes to the meridian of the place for which the tables are made) what part of the equator is intercepted between that meridian and the equinoctial point γ . The tables for the stars shew the equatorial arcs contained between the point γ and the section of circles of right ascension, passing through those stars: The measures of the arcs of right ascension are reduced to time.

There are few days when one or more stars do not come to the meridian with the Sun, and then they have the same right ascension with him: Also, at some time of the year, the Sun must have the same right ascension which any proposed star has; though at other times he may have a less, and so precedes or comes to the meridian before that star; or a greater, and so follows the star, and comes to the meridian later. And hence is derived the following method

OF FINDING THE CULMINATING OF THE STARS.

134. To find the time when any star in the table will be on the meridian.

RULE. Subtract the Sun's right ascension for the proposed day from the right ascension of the given star; the difference will be the time of the star's culminating nearly. Say as 24^h is to the daily change of the Sun's right ascension, so is the time of culminating, nearly, to a fourth number; which being subtracted from the time of culminating nearly, will give the true time of the star's culminating. If this time be less than 12^h it happens in the afternoon; but if more than twelve hours, the excess above 12^h will shew the time next morning.

N. B. 24^h must be added to the star's right ascension, if the sun's right ascension be greatest.

EXAM.

EXAM I. *At what time will the star Arcturus come to the meridian of London on the 1st of September, 1780?*

Right ascen. of Arcturus	14 ^h 5' 42"
Sun's right ascension	10 44 34
Time of culmin. nearly	3 21 8
And 3 ^h 21 ^m $\frac{1}{4}$ give	30
True time of star's culm.	3 20 38

Ex. II. *On the 26th of Feb. 1780, at what hour will the star Virgin's Spike be on the meridian of London?*

Virg. Spike's right asc.	13 ^h 13' 38"
Sun's right ascension	22 37 10
Time of culm. nearly	14 36 28
And 14 ^h , 36 ^m $\frac{3}{8}$ give	2 18
True time of star's culm.	14 34 10

If the time of the star's culminating be wanted for any other meridian than that of Greenwich, or London, add the longitude in time to the time of culminating nearly, if the longitude be west, or take their difference if it be east, and use that sum or difference instead of the time of culminating nearly: observing, only, in the latter case, that if the longitude in time be greater than the time of culminating nearly, that the min. and sec. resulting from the proportion, must be added to the time of culminating nearly, instead of being subtracted from it.

EXAM. *On the 26th of February, 1784, what time will Sirius be on the meridian of a place which is in longitude 166° 30' E. of London?*

Rt. ascen. Sirius, 1780	6 ^h 35' 28"	}	Rt. asc. of Sirius, 1784	6 ^h 35' 39"
Precession for 4 years	+ 11		Sun's right ascen.	22 37 10
Time of culm. nearly	7 58 36		Time of culm. nearly	7 58 29
Long. in time	11 6 00		And 3 ^h 7 ^m , 4 ^s give +	29
Difference	3 7 24		True time of star's culm.	7 58 58

To find if any star in the table will be on or near the meridian at a given time, reckoned from the preceding noon.

RULE. To the given time add the Sun's right ascension for that time; the sum (rejecting 24 hours, if above) is the right ascension of the mid-heaven; which being sought among those of the stars, will shew what star will be on or near the meridian at the time proposed.

EXAM. I. *What star will be on the meridian of London about 10 o'clock at night on the 25th January, 1784?*

Given time 10 hours P. M.	10 ^h 0 ^m
Sun's right ascension at noon	20 31
And for 10 hours more	2
Sum (abating 24 hours)	6 33
answers to Sirius.	

Ex. II. *On the 10th May, 1784, what star will be on the merid. of Lond. about 30 min. after 4 in the morning?*

Given time	16 ^h 30'
Sun's right ascension at noon	3 12
And for 16 hours more	3
Right ascension mid-heaven	19 45
answers nearly to Altair.	

SECTION IV.

Of the Projection of the Sphere.

PROBLEM I.

135.

To project the sphere upon the plane of the solstitial colure, or upon the plane of the meridian of any place, those planes being supposed to coincide.

For this projection the eye is supposed to be in the first point of Aries, or the common intersection of the equator, ecliptic, and equinoctial colure; that being the pole of the plane of projection, or primitive circle. Pl. IV. Fig. 4.

1st. With the chord of 60 degrees describe a circle PESQ to represent the solstitial colure, the center of which γ is its pole. (IV. 62.)

2d. A diameter EQ will be the equator, and another ps at right angles to it will shew the equinoctial colure (IV. 60), or the axis of the world, the extremities of which p, s, will be the north and south poles.

3d. *For the parallels of declination.* On the primitive circle, beginning at E and Q, apply the chords of the given degrees of declination, suppose every 10 degrees, and also the distances of the tropics and polar circles from the equator, namely, $23\frac{1}{2}^\circ$ and $66\frac{1}{2}^\circ$. Then from the center γ in the axis ps produced, apply the respective secants of the complements of the degrees laid on the primitive (IV. 58), and these will give the centers of the corresponding parallels of the declination; from which centers, with the extents to the several divisions in the circumference, describe the small circles IO, IO; 20, 20; &c. and these will be the parallels of declination required: Among which *a* α , *b* ν are the tropics of Cancer and Capricorn; and *cc* *dd*, the arctic and antarctic polar circles.

4th. *For the circles of right ascension, or hour circles.* In the diameter EQ produced, lay off from the center γ both ways the tangents of 15° , 30° , 45° , 60° , 75° , respectively, and they will give the centers of circles, to be described through p and s, and cutting the equator in the points representing the 24 hours; the solstitial colure being the twelve o'clock, and the equinoctial colure, ps, the six o'clock hour circles. And in like manner may any other of this kind of circles be drawn. (IV. 75)

5th. *The ecliptic* $\mathcal{S}$ ν is drawn, making with the equator an angle of $23\frac{1}{2}^\circ$; the poles of which *c*, *d*, are the intersections of the polar circles with the solstitial colure.

6th. *Parallels of celestial latitude* are drawn parallel to the ecliptic, in the same manner as the circles of declination are drawn parallel to the equator.

7th. *Circles of celestial longitude* are described through *c*, *d*, the poles of the ecliptic in the same manner as the circles of right ascension were described through p, s, the poles of the equator; and thus were the divisions of the ecliptic found that are marked with the signs.

8th. *The horizon* is represented by drawing a diameter HR, making an angle with the axis ps, equal to the latitude of the place; and the poles of the horizon z, n, the zenith and nadir, are at 90° dist. from the circle HR.

9th. *Azimuth, or vertical circles*, making any angle with the meridian, are described like circles of right ascension: Thus zN is the prime vertical, and zAN is another azimuth, 45° from the south.

10th. *Almi-*

10th. *Almicanters, or parallels of altitude*, are in this projection drawn parallel to the horizon, in like manner as the circles of declination were drawn parallel to the equator.

136.

PROBLEM II.

To project a sphere upon the plane of the horizon.

In this projection, the eye is supposed in the nadir, one of the poles of the horizon or plane of projection. Plate IV. Fig. 5.

1st. *The horizon* is represented by the primitive circle, where the upper XII is the north, the lower XII the south, E the west, and Q the east points.

2d. *The azimuth circles* are represented by diameters drawn through z, the center or pole of the horizon: Thus the diameter XII, XII, is for the meridian, and EZQ for the prime vertical; and other azimuth circles, forming any angle with the meridian, are readily drawn by laying off their distances in the primitive from the north or south points.

3d. *Parallels of altitude* are concentric to the primitive, and are described about the pole z, with the half tangent of their distance from it: Thus the small circle, the diameter of which is *ab*, is a parallel of altitude 10° above the horizon, or at 80° distant from its pole z.

4th. The distance of the *equinoctial* from the zenith is equal to the latitude of the place, and therefore this circle makes with the horizon an angle, which is measured by the complement of the latitude; then setting off from the center z in z XII continued, the tangent of 50° (the latitude in this example being 40°), it will give the center of the circle EAQ, representing the equinoctial; and the half tangent of 50° , set the same way from z, will give P, the pole of the world.

5th. *The six o'clock hour circle* passes through the poles of the world, making with the horizon an angle equal to the measure of the latitude; therefore taking in the meridian from z toward A, the tangent of the latitude 40° , it gives G, the center of the six o'clock hour circle EPQ.

6th. *The hour circles* pass through the poles of the world, and make with one another angles of 15 degrees: Therefore (IV. 55) in a line DE, drawn through G, at right angles to the meridian, set off on both sides of G, the tangents 15° , 30° , 45° , 60° , 75° , to the radius PG, and they will give the centers of the several hour circles passing through P, cutting the horizon and equinoctial in the hour points.

7th. *The polar circles, tropics, and other circles of declination*, are described parallel to the equinoctial, about its pole P, at given distances from it, either by finding the centers of such parallels, as shewn in B. IV. 66; or by setting off on each side of z the half tangents of their greatest and least distances from z; then the middles of those intervals are the centers sought. Thus; the arctic circle is distant from P $23\frac{1}{2}^\circ$; then to and from $zP = 50^\circ$, add and take $23\frac{1}{2}^\circ$; there remain $73\frac{1}{2}^\circ$ and $26\frac{1}{2}^\circ$; the half tangents of these set off from z give p and q; then a circle described on the diameter pq is the arctic circle.

In like manner will the centers of the tropic of Cancer $c \in c$, and of Capricorn $d \in d$, be obtained.

8th. *The northern portion of the ecliptic* $\gamma \in \epsilon$ is described from a center distant from z toward P, the tangent of $73\frac{1}{2}^\circ = \angle$ the ecliptic makes with the horizon.

9th. Cir-

9th. *Circles of longitude* $\gamma p \triangle, p \delta, p \Pi, p \Omega, p \mathcal{M}$, are described thro' p , the pole of the ecliptic, from centers in the line BFC ; in like manner as the hour circles were described through P , the pole of the equator.

10th. *Circles of celestial latitude*, $VIII q IX$, are described about p , as the circles of declination were described about P , the pole of the equinoctial.

PROBLEM III.

137.

To project the sphere upon the plane of the equator.

In this projection the eye is supposed to be in one of the poles of the equator, suppose in the south pole, and projecting the north hemisphere. Plate IV. Fig. 6.

1st. *The equator* is represented by the primitive circle, the center and pole of which is P .

2d. *The hour circles* are expressed by diameters, making angles of 15° with one another; of which $XII P XII$ is the meridian, or solstitial colure, and $VI P VI$ the 6 o'clock circle, or equinoctial colure.

3d. *Circles of declination* are circles parallel and concentric to the equator, described from its center with radii equal to the half tangents of their several distances from the pole P , or half co-tangents of their degrees of declination; Thus pq the arctic circle, and $a \odot$ the tropic of Cancer, are described with the half tangents of $23\frac{1}{2}^\circ$ and $66\frac{1}{2}^\circ$ respectively; and so of the others.

4th. *The ecliptic* making an angle of $23\frac{1}{2}^\circ$ with the equator; the tangent of these degrees laid from P toward a will give the center for describing the ecliptic $\gamma \odot \triangle$, the pole of which p is in the polar circle.

5th. *Circles of longitude* are described through p , the pole of the ecliptic, in like manner as the hour circles were described through P , the pole of the equator in the last problem; and thus were the divisions $\delta, \Pi, \Omega, \mathcal{M}$, obtained.

6th. *Circles of celestial latitude* are projected in the same manner as the circles of declination in the last problem.

7th. *The horizon of any place*, suppose of London, being inclined to the equator in an angle equal to the co-latitude, $38^\circ 28'$; the tangent of this laid from P toward $\odot$, and the half tangent laid from P to z , will give the center, and z the pole of the horizon HOR .

8th. *The prime vertical* HZR making an angle with the equator equal to $51^\circ 32'$, the latitude of the place, its center is found by laying the tangent of $51^\circ 32'$ from P toward o .

9th. *Azimuth circles*, making given angles with the meridian zo , are thus described: In a line drawn through the center of the prime vertical, at right angles to the meridian, take distances from that center, equal to the tangents of the proposed azimuth angles, the semidiameter of the prime vertical being the radius, those distances give the centers sought; and thus was the azimuth circle ZA described.

10th. *Parallels of altitude* are described about z , the pole of the horizon, at the distances of the co-altitudes, in the same manner as the circles of declination were described about P , the pole of the equator in the last problem; and thus was the small circle $v b VII$ described at 10° distance from the horizon, or 80° distant from its pole z .

138, PRO-

138.

PROBLEM IV.

To project the sphere upon the plane of the ecliptic.

The eye is here supposed to be in one of the poles of the ecliptic, and thence viewing the northern hemisphere. Plate IV. Fig. 7.

1st. *The ecliptic* is here represented by the primitive circle, the center of which p is its pole.

2d. *Circles of longitude* are here represented by diameters; those that make angles of 30° with one another, being drawn through the divisions marked with the signs of the zodiac.

3d. *Parallels of celestial latitude* are circles described about p , concentric to the ecliptic; such is the small circle, the diameter of which is ab , representing the parallel of 10° of latitude.

4th. *The equator* making an angle with the ecliptic of $23\frac{1}{2}^\circ$; therefore the tangent of this inclination laid from p toward $\odot$ will give the center of the equator γ XII $\simeq$; and the half tangent of $23\frac{1}{2}^\circ$ laid from p the same way, gives P for the pole of the equator.

5th. *The equinoctial colure*, which here makes the *six o'clock circle*, makes an angle with the ecliptic of $66\frac{1}{2}^\circ$; therefore the tangent of $66\frac{1}{2}^\circ$ laid from p towards $\vee$, gives the center of the 6 o'clock circle γ P $\simeq$.

6th. *Hour circles* passing through P , and making angles of 15° with one another, are described from centers, found in a right line passing through the center of γ P $\simeq$, and drawn at right angles to the solstitial colure γ p $\vee$; by laying off in that line the tangents of 15° , 30° , 45° , 60° , 75° , reckoned from the center of γ P $\simeq$, on both sides, the semi-diameter of this circle being the radius. These hour circles cut the equator in the hour points.

7th. *Parallels of declination*, such as the tropic of Cancer, and the arctic circle, the diameters of which are 12 , 12 , and pq , are described by laying off from p the half tangents of their greatest and least distances: Thus q being distant from p 47° , makes $pq = \frac{1}{2}$ tangent of 47° . the middle of pq will be the center of the polar circle.

8th. *The horizon* HOR is to make an angle with the ecliptic equal to the difference between the co-latitude and the obliquity of the ecliptic, when P is projected to the north of p ; otherwise that angle is equal to the sum of those quantities. And for London, where the said difference $= (38^\circ 28' - 23^\circ 28') = 15^\circ 00'$, the tangent of $15^\circ 00'$, gives the center of HOR, and the half tangent gives z the zenith.

9th. *The prime vertical* HZR is described by laying from p , toward $\odot$, the co-tangent of pz for a center.

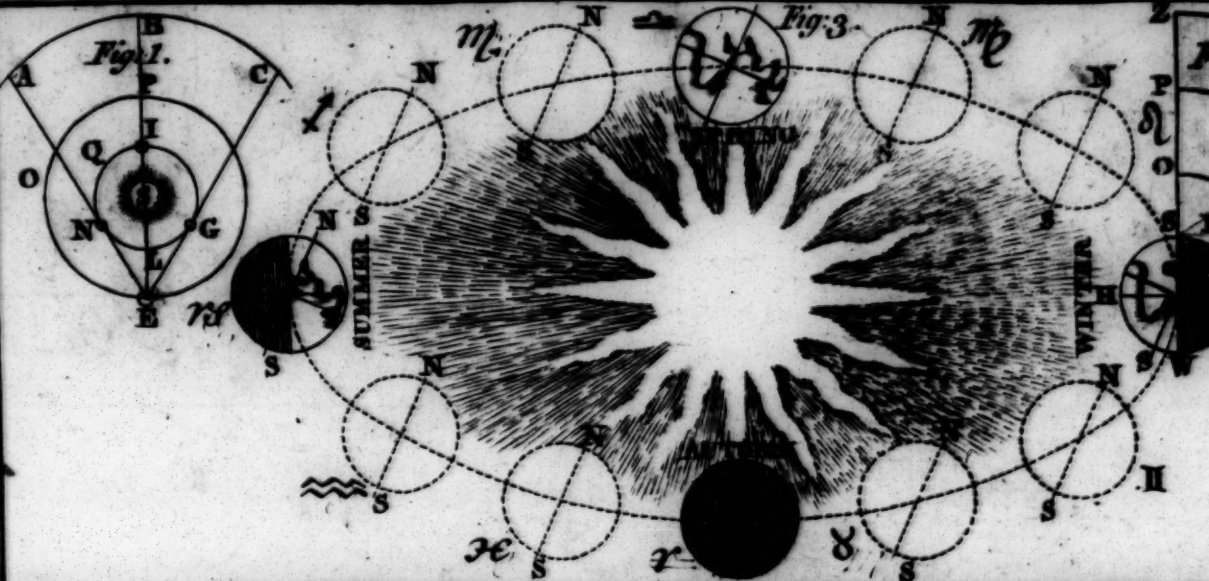
10th. *Azimuth circles* are described through z , making given angles with the meridian zo , by finding their centers in a line drawn through the center of HZR, in the manner described for the hour circles, Prob. II.

11th. *Parallels of altitude* are represented by describing small circles parallel to the horizon HOR, at given distances from it; or, which comes to the same, describing small circles about the pole z , at distances equal to the complements of the given altitudes: And thus the circle cdc was described for a parallel of 33° of altitude.

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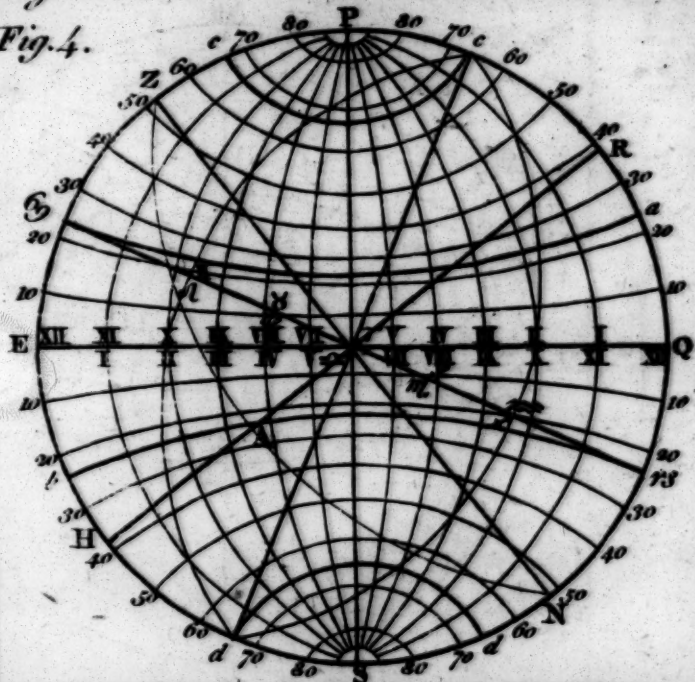
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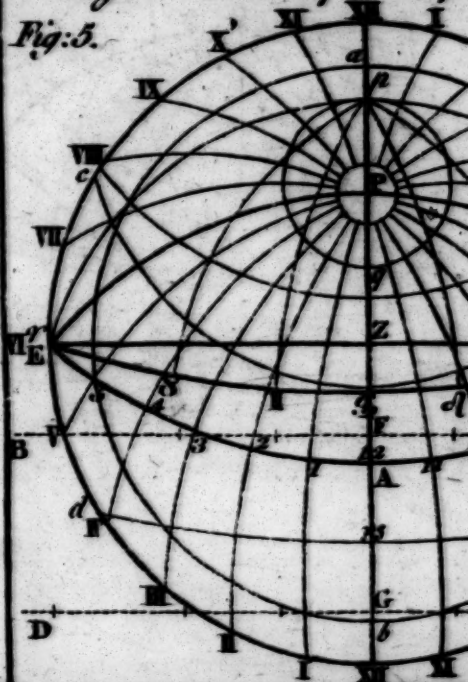
Projection on the Solstitial Colure or Meridian

Fig. 4.



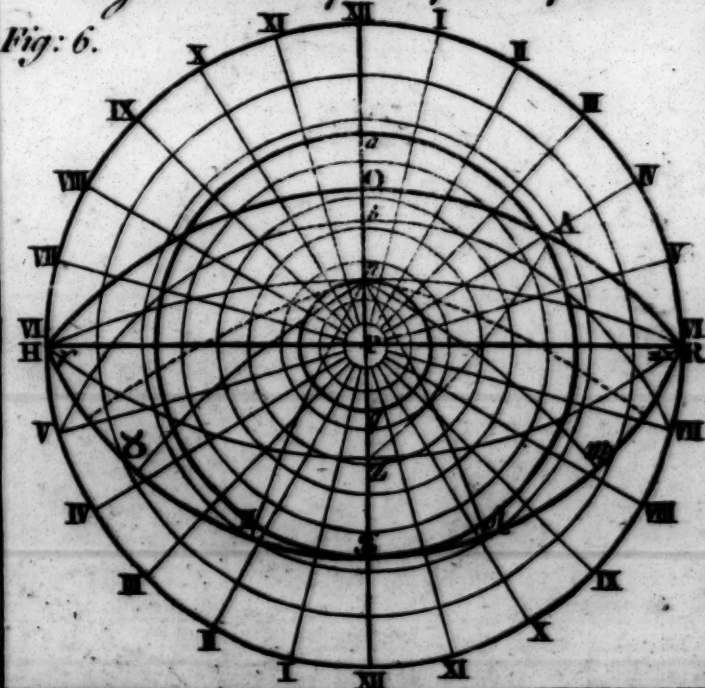
Projection on the plane of

Fig. 5.



Projection on the plane of the Equator.

Fig. 6.



Projection on the plane of

Fig. 7.

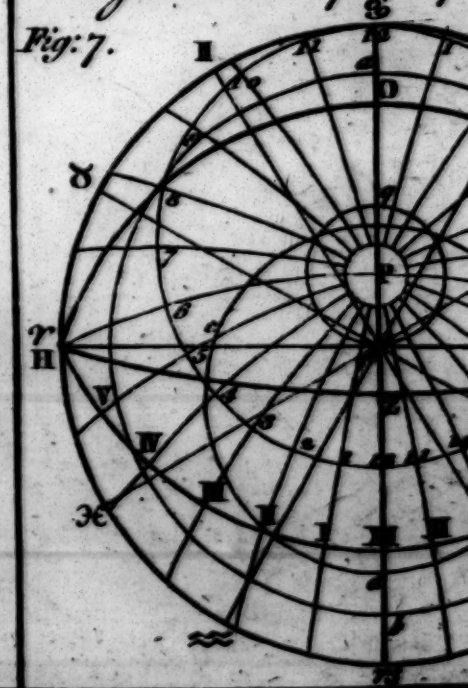
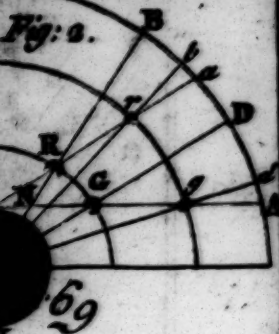
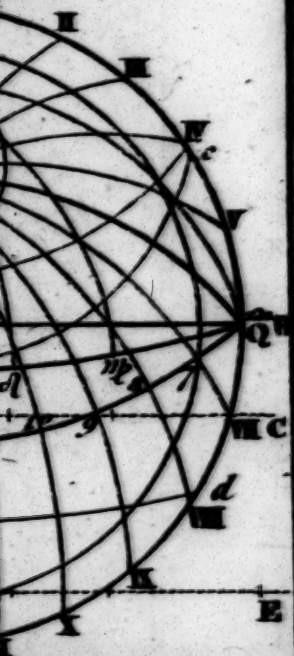


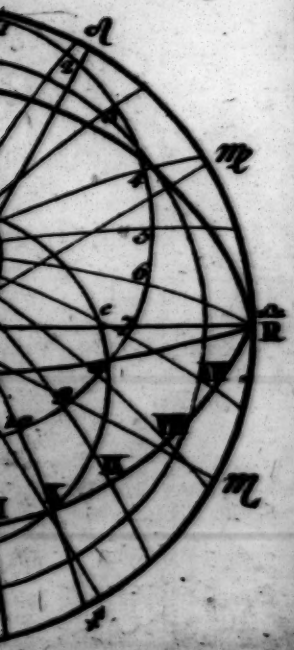
Fig: 9.

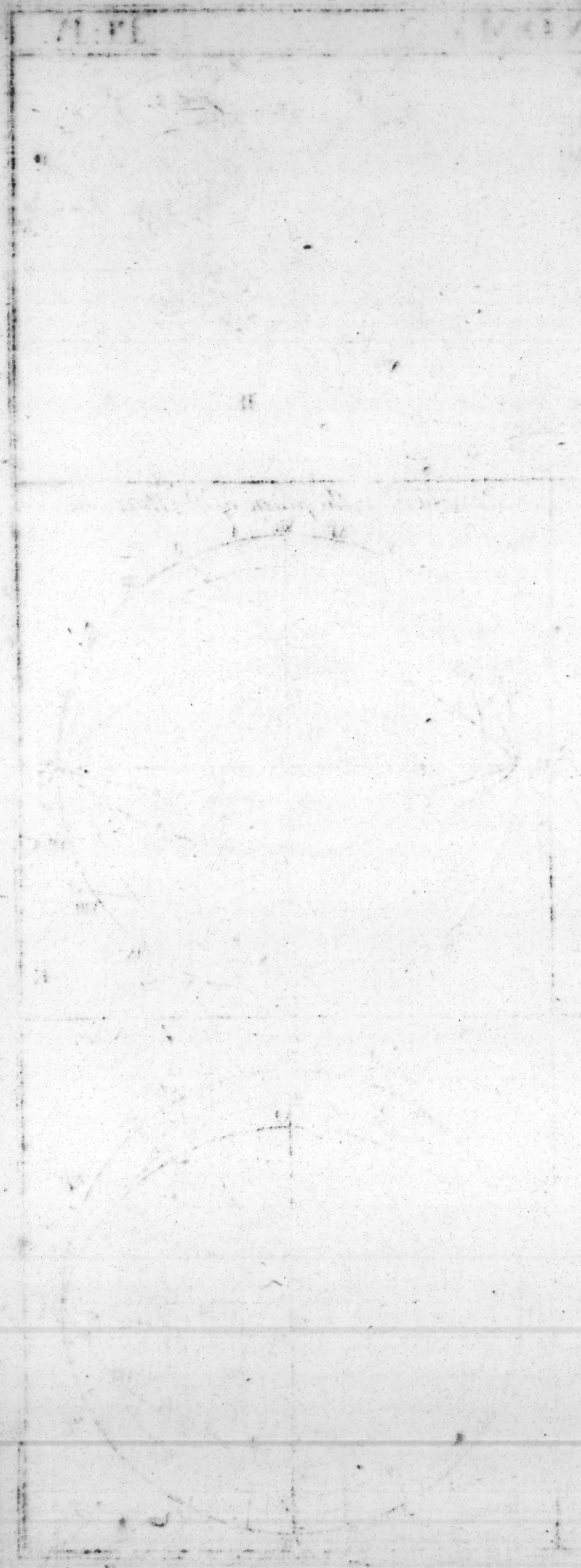


of the Horizon.



of the Ecliptic:





SECTION V. Problems of the Sphere.

PROBLEM V.

Pl. V.

139.

Given the Sun's longitude, and the obliquity of the ecliptic;
Required the Sun's right ascension and declination.

EXAM. Let the obliquity of the ecliptic, or the Sun's greatest declination, be $23^{\circ} 28'$, and the Sun's place $13^{\circ} 16'$ in Taurus: Required the rest.

CONSTRUCTION.

In the primitive circle PESQ, representing the solstitial colour, the center of which is γ , draw a diameter EQ for the equator, and at right angles to EQ draw a diameter PS for the equinoctial colour: Make $ES = 23^{\circ} 28'$, and draw a diameter $\odot \Psi$ for the ecliptic, in which (IV. 71.) take $\gamma \odot = 43^{\circ} 16'$ for the Sun's distance from the point γ : Through $\odot$ describe a circle of right ascension.

COMPUTATION. See Book IV. Art. 130, 131.

In the right angled spheric triangle $\gamma \odot B$.

Given Sun's longitude $\gamma \odot = 43^{\circ} 16'$ } Req. right ascen. γB .
Obliquity of the eclip. $\angle \odot \gamma B = 23^{\circ} 28'$ } declin. $B \odot$.

To find the declination.

As radius	= R	10,00000
To f. Sun's lon.	= $43^{\circ} 16'$	9,83594
So f. ob. eclip.	= $23^{\circ} 28'$	9,60012
To f. Sun's decl.	= $15^{\circ} 50'$	9,43606

To find the right ascension.

As radius	= R	10,00000
To t. Sun's lon.	= $43^{\circ} 16'$	9,97371
So co-f. obl. ecl.	= $23^{\circ} 28'$	9,96251
To t. rt. ascen.	= $40^{\circ} 48'$	9,93622

140. While the sun is moving from γ to $\odot$, or is in the first quadrant of the ecliptic, the given longitude is the hypotenuse in the triangle $\gamma \odot B$, the declination $B \odot$ is north, and γB is the right ascension.

When the Sun has past the solstice $\odot$, and is descending toward $\sphericalangle$, he is then said to be in the second quadrant, and his longitude or distance from γ being taken from 180° , the remainder $\sphericalangle \odot$ becomes the hypotenuse, and the declination is still north; but the arc $B \sphericalangle$ found for the right ascension is only the supplement, and must therefore be taken from 180° .

The Sun having past the point $\sphericalangle$, and descending toward Ψ , has got into the third quadrant; the longitude then, reckoned from γ , will be greater than 180° : In this case the excess above 180° , or the distance the Sun is removed from $\sphericalangle$, will be the hypotenuse $\sphericalangle \odot$; the declination will be south; and the arc $\sphericalangle A$, found for the right ascension, must be added to 180° , to give the right ascension estimated from γ .

When the Sun has past the solstice Ψ , and is ascending toward γ , he is then in the fourth quadrant; therefore the longitude is greater than 270° , and must be taken from 360° , to give the hypotenuse $\sphericalangle \odot$. Here the declination is south, and the right ascension $\sphericalangle A$, found by the proportion, must be taken from 360° , to give the right ascension from γ .

At equal distances from the equinoctial points γ or $\sphericalangle$, the Sun will have equal quantities of declination; but will be of different names, according as it is on the north or south sides of the equinoctial.

141.

PROBLEM VI.

Pl. V.

Given the obliquity of the ecliptic, and the Sun's declination;
Required the Sun's longitude and right ascension.

EXAM. *The obliquity of the ecliptic, being $23^{\circ} 28'$, what is the Sun's longitude and right ascension when he has $20^{\circ} 43'$ of north declination?*

CONSTRUCTION.

Having described the solstitial colure, and drawn the equator EQ, the axis PS, and the ecliptic $\propto \wp$, as before; make EN, QN, equal to the given declination, and (3d 133) describe the parallel of declination nn, its intersection with the ecliptic gives $\odot$ the Sun's place; through P, $\odot$, s, describe the circle of right ascension P $\odot$ s.

COMPUTATION.

In the right-angled spheric triangle $\gamma \odot B$.

Given the ob. eclip. $\angle \odot \gamma B = 23^{\circ} 28'$ } Required Sun's long. $\gamma \odot$.
the Sun's decl. $\odot B = 20^{\circ} 43'$ } rt. ascen. γB .

To find the Sun's longitude.

As f. obliq. eclip. $= 23^{\circ} 28'$ 0,39988
To fin. Sun's decl. $= 20^{\circ} 43'$ 9,54869
So radius $= R$ 10,00000

To fin. $\odot$ longit. $= 62^{\circ} 40'$ 9,94857

To find the Sun's right ascension.

As radius $= R$ 10,00000
To co-t. obl. eclip. $= 23^{\circ} 28'$ 10,36239
So tan. Sun's dec. $= 20^{\circ} 43'$ 9,57772

To fin. rt. ascen. $= 60^{\circ} 36'$ 9,94011

Therefore the Sun is in $\pi 2^{\circ} 40'$, or in $\propto 27^{\circ} 20'$, according as the time of the year is before or after the summer solstice.

142.

PROBLEM VII.

Pl. V.

Given the obliquity of the ecliptic, and the Sun's right ascension;
Required the Sun's longitude and declination.

EXAM. *When the Sun's right ascension is $60^{\circ} 31'$, what is the longitude and present declination, the obliq. of the eclip. being $23^{\circ} 28'$?*

CONSTRUCTION.

The solstitial colure, equator, axis, and ecliptic, being described as before, make $\gamma B =$ given right ascension (4th 133), and describe the circle PBS, cutting the ecliptic in $\odot$ the Sun's place.

COMPUTATION.

In the right-angled spheric triangle $\gamma \odot B$; the leg γB and $\angle \odot \gamma B$ being known, the hypoth. $\gamma \odot$, and other leg $\odot B$, are found as in Art. 137, 138. Book IV.

As rad. : co-f. ob. eclip. : : co-t. rt.	As rad. : tan. obl. eclip. : : fin. rt.
[af. : co-t. $\odot$ long.]	[af. : tan. decl.]
As rad. : co-f. $23^{\circ} 28'$: : co-t. $60^{\circ} 31'$	As rad. : tan. $23^{\circ} 28'$: : fin. $60^{\circ} 31'$
[: co-t. $62^{\circ} 35'$]	[: tan. $20^{\circ} 42'$]

Three other problems may be formed out of the four things concerned, or obliquity of the ecliptic, declination, longitude, and right ascension: But these being of little more importance than as an exercise for right-angled spheric triangles, they are therefore omitted.

143. PRO-

143.

PROBLEM VIII.

Pl. V.

Given the latitude of the place, and the Sun's declination;
Required the Sun's altitude and azimuth at 6 o'clock.

EXAM. *At London, in lat. $51^{\circ} 32'$ N. on the longest day, when the Sun's declination is $23^{\circ} 28'$: Required the Sun's altitude and azimuth at 6 o'clock in the morning or evening.*

CONSTRUCTION.

Describe the meridian, draw the horizon HR, and prime vertical ZN; make RP = latitude $51^{\circ} 32'$ N.; draw the 6 o'clock hour circle PS, the equator EQ, the $23^{\circ} 28'$ N. parallel of declination nm , cutting the 6 o'clock hour circle PS in $\odot$; and through z, $\odot$, N, describe the azimuth circle z $\odot$ N, cutting the horizon in A; then the things given and required fall in either of the triangles z $\odot$ P or $\gamma \odot$ A, they being supplemental triangles one to the other.

COMPUTATION.

In the spheric triangle z $\odot$ P, right-angled at P.

Given the co-latit. zP = $38^{\circ} 28'$ } Required the co-altitude z $\odot$.
the co-decl. $\odot$ P = $66^{\circ} 32'$ } the azimuth $\angle \odot$ zP.

Or in the spheric triangle γ A $\odot$, right-angled at A.

Given the latit. A $\gamma \odot$ = $51^{\circ} 32'$ } Required the altitude A $\odot$.
the decl. $\gamma \odot$ = $23^{\circ} 28'$ } the co-azimuth γ A.

To find the altitude A $\odot$.

As radius	= R	10,00000
To fin. decl.	= $23^{\circ} 28'$	9,60012
So fin. lat.	= $51^{\circ} 32'$	9,89375
To fin. alt.	= 18 10	9,49387

To find the azimuth AR.

As radius	= R	10,00000
To co-f. lat.	= $51^{\circ} 32'$	9,79383
So tan. decl.	= $23^{\circ} 28'$	9,63761
To co-t. azim.	= 74 53	9,43144

For the arc AR measures the $\angle$ RZA, the azimuth.

(IV. 9)

144. On the shortest day at London, the parallel of S. declination cuts the 6 o'clock hour circle below the horizon; and as the triangles γ A $\odot$, γ a $\odot$, are congruous, the depression below the horizon, on the shortest day at 6 o'clock, will be equal to the altitude at the same hour on the longest day; and the azimuth will also be equal, if estimated from the south.

So that on the 21st of June, at London, the Sun will bear N. $74^{\circ} 53'$ E. at 6 o'clock in the morning, and N. $74^{\circ} 53'$ W. at 6 in the evening; but on the 21st of December, at the same hours, it will bear S. $74^{\circ} 53'$ E. and S. $74^{\circ} 53'$ W.

From a due consideration of this Problem it is evident, that as the declination increases, the altitude increases and the azimuth lessens; and the contrary happens while the declination is diminishing: So that on the days of the equinoxes, on which the Sun has no declination, the altitude at 6 o'clock will be nothing, or the Sun will be in the horizon; and the azimuth being then 90 degrees, the Sun will be due east in the morning, and west in the evening; that is, on the days of the equinoxes the Sun rises and sets at fix, in the east and west points of the horizon.

Q 2

145. PRO.

145.

PROBLEM IX.

Pl. V.

Given the latitude of the place, and the Sun's declination;
Required the altitude and hour when the Sun is due east or west.

EXAM. *At London, in latitude $51^{\circ} 32' N.$, what is the Sun's altitude, and the hour when he is due east or west, on the longest day, or when the declination is $23^{\circ} 28' N.$?*

CONSTRUCTION.

Describe the primitive circle to represent the meridian of London, draw the horizon HR , and the prime vertical ZN ; make $RP = 51^{\circ} 32'$, the given latitude, draw the 6 o'clock hour circle PS , the equator EQ , the parallel of declination nm ($3d$ 135), cutting the prime vertical in $\odot$, and through $P\odot$ describe (II. 72) the hour circle $P\odot s$, cutting the equator in A .

Here the things concerned in the Problem fall in either of the triangles $PZ\odot$, or $\Upsilon A\odot$.

COMPUTATION.

In the spheric triangle $PZ\odot$, right-angled at z .

Given the co-latit. $PZ = 38^{\circ} 28'$ } Required the co-altitude $z\odot$
the co-decl. $P\odot = 66^{\circ} 32'$ } the hour fr. noon $\angle ZP\odot$.

Or in the spherical triangle $A\odot$, right-angled at A .

Given the latit. $\angle A\Upsilon\odot = 51^{\circ} 32'$ } Required the altitude $\Upsilon\odot$
the decl. $A\odot = 23^{\circ} 28'$ } the hour after 6 ΥA .

To find the altitude $\Upsilon\odot$.

As f. lat. $\angle A\Upsilon\odot = 51^{\circ} 32'$ 0,10625
To fin. decl. $A\odot = 23^{\circ} 28'$ 9,60012
So radius $= R$ 10,00000

To fin. alt. $\Upsilon\odot = 30^{\circ} 34'$ 9,70637

To find the hour after 6.

As radius $= R$ 10,00000
To co-t. lat. $A\Upsilon\odot = 51^{\circ} 32'$ 9,90009
So tan. decl. $A\odot = 23^{\circ} 28'$ 9,63761

To f. h. fr. 6. $A\Upsilon = 20^{\circ} 11'$ 9,53770

Which $20^{\circ} 11'$ converted into time (132), gives 1 h. 20 m. 44 s. for the time after 6 in the morning, and before 6 in the evening, when the Sun will appear due east or west; which will be at 7 h. 20 m. 44 s. in the morning, and 4 h. 39 m. 16 s. in the afternoon.

Or, the compl. of $20^{\circ} 11'$, viz. $69^{\circ} 49'$ put into time, which gives 4 h. 39 m. 16 s., shews the time before and after noon, when the Sun will be due east or west.

146. This Problem worked for the shortest day, namely, in the $\Delta \Upsilon a\odot$, which is congruous to $\Upsilon A\odot$, would give the Sun's depression at the time when he was east or west, which would be before 6 in the morning, and after 6 in the evening, by as much as was found above, viz. 1 h. 20 m. 44 s.

By this Problem it appears, that when the latitude of the place, and the Sun's declination, have the same name; then, the greater the declination and latitude, the greater the altitude and time from 6: And having contrary names, the same things happen; but with this difference, that in the former case the days lengthen on account of the increase of the latitude and declination; whereas in the latter case the days shorten on that account.

147. PRO.

147.

PROBLEM X.

Pl. V.

Given the latitude of a place and the Sun's declination ;
Required his amplitude and ascensional difference.

EXAM. At London, lat. $51^{\circ} 32' N.$ on the 21st of June, being the longest day, when the Sun's declination is $23^{\circ} 28' N.$ How far from the north does the Sun rise and set, at what time, and what is the length of the day and night ?

CONSTRUCTION.

Let the primitive circle represent the meridian of the place, and the diameter HR the horizon ; from R, the north point, take $RP = 51^{\circ} 32'$ for the latitude, draw the axis, or 6 o'clock hour circle PS, and at right angles to it draw the equator EQ ; make $En, Qm = 23^{\circ} 28'$, the declination, and (3d 135) describe the parallel of declination nm , cutting the horizon in $\odot$, the place of the Sun at its rising and setting ; through which describe (II. 72) the hour-circle $P\odot S$.

COMPUTATION.

Now as the arc $QR =$ co-latitude, measures the $\angle Q\gamma R$.

In the spheric triangle $\gamma \odot A$, right-angled at A.

Given Sun's decl. $A\odot = 23^{\circ} 28'$ } Required the amplitude $\gamma \odot$
co-latit. $\angle A\gamma \odot = 38^{\circ} 28'$ } the ascen. diff. γA .

To find the amplitude $\gamma \odot$.

As fin. $\angle \gamma \odot$, co-l. $= 38^{\circ} 28'$	0,20617	This $39^{\circ} 48'$ is the amplitude reckoned from the east or west points of the horizon : But its complement $50^{\circ} 12'$ shews how far from the north the sun rises or sets on the longest day at London.
To fin. decl. $A\odot = 23^{\circ} 28'$	9,60012	
So radius $= R$	10,00000	
To fin. amp. $\gamma \odot = 39^{\circ} 48'$	9,80629	

To find the ascensional difference γA .

As radius $= R$	10,00000	Which $33^{\circ} 07'$, converted into time (132) gives 2 h. 12 m. 28 s. for the time which the sun rises before, and sets after, the hour of six on the longest day.
To t. lat. $\angle P\gamma \odot = 51^{\circ} 32'$	10,09991	
So tan. decl. $A\odot = 23^{\circ} 28'$	9,63761	
To f. af. diff. $\gamma A = 33^{\circ} 07'$	9,73752	

Suppose rs to be a parallel of declination as far south, as $m n$ is north ; then the hour circle PBS , passing through $\odot$ the place of the sun at its rising or setting, will form a triangle $\gamma \odot B = \Delta \gamma \odot A$, where the amplitude is to the southward of the east and west points.

148. Hence it is evident, that when the latitude and declination have the same name, the Sun rises before, and sets after 6 ; But when they are of contrary names, the Sun rises after, and sets before 6.

149. And as the Sun describes the parallel of declination $n m$ in 24 hours, being at n when it is noon, and at m when it is midnight; therefore the time in passing from m to $\odot$, or the time of rising being doubled, gives the length of the night; and the time of setting being doubled, must give the length of the day.

Then to and from	6 ^h	0 ^m	0 ^s
Add and subtract the ascen. diff.	2	12	28
Sum, gives $\odot$ setting	8	12	28
Diff. gives $\odot$ rising	3	47	32
Length of day is	16	24	56
Length of night is	7	35	04

But when it is the shortest day at London, which is, when the Sun has $23^{\circ} 28'$ south declination; then the lengths of the day and night change places; the day being 7 h. 35 m. 04 s. long, and the night 16 h. 24 m. 56 s.

150. When the latitude and declination have the same name, the difference between the right ascension and the ascensional difference, is the oblique ascension; and their sum is the oblique descension.

But when they are of contrary names, their sum is the oblique ascension, and their difference is the oblique descension.

151. When the declination is equal to the co-latitude of any place (which can only happen to places within the polar circles), then the parallel of declination will not cut the horizon, and consequently the Sun will not set in those places during the time his declination exceeds the co-latitude: And the same may be said of all those stars, the polar distance of which is less than the latitude of the place; or, which is the same thing, that have declinations less than the co-latitude, for those stars will never descend below the horizon of that place. But this is to be understood only when the Sun or stars are in the same hemisphere with the given place; for when the Sun or stars are in a contrary hemisphere to any place, the co-latitude of which does not exceed the declination of those celestial objects, then they will never rise above the horizon of that place, and consequently are never visible there.

152.

PROBLEM XI.

Pl. V.

Given the latitude of a place, the Sun's declination and altitude;
Required the hour from noon, and the sun's azimuth.

EXAM. In the latitude of $51^{\circ} 32'$ N. the Sun's altitude was observed to be $46^{\circ} 20'$, when his declination was $23^{\circ} 28'$ N. What was the Sun's azimuth, and the hour when the observation was made?

CONSTRUCTION.

Let the primitive circle $ZRNH$ represent the meridian of London, HR the horizon, ZN the prime vertical; make $RP = 51^{\circ} 32'$ the height of the pole at London; draw the axis PS , and the equator EQ ; lay off the declination En , Qm , $23^{\circ} 28'$ N. the altitude HR , RS , $46^{\circ} 20'$; and (IV. 68) describe the parallel of declination nm , and the parallel of altitude rs , cutting one another in $\odot$, the place of the Sun at that time; through z , $\odot$, N , describe an azimuth circle $z\odot N$, and through P , $\odot$, s , describe an hour circle $P\odot s$: Then the angles $\odot ZP$, $\odot Pz$, being measured (IV. 72), will give the azimuth and hour from noon required.

COMPUTATION.

In the oblique-angled spheric triangle $P\odot z$.

Given the co-latitude $zP = 38^{\circ} 28'$ } Required the azim. $\angle \odot ZP$.
the co-alt. or zen. dist. $z\odot = 43^{\circ} 40'$ } and the h. fr. noon $\angle \odot Pz$.
the co-dec. or pol. dist. $P\odot = 66^{\circ} 32'$ } See Art. 167. Book IV.

To find the azimuth $\angle \odot ZP$.

Here $z\odot = 43^{\circ} 40'$ $zP = 38^{\circ} 28'$ $z\odot - zP = 5^{\circ} 12' = D$ $P\odot = 66^{\circ} 32'$ $71^{\circ} 44'$ $35^{\circ} 52'$ $2) \quad \quad \quad$ $61^{\circ} 20'$ $30^{\circ} 40'$	Then Co-ar. fin. co-lat. $= 38^{\circ} 28'$ 0,20617 Co-ar. fin. co-alt. $= 43^{\circ} 40'$ 0,16086 Sin. $\frac{1}{2}$ sum co-decl. & D $= 35^{\circ} 52'$ 9,76782 Sin. $\frac{1}{2}$ diff. co-decl. & D $= 30^{\circ} 40'$ 9,70761 The sum of the four logs. $19,84246$ The $\frac{1}{2}$ sum gives $56^{\circ} 31\frac{1}{2}'$ $9,92123$ Which doubled, gives $113^{\circ} 03'$ for the azimuth sought, reckoning from the north.
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To find the hour from noon, $\angle \odot Pz$.

Here $P\odot = 66^{\circ} 32'$ $Pz = 38^{\circ} 28'$ $P\odot - Pz = 28^{\circ} 4' = D$ $z\odot = 43^{\circ} 40'$ $71^{\circ} 44'$ $35^{\circ} 52'$ $2) \quad \quad \quad$ $15^{\circ} 36'$ $7^{\circ} 48'$	Then Co-ar. fin. co-decl. $= 66^{\circ} 32'$ 0,03749 Co-ar. fin. co-lat. $= 38^{\circ} 28'$ 0,20617 Sin. $\frac{1}{2}$ sum co-alt. & D $= 35^{\circ} 52'$ 9,76782 Sin. $\frac{1}{2}$ diff. co-alt. & D $= 7^{\circ} 48'$ 9,13263 The sum of the four logs. $19,14411$ The $\frac{1}{2}$ sum gives $21^{\circ} 55'$ $9,57206$ This doubled, gives $43^{\circ} 50'$ for the measure of the hour from noon, which is 2 h. 55 m. 20 s.
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Hence it appears, that the observation was made either at 9 h. 4 m. 40 s. in the morning, or at 2 h. 55 m. 20 s. in the afternoon.

The azimuth being first found, the hour from noon might have been found by the proportion between opposite sides and angles.

Had the declination and latitude been of contrary names, the same kind of operation would have been used to find the things required, only the side $\odot P$ would have been obtuse; by adding the declinat. to 90° , instead of subtracting it, as in the case of the lat. and decl. having like names.

153.

PROBLEM XII.

PL. V.

Given the latitude of the place, and the Sun's declination;
Required the time when the twilight begins and ends.

EXAM. *At what time does the twilight begin and end at London, when the Sun's declination is $15^\circ 12' N$. the latitude of the place being $51^\circ 32' N$.*

CONSTRUCTION.

Let the circle $ZRNH$ represent the meridian of the place, HR the horizon, ZN the prime vertical, and ts the Crepusculum, or small circle parallel to the horizon described at 18 degrees below it (IV. 68); lay off the latitude RP , draw the axis PS , the equator EQ , and describe the parallel of declination nm , and where nm cuts ts in $\odot$, is the Sun's place at the time of the beginning or end of the twilight; through $\odot$ describe (II. 72) the vertical circle $z\odot N$, and the hour circle $P\odot S$; then the $\angle ZP\odot$ being measured (IV. 72) will give the time before or after noon as required.

COMPUTATION.

In the oblique-angled spheric triangle $z\odot P$.

Given the co-lat. $zP = 38^\circ 28'$ } Req. the hour from noon = $\angle ZP\odot$.
the polar dist. $P\odot = 74^\circ 48'$ } The manner of solution is the
the zenith dist. $z\odot = 108^\circ 00'$ } same as in last Problem.

Here $P\odot = 74^\circ 48'$
 $PZ = 38^\circ 28'$
 $P\odot - PZ = 36^\circ 20' = D$
 $\odot Z = 108^\circ 00'$

$144^\circ 20'$ $72^\circ 10'$
2) $71^\circ 40'$ $35^\circ 50'$

Then Co-ar. fin. polar dist. $= 74^\circ 48' 0,01547$
Co-ar. fin. co-latit. $= 38^\circ 28' 0,20617$
Sine $\frac{1}{2}$ sum. of zen. d. & $D = 72^\circ 10' 9,97861$
Sine $\frac{1}{2}$ diff. of zen. d. & $D = 35^\circ 50' 9,76747$

The sum of these four logs. $19,96772$

The half sum gives $74^\circ 28\frac{1}{2}'$ $9,98386$

Which doubled, gives $148^\circ 57'$ for $\angle ZP\odot$.

And $148^\circ 57'$ reduced to time gives 9h. 55m. 48s. either before or after noon; that is, the twilight begins at 2h. 04m. 12s. in the morning, and ends at 9h. 55m. 48s. in the evening on the given day, at London.

154. When the declination becomes greater than the difference between the co-latitude and 18 degrees, then the parallel of declination nm will not cut the parallel ts 18 degrees below the horizon, and consequently at that time there will be no night at that place, but the twilight will continue from Sun-setting to Sun-rising; and on this account it is, that from the 22d of May to the 21st of July nearly, there is no total darkness at London, the Sun's declination during that interval being greater than $20^\circ 28'$, which is the difference between 18° and $38^\circ 28'$, the complement of the latitude.

155. PRO-

PROBLEM XIII.

Pl. V.

155.

Given the time of the year, the latitude of a place, and the altitude of a known fixed star;

Required the hour of the night when the observation was made.

EXAM. *Some time in the night, on the 1st of September 1780, suppose the star Arcturus, the declination of which is $20^{\circ} 30' N$. should be observed at London to be $27^{\circ} 12'$ above the horizon: At what hour would the observation be made?*

CONSTRUCTION.

Describe the meridian of the place, draw the horizon HR , the zenith and nadir of which are z and N , and describe the parallel of altitude rs at $27^{\circ} 12'$ above the horizon; take P the north pole $51^{\circ} 32'$ above the horizon for the latitude of the place, and s the south pole as much below the horizon; draw the equator EQ , and describe (3d 135) the star's parallel of declination nm ; and where this parallel nm cuts the former rs in $*$, is the position of the star at the time of observation; describe (II. 72) the vertical circle $z*N$, and the hour circle $P*s$, and the angle $zP*$ being measured (IV. 72) gives the hour from, or to, the time of the star's culminating.

COMPUTATION.

In the oblique-angled spheric triangle $P*z$.

Given the co-latitude $PZ = 38^{\circ} 28'$
 the co-altitude $z* = 62 48$ } Required $\angle zP*$, or the hour
 the polar dist. $*P = 69 30$ } from culminating.

Here $P* = 69^{\circ} 30'$
 $PZ = 38 28$

$P* - PZ = 31 \quad z = D$
 $P* = 62 48$

$93 50 \quad 46^{\circ} 55'$
 $2) \quad 31 46 \quad 15 53$

Then Co. ar. fin. co-lat. $= 38^{\circ} 28' \quad 0.20617$
 Co. ar. fin. pol. dist. $= 69 30 \quad 0.02841$
 Sin. $\frac{1}{2}$ sum zen. dist. and $D = 46 55 \quad 9.86354$
 Sin. $\frac{1}{2}$ diff. zen. dist. and $D = 15 53 \quad 9.43724$

The sum of the four logs. 19.53536

The $\frac{1}{2}$ sum gives $35^{\circ} 51' \quad 9.76768$

Which doubled, gives $71^{\circ} 42' = \angle zP*$.

This $71^{\circ} 42'$ turned into time (132) gives 4 h. 46 m. 48 s. for the time which has elapsed since the star was on the meridian.

Now, at the time of observation, September 1st, 1780.

The right ascension of Arcturus was

The right ascension of the Sun at noon †

$14^h \quad 5' \quad 42''$ (133)

$10 \quad 44 \quad 34$

Time of culminating nearly

And 24^h is to $3' 37''$ as $3^h 21 \frac{1}{4}'$ is to

$3 \quad 21 \quad 08$

30

The star souths, or culminates at

The time that the star has passed the meridian

$3 \quad 20 \quad 38$

$4 \quad 46 \quad 48$

The sum is the hour of the night

$8 \quad 07 \quad 26 \text{ P. M.}$

† Astronomical tables at the end of Book V.

And

And whether to subtract or add will always be known by the star's being in the eastern part of the horizon, or ascending; or by being in the western part of the horizon, or descending.

156.

PROBLEM XIV.

Pl. V.

Given the obliquity of the ecliptic, and a star's right ascension and declination;

Required its latitude and longitude.

EXAM. *What is the latitude and longitude of a star, its right ascension being 16 h. 14 m. its declination $25^{\circ} 51' N$. and the obliquity of the ecliptic $23^{\circ} 28'$?*

CONSTRUCTION.

Let the primitive circle represent the solstitial colure, in which draw the equator EQ , mark its poles $P S$, and describe (3d 135) the parallel of the star's declination nm .

The right ascension 16 h. 14 m. $= 243^{\circ} 30'$, which being $63^{\circ} 30'$ above 180° , falls in the third quadrant; therefore make (IV. 75.) $\angle a = 63^{\circ} 30'$, describe (4th 135) the circle of right ascension, cutting the parallel nm in $*$, the point of the heavens representing the star.

Make $\angle \varepsilon = 23^{\circ} 28'$, the obliquity of the ecliptic, draw the ecliptic $\varepsilon \nu s$, find its poles p, q , and through $p, *, q$, describe a circle of longitude; then the arc $p *$ measured (IV. 70) will give the co-latitude, and the $\angle p p *$ will shew the longitude.

COMPUTATION.

In the oblique-angled spheric triangle $p p *$.

Given the obliq. ecliptic $p p = 23^{\circ} 28'$ } Required the co-lat. $p *$.
 the co-declination $p * = 64^{\circ} 09'$ } and the longit. $\angle p p *$.
 the right ascen. $\angle p p * = 243^{\circ} 30'$ } See art. 150, 151. B. IV.

To find the latitude.

As radius $= R$
 To co-f. rt. asc. $= 26^{\circ} 30'$
 So tan. co-decl. $= 64^{\circ} 09'$
 To tan. 4th arc $= 61^{\circ} 34'$
 Obl. eclipt. $= 23^{\circ} 28'$
 Fifth arc $= 38^{\circ} 06'$

10,00000 As co-f. 4th arc $= 61^{\circ} 34'$ 0,32227
 9,95179 To co-f. 5th arc $= 38^{\circ} 06'$ 9,89594
 10,31471 So fine decl. $= 25^{\circ} 51'$ 9,63950
 10,26650 To fin. lat. $= 46^{\circ} 06\frac{1}{2}'$ 9,85771

Here the star's latitude is $46^{\circ} 06\frac{1}{2}' N$. because the declinat. is N., and greater than the obliquity of the ecliptic.

To find the longitude.

As fine 5th arc $= 38^{\circ} 06'$
 To fine 4th arc $= 61^{\circ} 34'$
 So tan. rt. asc. $= 26^{\circ} 30'$
 To tan. longit. $= 234^{\circ} 36'$

0,20969 Here the longitude $144^{\circ} 36'$ being
 9,94417 added to 90° , gives $234^{\circ} 36'$ for the
 9,69774 star's longitude, reckoning from the
 first point of Aries.
 9,85160

For the right ascension being in the third quadrant, the star is there also.

Now in $234^{\circ} 36'$, are 7 signs $24^{\circ} 36'$; that is the star's place, or longitude is $24^{\circ} 36'$ in the 8th sign, or $24^{\circ} 36'$ in m .

By the precession of the equinoxes, the fixed stars, although they always keep the same latitudes, yet are continually altering their longitude,
 right

right ascension, and declination; the alteration in longitude is uniformly 50 seconds and 3-10ths yearly (22), but that of the right ascension and declination is constantly varying: So that many stars, which once had north declination, come to have south; while others change from S. to N. declination.

157.

PROBLEM XV.

Pl. V.

Given the right ascensions and declinations of two fixed stars;
Required their distance.

EXAM. *What is the distance between the fixed stars, Betelguese in the east shoulder of Orion, and Aldebaran in Taurus; the former having $7^{\circ} 21' N.$ declinat. with 5 h. 43 m. 16 s. right ascension; and the other $16^{\circ} 03' N.$ declinat. with 4 h. 23 m. 20 s. of right ascension.*

CONSTRUCTION.

As Aldebaran precedes Betelguese in right ascension, let the primitive circle represent the circle of right ascension passing through Betelguese; describe the circle of right ascension PAS, making with PBS an angle of $19^{\circ} 59'$ (IV. 75) equal to the difference between the given right ascensions.

Describe the parallels of declination Bm, rs, at the given distances $16^{\circ} 03' N.$ and $7^{\circ} 21' N.$ (3d 133); and the intersections A, of Aldebaran's declination, and B, of Betelguese's, with their respective circles of right ascension, will be the positions of those stars from one another: Then draw a great circle BAC, through B and A, and the intercepted arc BA (measured by Art. 70. Book IV.) shews the distance of those two stars.

COMPUTATION.

(IV. 151)

In the oblique-angled spheric triangle PAB.

Given the co-decl. of Ald. $PB = 73^{\circ} 57'$
the co-decl. of Betelguese $PA = 82^{\circ} 39'$
diff. of right ascen. $\angle APB = 19^{\circ} 39'$ } Required their dist. AB.

Radius	= R	10,00000	Co-f. 4th arc	$82^{\circ} 11'$	0,86645
To co-f. diff. rt. af.	$19^{\circ} 59'$	9,97303	To co-f. 5th arc	8 14	9,99550
As co-t. Betelg. dec.	7 21	10,88944	As fin. Betelg. dec.	7 21	9,10697
To tang. 4th arc	82 11	10,86247	To co-f. dist.	21 25	9,96892
Aldeb. co-decl.	73 57		The same result would have come out, had the declination of Aldebaran been used in the proportions.		
Remains 5th arc.	8 14				

158.

PROBLEM XVI.

Pl. V.

Given the latitudes and longitudes of two known fixed stars;
Required their distance.

EXAM. *Aliath, in the Great Bear, Lon. $\text{m} 5^{\circ} 49'$ Lat. $54^{\circ} 18' N.$
Arcturus, in Bootes, Lon. $\text{z} 21^{\circ} 10'$ Lat. $30^{\circ} 54' N.$*

The construction of this Problem is like that of the last; only instead of circles of right ascension read circles of longitude, and use parallels of latitude instead of parallels of declination.

The

The computation is also like that of the last, there being given two co-latitudes and the included angle, which is the difference between the given longitudes: Thus Arcturus's long. is $6^{\circ} 21' 10''$, and Aliath's is $5^{\circ} 5' 49''$; their difference is $1^{\circ} 15' 21''$, or $45^{\circ} 21'$.

Hence the distance will be found to be $39^{\circ} 45'$.

159.

PROBLEM XVII.

Pl. V.

Given the latitude and longitude of a fixed star, and also the obliquity of the ecliptic;

Required the right ascension and declination of that star.

EXAM. Suppose the latitude of a star is $7^{\circ} 09' N.$ its longitude γ $29^{\circ} 01'$: What is the right ascension and declination of that star, the obliquity of the ecliptic being $23^{\circ} 28'$?

The construction of this Problem is much like that of Prob. XIV.; only here the intersection of a parallel of latitude cb , with a circle of longitude pAq , will give the place of the star.

The computation is also as in Prob. XIV; for here are given $pp = 23^{\circ} 28'$, $pA = 82^{\circ} 51'$, and the $\angle pPA = 60^{\circ} 59'$, the longitude from the first point of $\odot$, to find PA the co-declination, and $\angle pPA$ the right ascension.

The declination of the star will be found to be $17^{\circ} 49' N.$ (IV. 151)
And the right ascension will be $24^{\circ} 19'$. (IV. 150)

160.

PROBLEM XVIII.

Pl. V.

Given the meridional altitude of any celestial object, suppose a comet, its distance from a known star, and the latitude of the place;

Required the declination and right ascension of that comet.

EXAM. Suppose a comet was observed on the meridian at London, when its altitude was $51^{\circ} 55'$, and its distance from the star Arcturus was $59^{\circ} 47'$: What was the declination and right ascension of the comet at that time?

CONSTRUCTION.

In the primitive circle, representing the meridian of the place, draw the horizon HR and prime vertical ZN ; lay off the given latitude $RP = 51^{\circ} 32'$, draw the axis PS , the equator EQ , and (3d 135) $n m$ Arcturus's parallel of declination $= 20^{\circ} 21'$. From the south point of the horizon lay off the given altitude of the comet $= 51^{\circ} 55'$ from H to o : About the point o as a pole, at the given distance between the comet and Arcturus, describe (IV. 68) a small circle aa cutting the parallel $n m$ in $*$, the position of Arcturus at that time: Describe the circle of right ascension $p * s$, and a great circle through o and $*$.

COMPUTATION.

Since $HE = 38^{\circ} 28'$, the co-lat. and the alt. $HO = 51^{\circ} 55'$, then $EO = (HO - HE =) 13^{\circ} 27'$, is the decl. sought; which is north, as the altitude exceeds the co-lat.; consequently the polar distance $OP = 76^{\circ} 33'$.

Then

Then in the triangle $p*o$ are given the three sides to find the $\angle op*$, the difference between the right ascensions of the comet and Arcturus. And (IV. 154) the $\angle op*$ will be found $= 62^\circ 24' = 4h. 9m. 36s.$ which is the difference of their right ascensions: Now if Arcturus had passed the meridian, the right ascension of the comet was $18h. 15m. 16s.$ but if Arcturus had not passed the meridian, the right ascension of the comet was $9h. 56m. 4s.$; it being, in the former case, equal to the sum, and in the latter to the diff. of Arcturus's right ascen. and the $\angle op*$.

161.

PROBLEM XIX.

PL V.

Given the latitude of a place, the Sun's declination and Azimuth;
Required his altitude and the time of the observation.

EXAM. In the latitude of $13^\circ 30' N.$ and when the Sun has $23^\circ 28' N.$ declination: What is the Sun's altitude and time of the day, when he is seen on the ENE azimuth circle?

CONSTRUCTION.

Let the primitive circle represent the meridian of the place, in which HR represents the horizon, and ZN the prime vertical; make RP equal to the latitude, draw the 6 o'clock hour circle PS, the equator EQ, and (3d 153) the parallel of $23^\circ 28'$ of declination nm : The tangent of $67^\circ 30'$ being laid from the center a toward H, gives the center of the vertical circle ZDN, which cuts the parallel nm in the points A and B; and shews that at two distant times in the forenoon the Sun will have the azimuth proposed: Through the point A and B describe the hour circles PAS, PBS (II. 72); the angles ZPA, ZPB, shewing the times from noon, may be measured by Art. 72. Book IV.; and the altitudes DA, DB, by Art. 70. Book IV.

COMPUTATION. See Art. 146. Book IV.

In the spheric triangle PZA, or PZB, there are known,

The co-lat. $PZ = 76^\circ 30'$; the co-decl. PA , or $PB = 66^\circ 32'$; the azim. $\angle PZD = 67^\circ 30'$.

To find ZA, or ZB, and the $\angle ZPA$, or $\angle ZPB$.

As radius : co-f. azimuth :: co-t. lat. : to tan. of a 4th arc $M = 57^\circ 54'$.

And as fin. lat. : fin. decl. :: co-f. M to co-f. of a 5th arc $N = 24^\circ 56'$.

Then $M + N$, or $57^\circ 54' + 24^\circ 56' = 82^\circ 50' = ZA$, is the comp. of least alt.

And $M - N$, or $57^\circ 54' - 24^\circ 56' = 32^\circ 58' = ZB$, is the comp. of the gr. alt.

Therefore, when the Sun has $7^\circ 10'$, or $57^\circ 02'$, of alt. he is on the given azimuth.

Again, As co-f. dec. : fin. azim. :: co-f. least alt. : fin. hour fr. noon $87^\circ 55'$.

And as co-f. dec. : fin. azim. :: co-f. greater alt. : fin. h. fr. noon $33^\circ 14'$.

But $87^\circ 55' = 5^h 51^m 40^s$; and $33^\circ 14' = 2^h 12^m 56^s$, the respective times from noon.

Consequently the Sun will be seen on the ENE azimuth at 6 h. 08 m. 20 s. and again at 9 h. 47 m. 4 s. both in the morning: Also, in the afternoon he will be on the WNW azimuth at 2 h. 12 m. 56 s. and at 5 h. 51 m. 40 s.

162. Now to find at what time, and at what altitude, the greatest azimuth will happen at that place on the said day;

As the azimuth circle in this case is to touch the parallel nm , therefore the greatest distance of the azimuth from the equator will be $23^\circ 28'$; and as their poles must be at the same distance (IV. 33) therefore a small circle rs described about the pole s , at the distance of $23^\circ 28'$ (IV. 66),

its

its intersection p with the horizon, is the pole of the circle zcn ; then describe an hour circle pcs through p .

In the spheric triangle zpc right angled at c .

Given the co-lat. $pz = 76^\circ 30'$, and the co-decl. $pc = 66^\circ 32'$. (IV. 34)

Required the greatest azimuth. $\angle pzc = 70^\circ 37'$, the dist. $zc = 54^\circ 08'$, and the hour from noon $= 56^\circ 26' = 3$ h. 45 m. 44 s.

So that the azimuth is altered only $3^\circ 7'$ in 2 h. 6 m.; and consequently the variation of the compass may be observed with more certainty in the torrid zone than elsewhere.

163.

PROBLEM XX.

Pl. XIV. Fig. 1.

In the latitude of $20^\circ 00'$ N. stands a horizontal dial, the gnomon of which is perpendicular to the plane of the horizon: It is required to know at what hour in the afternoon on the longest day, the shadow of that gnomon shall stand still; and how many degrees shall the shadow run back.

CONSTRUCTION. Let the circle $zRNH$ be the meridian of the place, HR the horizon, the poles of which are z, N ; $RP = 20^\circ$, the latitude P and s the north and south poles: About P describe the tropic of Cancer aa , cutting the horizon in L ; about s , a small circle being described at the dist. of $23^\circ 28'$, the complement of the dist. of aa from P , its intersection p with the horizon, is the pole of the azimuth circle which will touch the parallel aa in $\odot$, the place of the Sun when he has the greatest azimuth that day: Through $z, \odot, N$, describe a vertical circle cutting the horizon in K ; and through $\odot$ and L describe the hour circles $P\odot s, PLS$.

Then will the $\angle ZP\odot$ be equivalent to the hour when the shadow will stand still; and KL , the difference between the measures of the azimuth and amplitude, will shew how much the shadow will run back.

COMPUTATION. In the right-angled spheric triangle PRL .

Given $\angle R = 90^\circ 00'$
 lat. $= RP = 20^\circ 00'$
 co-decl. $= PL = 66^\circ 32'$
 Required. ampl. $= RL = 64^\circ 56'$

In rt. angl. spheric triangle $P\odot z$.

Given $\angle \odot = 90^\circ 00'$
 co-lat. $= zP = 70^\circ 00'$
 co-decl. $= P\odot = 66^\circ 32'$
 Required hour $= \angle ZP\odot = 33^\circ 06'$
 azimuth $= \angle \odot zP = 77^\circ 28'$

Then $33^\circ 02' = 2$ h. 12 m. 08 s.: And $77^\circ 28' - 64^\circ 56' = 12^\circ 32'$.

So that the shadow will stand still at 2 h. 12 m. 08 s. and will run back $12^\circ 32'$.

164.

PROBLEM XXI.

Pl. XIV. Fig. 2.

A comet, the declination of which was $47^\circ 00'$ N. was observed to be distant from a star, to the eastward of it, $49^\circ 00'$; the star's declination was $36^\circ 00'$ N. and its right ascension $45^\circ 00'$: What was the latitude and longitude of that comet?

CONSTRUCTION.

On the plane of the solstitial colure, where P and E are the poles of the equator and ecliptic, put the star at A by its right ascension and declination: About P and A describe small circles, at the distances of the comet from those points, their intersection $\odot$ gives the place of the comet; describe great circles through $A\odot, P\odot, E\odot$, then $E\odot$ will be the co-latitude, and $\angle PE\odot$ the co-longitude; and their measures may be obtained from Articles 70 and 72 of Book IV.

COMPU-

COMPUTATION. In the triangle
 $\triangle APO$.
 Given the $\star$'s co-decl. $PA = 54^\circ 00'$
 comet's co-decl. $PO = 43^\circ 00'$
 their distance $AO = 49^\circ 00'$

Req. com. rt. asc. fr. $\star \angle APO = 65^\circ 48'$
 Then $65^\circ 48' - 45^\circ 00' = 20^\circ 48'$.
 And $90^\circ 00' - 20^\circ 48' = 69^\circ 12' = \angle POE$

In the triangle $EP\odot$.
 Given comet's co-decl. $PO = 43^\circ 00'$
 obliq. of ecliptic $PE = 23^\circ 28'$
 comet's rt. asc. $\angle EP\odot = 69^\circ 12'$

Req. comet's co-lat. $E\odot = 39^\circ 54'$
 and co-longit. $\angle PE\odot = 83^\circ 42'$
 Which being taken from 90° , leaves
 $\gamma 6^\circ 18'$ for the long. required.

165.

PROBLEM XXII. Pl. XIV. Fig. 3.

At London, on the 10th of December 1780, at what time of the night will the stars Aldebaran and Rigel be on the same azimuth circle?

Aldeb. decl. $= 16^\circ 03' N.$; right asc. $= 4 h. 23 m. 19 s.$ Rigel's decl. $= 8^\circ 28' S.$; right asc. $= 5 h. 03 m. 58 s.$

Their difference of right ascensions is $40 m. 39 s.$ or $10^\circ 10'$.

CONSTRUCTION. On the plane of the equinoctial put Aldebaran at A , and Rigel at B , by their right ascensions and declinations, then a great circle through B and A will be the azimuth they are on at the time sought; and the parallel of London's lat. described about P , will cut the azimuth circle BA in z the zenith, through which draw the meridians PZ, PZ .

Here the nearest intersection to the stars is taken for the zenith, for as the stars are both above the horizon, the greatest zenith distance is less than 90 degrees.

COMPUTATION. In the $\triangle APB$.
 Given B 's co-decl. $PB = 98^\circ 28'$
 A 's co-decl. $PA = 73^\circ 57'$
 diff. rt. asc. $\angle APB = 10^\circ 10'$

In the triangle APZ .
 Given A 's co-decl. $PA = 73^\circ 57'$
 the co-lat. $PZ = 38^\circ 28'$
 the suppl. of BAP or $\angle PAZ = 23^\circ 02'$

Required the $\angle BAP = 156^\circ 58'$ Req. $\angle APZ = 142^\circ 35'$, or $= 24^\circ 01'$

Now the star Aldebaran comes to the meridian at $11 h. 8 m. 27 s.$ in the evening; which lessened by $1 h. 36 m. 4 s.$ ($24^\circ 01'$) gives $9 h. 32 m. 23 s.$ for the time in the evening when those stars will be on the same azimuth.

166.

PROBLEM XXIII. Pl. XIV. Fig. 4.

At what time in the evening will the stars Betelguese and Pollux have one common altitude above the horizon of London, on the 10th of December 1780?

Betelguese right ascen. $= 5 h. 43 m. 16 s.$; decl. $= 7^\circ 21' N.$ Pollux right. ascen. $= 7 h. 31 m. 51 s.$; decl. $= 28^\circ 32' N.$

Their difference of right ascension is $1 h. 48 m. 35 s.$ $= 27^\circ 09'$.

CONSTRUCTION. On a primitive circle, where any point P represents the pole of the equinoctial, put the star Pollux at B , and Betelguese at A , by their declination and difference of right ascensions; through A, B , describe a great circle; through c , the middle of AB , describe a great circle at right angles to AB , and cutting PA in D ; then a small circle described about P , at the distance of $38^\circ 28'$, the co-lat. will cut the circle CD in z the zenith.

For the stars A and B having a common altitude, are equally distant from z .

COMPUTATION. In the triangle APB.

Given A's co-decl.	PA = 82° 39'	} Required the	∠ BAP = 47° 01'
B's co-decl.	PB = 61 28		AB = 33 14
diff. rt. asc. ∠ APB = 27 09			AC = 16 37

In the triangle ACD.

Given the	∠ C = 90° 00'	} Required the	∠ D = 45° 30'
	∠ A = 47 01		AD = 23 38
	AC = 16 37		Theref. PA—AD=PD = 59 01

In the triangle PDZ.

Given the co-lat.	PZ = 38° 28'	} Required	∠ ZPD = 48° 56'	
	PD = 59 01		Or it is	= 75 46
	∠ PDZ = 45 30			

Now the star Betelgeuse comes to the meridian at 12 h. 28 m. 8 s. that is between twelve and one o'clock in the morning (133); from which take 3 h. 15 m. 44 s. as the stars are to the east of the meridian, and it leaves 9 h. 12 m. 24 s. in the evening, for the time when those stars have the same altitude.

167. PROBLEM XXIV. Pl. XIV. Fig. 5.

Wanting to know the latitude and longitude of a comet c, its distance from two known stars A and B were observed, and are as follows:

A's lat. = 49° 12' N. Lon. = 16° 39' X; distance from c = 49° 05'.

B's lat. = 30 05 N. Lon. = 2 48 II; distance from c = 45 57.

Hence the place of the comet c is required.

CONSTRUCTION. On the plane of the solstitial colure, where E is the pole of the ecliptic, put the stars A and B by their latitudes and longitudes, and describe a great circle through A and B; then small circles described about A and B as poles, at the respective distances of the comet, their intersection c will give its place; describe great circles through A, c; B, c; E, c; and E c will be the co-latitude, and from ∠ AEC will be obtained the longitude.

COMPUTATION. In the Δ ABE.

Given A's co-lat.	AE = 40° 48'
B's co-lat.	BE = 59 55
diff. longit. ∠ AEB = 76 09	

In the triangle ABC.

Given the distance	AB = 59° 01'
distance	AC = 49 05
distance	BC = 45 57

Required	AB = 59 01
and ∠ EAB = 78 31	

Required the	∠ BAC = 56 27
Then ∠ EAB + ∠ BAC = ∠ EAC = 134° 58	

In the triangle CEA.

Given A's co-lat.	AE = 40° 48'	} Required the co-lat.	EC = 81° 33'
distance	AC = 49 05		diff. long. AEC = 32 43
and the ∠ EAC = 134 58			Hence lat. is 8° 27' N. lon. 19° 22' in V

168. PROBLEM XXV. Pl. XIV. Fig. 6.

The distance of the star c being observed from two stars A and B, the latitude and distance of which are known, and also the longitude of one of them; thence to find the lat. and long. of c.

Suppose A's lat. to be 5° 30' N.; its dist. from c = 39° 40'; B's lat. 9° 57' N. its long. Taurus, 18° 16', and dist. from c 10° 7½'. And the distance of AB 44° 43': Required the long. and lat. of c.

CON.

CONSTRUCTION. On a circle of longitude, where E is the pole of the ecliptic, put the star B by its lat.; about the points B and E describe circles at the distances of A from those points, their intersection gives the place of A : Also circles described from A and B , at the distances of C respectively from them, their intersection is the place of C : Then describe the great circles EA , EC ; AC , AB ; BC ; and EC will be the co-lat. and $\angle BEC$ the longitude, of C from B .

COMPUTATION. In the $\triangle AEB$.		In the triangle ABC .	
Given A 's co-lat.	$AE = 84^\circ 30'$	Given the distance	$AB = 44^\circ 43'$
B 's co-lat.	$BE = 80^\circ 03'$	distance	$AC = 39^\circ 40'$
the distance	$AB = 44^\circ 43'$	distance	$BC = 10^\circ 07\frac{1}{2}'$
Required the $\angle ABE = 92^\circ 14'$		Required the $\angle ABC = 55^\circ 22'$	

In the triangle BEC .		In the triangle ABC .	
Given B 's co-lat.	$BE = 80^\circ 03'$	Required C 's co-lat.	$CE = 72^\circ 01'$
the distance	$BC = 10^\circ 07\frac{1}{2}'$	C 's lon. fr. B , $\angle BEC =$	$6^\circ 22'$
$(\angle ABE - \angle ABC) \angle CBE = 36^\circ 52'$		And its absolute long. $\odot 11^\circ 54'$	

169. **PROBLEM XXVI.** Pl. XIV. Fig. 7.

From the altitudes of two known fixed stars, and the altitude of a planet when in the same azimuth with one of these stars; to find the place of the planet.

EXAMPLE. Observed the Moon and Cor Leonis in the same azimuth, when the Moon's zenith distance was $36^\circ 37'$.

Cor Leonis's zen. dist. $= 45^\circ 00'$; decl. $13^\circ 02' N.$; rt. af. $9^h 56^m 39^s$.

Cor Hydra's zen. dist. $= 49^\circ 16'$; decl. $7^\circ 43' S.$; rt. af. $9^h 16^m 47^s$.

CONSTRUCTION. On the plane of the equinoctial, the pole of which is P , draw the colures, and in the solstitial, take E for the pole of the ecliptic; put the given stars at B and A by their declinations and right ascensions: About B and A as poles, with their respective zenith distances, describe circles cutting in Z the zenith; through Z and B describe an azimuth circle, and making ZD equal to the D 's zenith distance, it gives her place: Then describe the great circles ZA , AB , ED ; and the arc ED will be the co-latitude, and $\angle PED$ the longitude from the first point of $\odot$.

COMPUTATION. In the $\triangle ABP$.		In the triangle AZB .	
Given A 's co-decl.	$PA = 97^\circ 43'$	Given A 's zen. dist.	$ZA = 49^\circ 16'$
B 's co-decl.	$PB = 76^\circ 58'$	B 's zen. dist.	$ZB = 45^\circ 00'$
diff. rt. af.	$\angle BPA = 9^\circ 58'$	the side	$AB = 22^\circ 59\frac{1}{2}'$
Required the $\angle ABP = 153^\circ 57\frac{1}{2}'$		Required the $\angle ABZ = 89^\circ 38\frac{1}{2}'$	
the side $AB = 22^\circ 59\frac{1}{2}'$		Then $\angle ABP - \angle ABZ = \angle ZBP =$	
		$64^\circ 19' = \angle DBP$.	

In the triangle BDP .		In the triangle PDE .	
Given B 's co-decl.	$BP = 76^\circ 58'$	Given obl. of eclip.	$PE = 23^\circ 28'$
$(BZ - ZD =)$ side BD	$= 8^\circ 23'$	D 's co-decl.	$PD = 73^\circ 26\frac{1}{2}'$
$\angle PBD$	$= 64^\circ 19'$	D 's co-rt. af.	$\angle EPD = 128^\circ 43'$
Requ. D 's co-decl. $PD = 73^\circ 26\frac{1}{2}'$		Requir. D 's co-lat. $DE = 88^\circ 41'$	
D 's rt. af. fr. B , $\angle BPD = 7^\circ 53'$		D 's lon. fr. $\odot$, $\angle PED = 48^\circ 25'$	
Then $90^\circ + \angle BPD + \angle BPD =$		And its absolute long. is $\Omega 18^\circ 25'$	
$\angle EPD = 128^\circ 43'$.			

SECTION VI.

Of various methods to find the Latitude.

The usual way at sea to find the latitude is from the Sun's meridional altitude and declination; the manner of doing this will be particularly shewn in Book IX. But as it frequently happens at sea, that the meridian altitude cannot be taken, therefore the mariner should be furnished with other means to come at the knowledge of this most useful article. To help him in this point, and as a farther exercise in the Astronomy of the Sphere, the following Problems are collected together.

170.

PROBLEM XXVII.

Pl. V.

Given the Sun's declination and his amplitude;
Required the latitude of the place.

EXAM. *Being in a place where the compass had no variation, on a day when the Sun's declination was $15^{\circ} 12' N.$, I observed him to rise $62^{\circ} 30'$ from the north towards the east: Required the latitude of that place.*

CONSTRUCTION.

Having described the primitive circle, drawn the horizon HR, and (IV. 71) taken $RO = 62^{\circ} 30'$; then about O as a pole describe (IV. 66) a small circle, at the distance of $74^{\circ} 48' = \text{co-decl.}$ cutting the primitive in P, the place of the north pole: Draw the axis PS, the equator EQ, and the circle POS, cutting the equator in A.

COMPUTATION.

In the spheric triangle γOA right-angled at A.

Given the co-amp. $\gamma O = 27^{\circ} 30'$	Then $\text{f. } \gamma O : \text{rad.} :: \text{f. } AO : \text{f. } \angle A \gamma O.$ Hence the latitude will be $55^{\circ} 24' N.$
the declin. $AO = 15^{\circ} 12'$	
Required the co-latitude $A \gamma O.$	

171.

PROBLEM XXVIII.

Pl. V.

Given the Sun's declination, and his ascensional difference;
Required the latitude of the place.

EXAM. *When the Sun had $20^{\circ} 01' S.$ of declination S. he was observed to set at 4 h. 30 m.: Required the latitude of the place.*

As the ascensional difference is the time that the Sun rises or sets before or after 6 o'clock; therefore $6 \text{ h.} - 4 \text{ h. } 30 \text{ m.} = 1 \text{ h. } 30 \text{ m.} = 22^{\circ} 30' = \text{ascensional difference.}$

CONSTRUCTION. In the primitive circle representing the meridian of the place, draw the equator EQ, the axis PS, the parallel of declination ro , $20^{\circ} 01' S.$: Make $\gamma B = 22^{\circ} 30'$, the ascensional difference; describe the circle of right ascension PBS, cutting ro in O; then a diameter HR through O will be the horizon, and RP the lat. sought.

COMPUTATION. In the spheric triangle γBO , right-angled at B.

Given the asc. diff. $\gamma B = 22^{\circ} 30'$	Then $\text{rad.} : \cot. OB :: \text{f. } \gamma B : \cot. \angle O \gamma B.$ Or $\text{rad.} : \text{co-t. dec.} :: \text{fin. asc. diff.} : \tan. \text{lat.}$ Hence the lat. will be $46^{\circ} 25' N.$ being contrary to the decl. when the asc. diff. falls between noon and fix.
the declin. $OB = 20^{\circ} 01'$	
Required the co-lat. $\angle O \gamma B.$	

P R O B L E M XXIX.

Pl. V.

172.

Given the Sun's declination, and altitude at six o'clock;
Required the latitude of the place.

EXAM. *Being at sea, on a day when the Sun's declination was $20^{\circ} 04'$ N. his altitude at six o'clock in the evening was $18^{\circ} 45'$: What was the latitude of the place of observation?*

CONSTRUCTION. Having described the meridian, drawn the horizon HR, the prime vertical ZN, and the parallel st of $18^{\circ} 45'$ of altitude; from the center γ , with the half tangent of the declination $= 20^{\circ} 04'$, cut the parallel st in o : Through o draw the axis ps , and the azimuth circle ZON (II. 72), and the measure of RP will give the latitude sought.

COMPUTATION. In the spheric triangle γAO , right-angled at A .

Given the decl. $\gamma o = 20^{\circ} 04' N.$ Then $\sin. \gamma o : \text{rad.} :: \sin. AO : \sin. \angle o \gamma A$:
the alt. $AO = 18^{\circ} 45'$ Or $\sin. \text{decl.} : \text{rad.} :: \sin. \text{alt.} : \sin. \text{lat.}$

Required the lat. $= \angle o \gamma A$ Which is $69^{\circ} 32' N.$ as the decl. is $N.$

P R O B L E M XXX.

Pl. V.

173.

Given the Sun's declination, and his altitude when due east or west;
Required the latitude of the place.

EXAM. *In a place where the compass had no variation, the Sun was observed to be due east when his declination was $16^{\circ} 38' N.$, and his altitude $20^{\circ} 12'$: What is the latitude of that place?*

CONSTRUCTION. In the meridian HZRN, draw the horizon HR, the prime vertical ZN, and make $\gamma o =$ the half tan. of the alt. $20^{\circ} 12'$: About o as a pole, at the distance of $73^{\circ} 22'$, the co-decl. describe (IV. 66) a small circle, cutting the meridian in P the elevated pole; draw the axis ps , equator EQ , and through P, o, s , describe an hour circle Pos ; then the measure of PR shews the latitude.

COMPUTATION. In the spheric triangle γAO , right-angled at A .

Given the alt. $\gamma o = 20^{\circ} 12'$ Then $\sin. \gamma o : \text{rad.} :: \sin. AO : \sin. \angle A \gamma o$.
the decl. $AO = 16^{\circ} 38' N.$ Or $\sin. \text{alt.} : \text{rad.} :: \sin. \text{decl.} : \sin. \text{lat.}$

Required the latit. $= \angle A \gamma o$. Which is $56^{\circ} 00' N.$ as the decl. is $N.$

But had the declination been $S.$, the other intersection of the parallel circle and meridian must have been taken for the elevated pole, and the latitude would be south.

P R O B L E M XXXI.

Pl. V.

174.

Given the Sun's altitude and the hour of the day on either equinox;
Required the latitude of the place.

EXAM. *On the day the Sun entered the vernal equinox, his alt. was found $22^{\circ} 56'$ at 9 o'clock in the morning: In what lat. was that observation made?*

CONSTRUCTION. Describe the meridian, draw the horizon, the prime vertical, and (IV. 68) the parallel st of $22^{\circ} 56'$ of altitude; from the center γ , with the half tan. of $45^{\circ} = 3$ h., the time from 6 o'clock, cut st in o , and describe the vertical circle ZON, cutting the horizon in B .

COMPUTATION. In the spheric triangle γBO , right-angled at B .

Given the time aft. 6, $\gamma o = 45^{\circ} 00'$ As $\sin. \gamma o : \text{rad.} :: \sin. BO : \sin. \angle B \gamma o$.

the altitude $BO = 22^{\circ} 56'$ Or $\sin. \text{time à 6} : \text{rad.} :: \sin. \text{alt.} : \text{co-f. l.}$

Required the co-latitude $B \gamma o$. Which is $56^{\circ} 34'$.

175.

PROBLEM XXXII.

Pl. V.

Given the Sun's altitude, declination, and azimuth;
Required the latitude of the place.

EXAM. *Being at sea in a place where the compass had no variation, in the afternoon when the Sun was $42^{\circ} 30'$ high, his bearing was S. $57^{\circ} 45'$ W. and his declination $22^{\circ} 30'$ N.: What is the latitude of that place?*

CONSTRUCTION. Draw the meridian, the horizon HR, the prime vertical ZN, and the parallel st at $42^{\circ} 30'$ above the horizon (IV. 68): The tangent of $57^{\circ} 45'$ set from A towards R gives the center of the azimuth circle ZON, cutting the parallel of altitude st in o: About o as a pole (IV. 66), at the distance of $67^{\circ} 30'$, equal to the co-declination, describe a small circle, cutting the meridian in P, the place of the pole; then the measure of RP gives the latitude sought.

COMPUTATION. In the oblique-angled spheric triangle ZOP.

Given the zenith dist. $zo = 47^{\circ} 30'$
the polar dist. $po = 67^{\circ} 30'$ } Required the co-latitude pz.
the azimuth $\angle PZO = 122^{\circ} 15'$

As rad.	= R	10,00000	As fin. alt.	= $42^{\circ} 30'$	0,17032
To co-f. azim.	= $122^{\circ} 15'$	9,72723	To fin. decl.	= $22^{\circ} 30'$	9,58284
So co-t. alt.	= $42^{\circ} 30'$	10,03795	So co-f. 4th.	= $30^{\circ} 13'$	9,93658
To tan. 4th.	= $30^{\circ} 13'$	9,76518	To co-f. 5th.	= $60^{\circ} 42'$	9,68974

Then the difference between the 5th and 4th arcs, that is $30^{\circ} 13'$ taken from $60^{\circ} 42'$, the remainder $30^{\circ} 29'$ is the co-lat. Therefore $59^{\circ} 31'$ N. is the latitude sought.

176.

PROBLEM XXXIII.

Pl. V.

Given the Sun's declination, his altitude, and the hour of the day;
Required the latitude of the place.

EXAM. *Being at sea, the Sun's altitude was observed to be $37^{\circ} 20'$ at 9 h. 45 m. in the morning, his declination at that time being $22^{\circ} 30'$ N.: What is the latitude of the place of observation?*

CONSTRUCTION. In the meridian PFSQ, draw the equator EQ, axis PS, and parallel of declin. nm , $22^{\circ} 30'$ dist. from the equator (3d 135). Set off from the center A towards Q the tang. of $33^{\circ} 45' = 2$ h. 15 m., the distance between the time of observation and noon, which gives the center of the hour circle POS, cutting the parallel nm in o: about the point o as a pole, describe (IV. 66) at the dist. of $52^{\circ} 40'$, the zen. dist., a small circle, cutting the meridian in z the zenith; through z o describe an azimuth circle ZON; then the measure of ZE will give the lat. sought.

COMPUTATION. In the oblique-angled triangle ZOP.

Given the zenith distance $zo = 52^{\circ} 40'$
the polar distance $po = 67^{\circ} 30'$ } Required the co-lat. zp.
the hour from noon $\angle ZPO = 33^{\circ} 45'$

As rad. : co-f. hour A. M. :: co-t. decl. : tan. 4th arc $= 63^{\circ} 31'$.

As fin. decl. : fin. alt. :: co-f. 4th arc : co-f. 5th arc $= 45^{\circ} 02'$.

Their difference is the co-latitude $18^{\circ} 29'$. Therefore the lat. is $71^{\circ} 31'$ N.

177. PRO-

PROBLEM XXXIV.

Pl. V.

177.

Given the altitude of one of two known fixed stars, when they have the same azimuth;

Required the latitude of the place.

EXAM. Being at sea in an unknown latitude, I observed the star Schedar in Cassiopeia, and Almaach in Andromeda, to have the same azimuth, when the altitude of Schedar was $37^{\circ} 15'$: What is the latitude of that place?

CONSTRUCTION. Let the primitive circle represent the equator, the pole of which is P, and any point V the place where the right ascension begins, from whence lay off $Va = 27^{\circ} 37'$ for Almaach's right ascension, and $Vb = 7^{\circ} 2'$ for Schedar's; draw the circles of right ascension Pa, Pb: Describe (3d 137) Almaach's and Schedar's parallels of declination, cutting Pa, Pb, in A, B; A being Almaach, and B Schedar. A great circle passing through A and B (IV. 61) will be the azimuth they are on. About B at the distance of $52^{\circ} 45'$, Schedar's zenith distance, describe (IV. 66) a small circle, cutting the said azimuth circle in Z, the zenith of the place; draw PZ, which measured on the half tangents gives the co-latitude of the place of observation.

COMPUTATION.

1st. In the oblique-angled spheric triangle APB.

Given Almaach's co-dec. $PA = 48^{\circ} 44'$ Required the angle of position APB.
 Schedar's co-dec. $PB = 34^{\circ} 40'$
 their diff. of r. asc. $\angle APB = 20^{\circ} 33'$ For the solution, see IV. 165.

As rad.	$90^{\circ} 00'$	10.00000	As fin. 5th arc	$12^{\circ} 11'$	0.67563
To co-f. $\angle APB$	$20^{\circ} 35'$	9.97135	To fin. 4th arc	$46^{\circ} 51'$	9.86366
So is tan. AP	$48^{\circ} 44'$	10.05676	So is tan. $\angle APB$	$20^{\circ} 35'$	9.57466
To tan. 4th arc	$46^{\circ} 51'$	10.02811	To tan. $\angle B$	$52^{\circ} 23'$	10.11335
The side BP	$34^{\circ} 40'$				
The 5th arc	$12^{\circ} 11'$				

2d. In the oblique spheric triangle PBZ.

Given Schedar's co-dec. $PB = 34^{\circ} 40'$ Required the co-lat. PZ.
 Schedar's co-alt. $BZ = 52^{\circ} 45'$ For the solution, see IV. 165.
 angle of position PBZ = $52^{\circ} 23'$ Here $\angle PBZ = \text{sup. } \angle PBA$.

As rad.	= R	10.00000	As co-f. 4th arc	= $22^{\circ} 53'$	0.03560
To co-f. $\angle \text{posit.}$	= $52^{\circ} 23'$	9.78560	To co-f. 5th arc	= $29^{\circ} 52'$	9.93811
So co-t. Sch. decl.	= $55^{\circ} 20'$	9.83984	So fin. Sch. dec.	= $55^{\circ} 20'$	9.91512
To tan. 4th arc	= $22^{\circ} 53'$	9.62544	To fin. latit.	= $5^{\circ} 44'$	9.88883
Which taken from	$52^{\circ} 45' = BZ$				
Leaves 5th arc	$29^{\circ} 52'$				

178.

PROBLEM XXXV.

Pl. V.

Given the difference of time between the rising of two known stars;
Required the latitude of the place.

EXAM. *Being at sea in an unknown place, the star Aldebaran was observed to rise 3 h. 15 m. later than the bright star in Aries: Required the latitude of that place.*

Bright star in γ decl. $22^{\circ} 25' N.$; right ascen. 1 h. 54 m. 49 s. Aldebaran's decl. $16^{\circ} 03' N.$; right ascension 4 h. 23 m. 19 s.

CONSTRUCTION.

Let the primitive circle represent the equator, describe (3d 137) the parallels of declination of the two stars, that of Aries being $22^{\circ} 25' N.$, and of Aldebaran $16^{\circ} 03' N.$: Draw pa for the circle of right ascension passing through the star in γ , which suppose in A : From a lay off ab , $= 37^{\circ} 7\frac{1}{2}' = \text{diff. of right ascensions}$; and $ac = 48^{\circ} 45' = 3 \text{ h. } 15 \text{ m. the diff. of time between their rising}$; draw bp , cp , cutting the parallels of declination in B , C : Through the points B , C , describe (IV. 61) the great circle HOR ; draw po at right angles to HR ; then the measure of po will give the latitude sought.

COMPUTATION.

In the oblique-angled spheric triangle PBC .

Given B 's co-decl. $PB = 73^{\circ} 57'$	Rad.: co-s. $\angle BPC :: t. PB : t. M = 73^{\circ} 38'$
A 's co-decl. $PC = 67^{\circ} 35'$	From M take PC , leaves $N = 6^{\circ} 03'$
$\angle APC - \angle APB = \angle BPC = 11^{\circ} 37\frac{1}{2}'$	As fin. $N : f. M :: \tan. \angle CPB : t. \angle C =$
Required the angle PCB	Hence $\angle PCO = 61^{\circ} 54'$. [118° 06']

In the spheric triangle PCO , right-angled at O .

Given C 's co-decl. $PC = 67^{\circ} 35'$	As rad. : fin. $PC. :: \sin. \angle PCO : \sin.$
and the $\angle PCO = 61^{\circ} 54'$	[$PO = 54^{\circ} 38'$]
Required the latitude PO .	Therefore the latitude is $54^{\circ} 38'$.

179.

PROBLEM XXXVI.

Pl. V.

Two known fixed stars being observed to have the same altitude;
Required the latitude of the place of observation.

EXAM. *In the evening, the stars Capella and Procyon were observed at the same time to have each $38^{\circ} 00'$ of altitude: Required the latitude of the place where that observation was taken.*

Capella's decl. $= 45^{\circ} 45' N.$ right ascen. 5 h. 0 m. 28 s. Procyon's decl. $= 5^{\circ} 47' N.$ right ascen. $= 7 \text{ h. } 27 \text{ m. } 48 \text{ s.}$

CONSTRUCTION.

On the plane of the equator, represented by the primitive circle, describe (3d 137) the parallels of the given declinations; take $ab = 36^{\circ} 50'$ the diff. of the given right ascensions; draw pa , pb , then A represents Procyon, and B Capella.

About A and B as poles, describe (IV. 66) arcs of small circles, at the distance of 52° , the co-altitude, and their intersection gives Z the zenith of the place: Through the points A , B ; Z , A ; Z , B ; describe great circles (IV. 61) and draw ZP , the measure of which will give the co-latitude of the place of observation.

COMPU-

COMPUTATION.

In the oblique angled spheric triangle APB.

Given A's co-decl. $AP=84^{\circ} 13'$ Rad. : cof. $\angle P :: t. BP : t. M=37^{\circ} 57'$
 B's co-decl. $BP=44^{\circ} 15'$ And M tak. fr. PA leaves $N=46^{\circ} 16'$
 diff. rt. asc. $\angle APB=36^{\circ} 50'$ Sin. N : sin. M :: tan. $\angle P : \tan \angle BAP$
 Required the star's distance AB. $[=32^{\circ} 31'$
 And the angle BAP. $\text{Cof. } M : \text{cof. } N :: \text{cof. } BP : \text{cof. } BA=51^{\circ} 06'$

In the oblique angled spheric triangle AZB.

Given A's co-alt. $ZA=52^{\circ} 00'$ The angle BAZ will be found =
 B's co-alt. $ZB=52^{\circ} 00'$ $68^{\circ} 04'$.
 stars distance $AB=51^{\circ} 06'$ Then $\angle BAZ - \angle BAP = \angle PAZ =$
 Required the angle ZAB. $35^{\circ} 33'$.

In the spheric triangle AZP.

Given A's co-alt. $ZA=52^{\circ} 00'$ Rad. : cof. $\angle A :: t. ZA : t. M=47^{\circ} 29'$
 A's co-declin. $AP=84^{\circ} 13'$ M taken from PA leaves $N=36^{\circ} 44'$
 the angle $ZAP=35^{\circ} 30'$ Cof. M : cof. N :: cof. ZA : cof. PZ = $43^{\circ} 06'$
 Required the co-lat. ZP. Therefore the latitude is $46^{\circ} 54' N$.

180.

PROBLEM XXXVII.

Pl. V.

Given the altitudes of two known stars ;

Required the latitude of the place.

EXAM. The altitude of the Hydra's heart was observed to be $40^{\circ} 44'$, and of the Lion's heart $45^{\circ} 00'$ What is the latitude of the place of observation ?

Hydra's heart, decl. = $7^{\circ} 43' S$. ; right ascen. = 9 h. 16 m. 47 f.
 Lion's heart, decl. = $13^{\circ} 02' N$. ; right ascen. 9 h. 56 m. 39 f.

CONSTRUCTION.

If this problem is constructed on the plane of the equator, it will be in every respect like the last ; only the small circles, described about A and B, are to be unequally distant from their respective poles A, E.

COMPUTATION.

Here, as in the last, there will be three spheric triangles to work in ; namely, the triangles APB, ZAB, and ZPB.

In the triangle APB, where $AP=97^{\circ} 43'$, $BP=76^{\circ} 58'$, $\angle APB=9^{\circ} 58'$.

As rad. : cof. $\angle APB :: \tan. BP : \tan. M=76^{\circ} 46\frac{1}{2}'$. Then $AP-M=N=20^{\circ} 57\frac{1}{2}'$.

As sin. N : sin. M :: tan. $\angle APB : \tan. \angle BAP=25^{\circ} 34'$.

And as cof. M : cof. N :: cof. BP : cof. BA = $22^{\circ} 59'$.

In the triangle BAZ, where $AZ=49^{\circ} 16'$, $BZ=45^{\circ} 00'$, $AB=22^{\circ} 59'$.

The angle BAZ will be found equal to $68^{\circ} 56'$.

Then $\angle BAZ - \angle BAP = \angle PAZ = 43^{\circ} 22'$.

In the triangle APZ, where $AP=97^{\circ} 43'$, $AZ=49^{\circ} 16'$, $\angle PAZ=43^{\circ} 22'$.

As rad. : cof. $\angle PAZ :: \tan. AZ : \tan. M=40^{\circ} 10'$. Then $AP-M=N=57^{\circ} 33'$.

And as cof. M : cof. N :: cof. AZ : cof. PZ = $62^{\circ} 39'$.

Hence the latitude sought is $27^{\circ} 16' N$.

R 4

181. PRO.

181.

PROBLEM XXXVIII.

Pl. V.

Given the Sun's declination, two altitudes, and the time between the observations ;

Required the latitude of the place,

EXAM. On a day when the Sun's declination was $20^{\circ} 00' N.$, in the forenoon the Sun's altitude was observed to be $18^{\circ} 30'$, and three hours after his altitude was $44^{\circ} 00'$: What was the latitude of the place ?

CONSTRUCTION.

Let the primitive circle represent that hour circle on which the Sun was at the first observation, EQ being the equator, then Aa, the parallel of 20° of declination, gives A the Sun's place at first ; and as CQ is the tangent of 45° , Q will be the center of the hour circle Pbs three hours distant from the former, its intersection B with the parallel of declination, is the Sun's place at the second observation : About A as a pole, at the distance of $71^{\circ} 30'$, the first zenith distance, describe (IV. 66) a small circle ; about B, as a pole, at the distance of $46^{\circ} 00'$, the second zenith distance, describe (IV. 66) another small circle, cutting the former in z the zenith : Through z, A ; z, B ; A, B ; P, z ; describe (IV. 61) great circles ; then Pz is the co-latitude required.

COMPUTATION.

Here are three triangles to work in ; namely, APB, ABZ, BPZ.

In the isosceles spheric triangle APB.

Given	$AP = 70^{\circ} 00'$	Suppose the perpendicular Pb is drawn.
	$BP = 70^{\circ} 00'$	
	$\angle APB = 45^{\circ} 00'$	
Req.	$\angle ABP$ and AB.	
		Rad. : tan. Bpb :: cos. PB : co-t. $\angle PBA = 81^{\circ} 56'$.
		Rad. : sin. PB :: sin. $\angle Bpb$: sin. Bb = $21^{\circ} 04\frac{1}{2}'$
		Then Bb double gives $AB = 42^{\circ} 09'$.

In the oblique angled spheric triangle ABZ.

Given	$AZ = 71^{\circ} 30'$	Then working with the three sides, the angle ABZ will be found = $114^{\circ} 11'$.
	$BZ = 46^{\circ} 00'$	
	$AB = 42^{\circ} 09'$	
Required	$\angle ABZ$.	
		And $\angle ABZ - \angle PBA = \angle PBZ = 32^{\circ} 15'$.

In the oblique angled spheric triangle PBZ.

Given	$PB = 70^{\circ} 00'$	As rad. : cos. $\angle PBZ$:: tan. BZ : tan. M = $41^{\circ} 13'$.
	$BZ = 46^{\circ} 00'$	
	$\angle PBZ = 32^{\circ} 15'$	
Req.	the co-lat. Pz.	
		And $PB - M = N = 28^{\circ} 47'$.
		As cos. M : cos. N :: cos. BZ : cos. Pz = $30^{\circ} 59'$.
		Therefore the latitude is $54^{\circ} 01' N.$

182. If the Sun's altitude can be taken both before and after noon, when he has equal heights, then the time between these two observations being bisected, will give the time when the Sun was on the meridian : Now the co-declination, the co-altitude, and the time from noon at either observation being known, the latitude may be readily computed in one oblique angled triangle, in which are known two sides, and an angle opposite to one of them to find the other side, which is the co-latitude ; for which see the problem, art. 176,

183.

PROBLEM XXXIX.

Pl. V.

Given the Sun's declination, two altitudes, and the difference of the magnetic azimuths ;

Required the latitude of the place.

EXAM. On the 21st of May, the Sun's declination being $20^{\circ} 16' N.$; in the morning when the Sun was on the ESE point of the compass, his altitude was $43^{\circ} 30'$; and when he bore S. $20^{\circ} 30' E.$ his altitude was $58^{\circ} 30'$; What is the latitude of the place of observation ?

CONSTRUCTION.

Let the primitive circle represent the azimuth circle which the Sun was on at the greater altitude, $58^{\circ} 30'$, A being the Sun's place at that time, HR the horizon, and z the zenith ; draw *aa* a parallel of $43^{\circ} 30'$ of altitude, and (IV. 75) describe a vertical circle, making an angle with AZ, of $47^{\circ} 00'$, the difference of the observed azimuths ; the place where this cuts the parallel of altitude *aa*, gives B the Sun's place at the first observation : Then small circles being described about A and B, as poles at the distances of $69^{\circ} 44'$, the co-declination (IV. 66), their intersection will give P the place of the pole : Through A, B ; P, A ; P, B ; and z, P, describe great circles (IV. 61) ; then PZ is the co-latitude sought.

COMPUTATION.

1st. In the spheric triangle AZB ; Given AZ = A's co-alt. = $31^{\circ} 30'$
 BZ = B's co-alt. = $46 \ 30$
 AZB = diff. of az. = $47 \ 00$

Required $\angle ABZ$ = $45 \ 41$
 AB = $32 \ 17$

2^d. In the isosceles spheric $\triangle APB$: Given AP = A's co-decl. = $69 \ 44$
 BP = B's co-decl. = $69 \ 44$
 AB = distance = $32 \ 17$

Required $\angle ABP$ = $83 \ 52$

3^d. In the spheric triangle ZBP : Given BZ = B's co-alt. = $46 \ 30$
 PB = B's co-decl. = $69 \ 44$
 $\angle ZBP$ = $38 \ 11$

Requir. ZP = co-lat. = $39 \ 21$

Therefore the latitude of the place is $50^{\circ} 39' N.$

184.

PROBLEM XL.

Pl. V.

Two known stars being observed on the same azimuth, and two other known stars being observed on another azimuth, and the time between the observation being known; to find the latitude of the place.

EXAM. *The stars Aldebaran in Taurus, and Rigel in Orion, were observed on the same azimuth; and 2 h. 35 min. after, the stars Castor in Gemini and the Hydra's heart were also observed on another azimuth: What was the latitude of the place of observation?*

CONSTRUCTION.

On the plane of the equator put the stars Aldebaran and Rigel at A, B, also the stars Castor and Hydra at c, d, by the means of their right ascensions and declinations (2d and 3d of 137): Let the stars c, d, be removed forward $2^h 35^m$, or $38^\circ 45'$ to c, d; through A, B, and D, c, describe (IV. 61) great circles intersecting in z, the zenith of the place; draw the great circles AC and PZ; then the measure of PZ gives the co-latitude required.

COMPUTATION.

Aldebaran's declin.	= $16^\circ 03' \text{ N.}$	right ascen.	= $4^h 23^m 19^s = 65^\circ 50'.$
Rigel's	= $8^\circ 28' \text{ S.}$		= $5^\circ 03' 58'' = 75^\circ 59\frac{1}{2}'.$
Castor's	= $32^\circ 21' \text{ N.}$		= $7^\circ 20' 33'' = 110^\circ 08'.$
Hydra's heart	= $7^\circ 43' \text{ S.}$		= $9^\circ 16' 47'' = 139^\circ 12'.$

1st. In the triangle PAB, where $PB = 98^\circ 28'$, $PA = 73^\circ 57'$, $\angle APB = 10^\circ 09\frac{1}{2}'$.

Then the $\angle PAB$ will be $156^\circ 59'$, and the $\angle PAZ$, the suppl. = $23^\circ 01'$.

2d. In the triangle PCD, where $PD = 97^\circ 43'$, $PC = 57^\circ 39'$, $\angle CPD = 29^\circ 04'$.

Then the $\angle PCD$ will be $140^\circ 09'$, and the $\angle PCZ$, the suppl. = $39^\circ 51'$.

3d. In the triangle APC, where $AP = 73^\circ 57'$, $CP = 57^\circ 39'$, $\angle APC = 5^\circ 33\frac{1}{2}'$, which is the difference between 2 h. 35 m. and the difference of the right ascensions of A and c.

Then the $\angle PAC$ will be $16^\circ 12'$, $\angle PCA = 161^\circ 30'$, and $AC = 17^\circ 03'$.

Now $\angle PAC + \angle PAZ = \angle CAZ = 39^\circ 13'$; and $\angle PCA - \angle PCZ = \angle ACZ = 121^\circ 39'$.

4th. In the triangle ACZ, where $AC = 17^\circ 03'$; $\angle CAZ = 39^\circ 13'$; $\angle ACZ = 121^\circ 39'$.

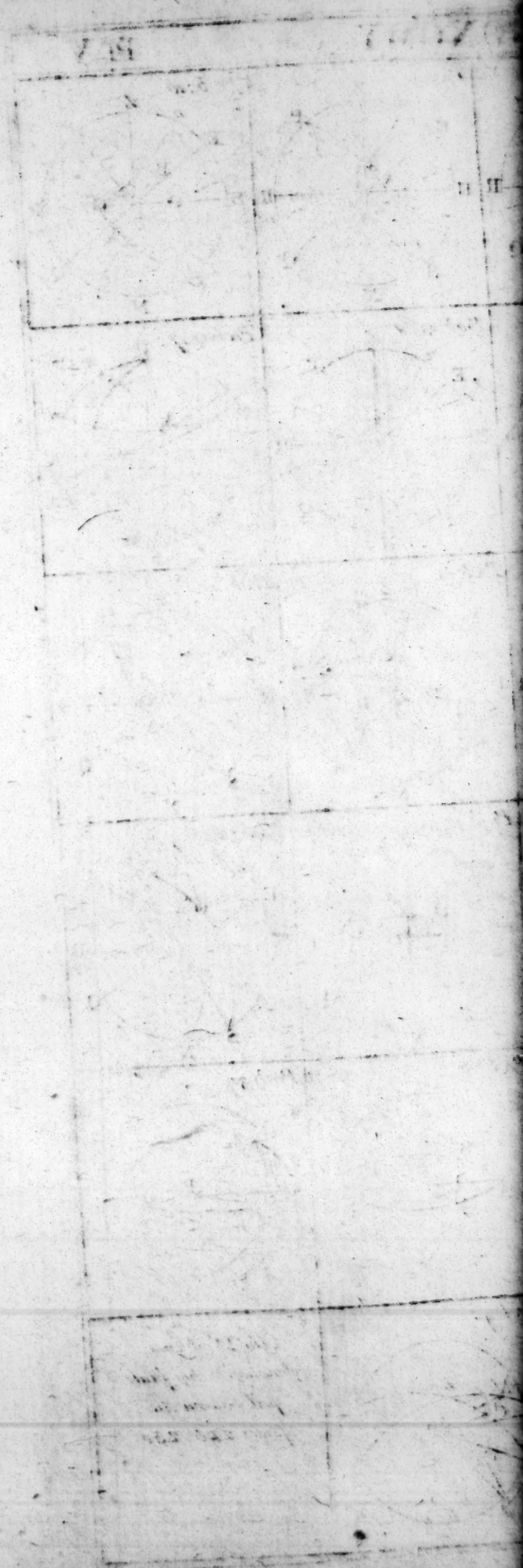
Then cz will be found equal to $28^\circ 26'$.

5th. In the triangle CPZ, where $CZ = 28^\circ 26'$, $CP = 57^\circ 39'$, $\angle PCZ = 39^\circ 51'$.

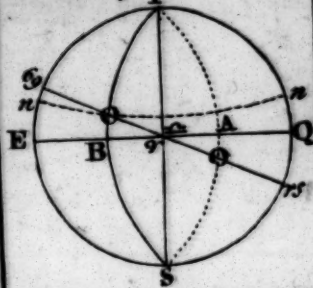
Then PZ will be found equal to $38^\circ 49'$.

And the latitude of the place of observation is $51^\circ 11' \text{ N.}$

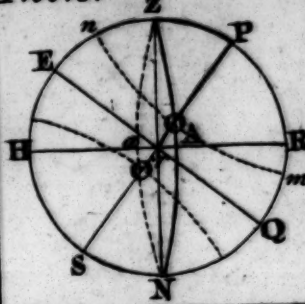
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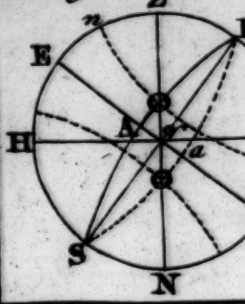
Prob: 5. 6. 7.



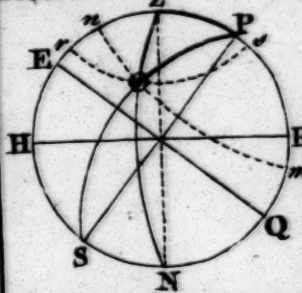
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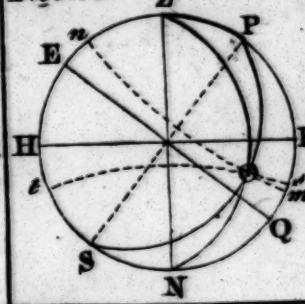
Prob: 9.



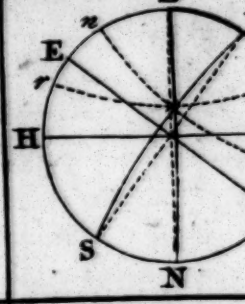
Prob: 11.



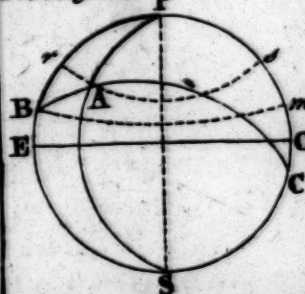
Prob: 12.



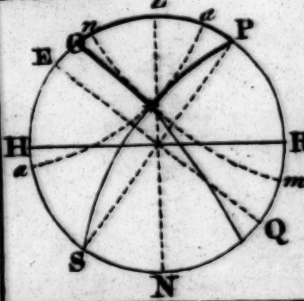
Prob: 13.



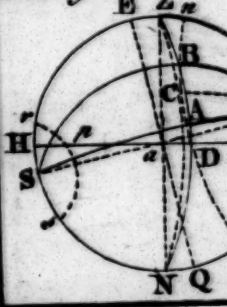
Prob: 15. 16.



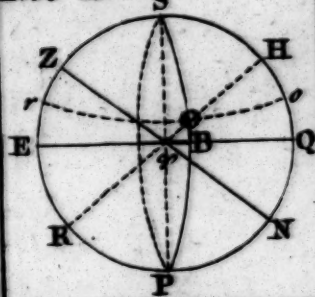
Prob: 18.



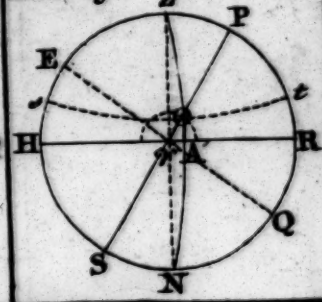
Prob: 19.



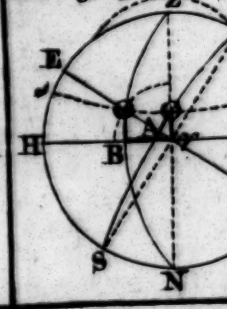
Prob: 28.



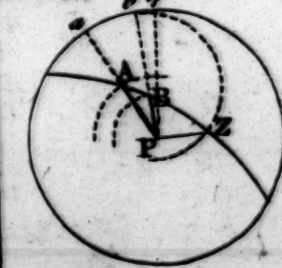
Prob: 29.



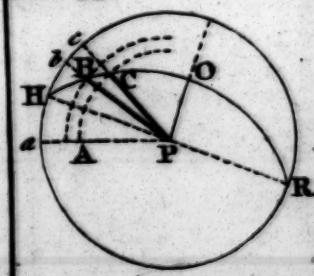
Prob: 30. 31.



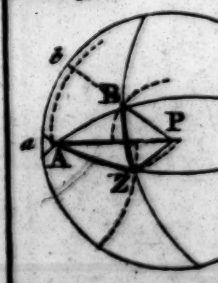
Prob: 34.



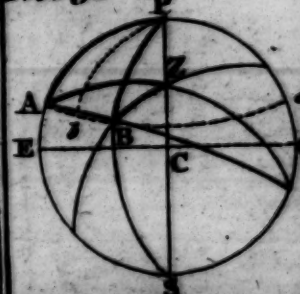
Prob: 35.



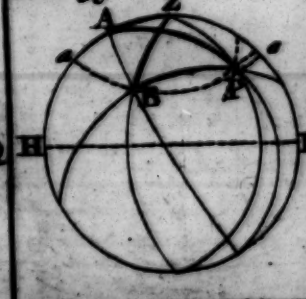
Prob: 36.



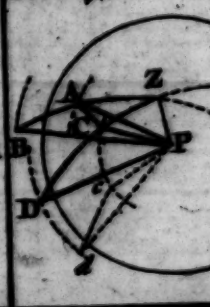
Prob: 38.



Prob: 39.

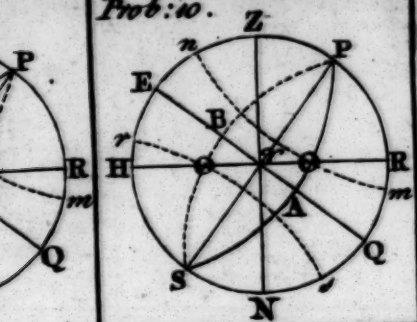


Prob: 40.

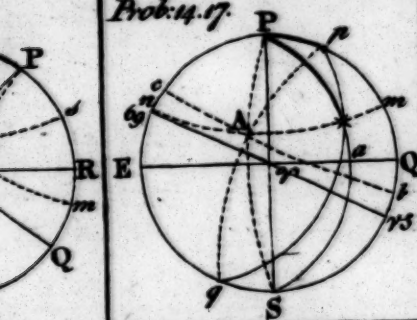


Pl. V.

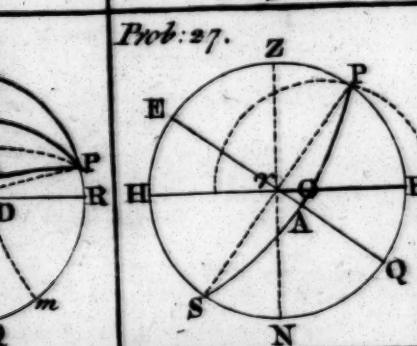
Prob: 10.



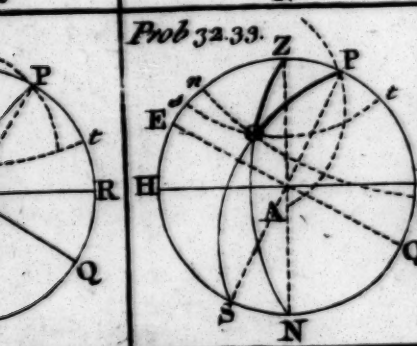
Prob: 14. 17.



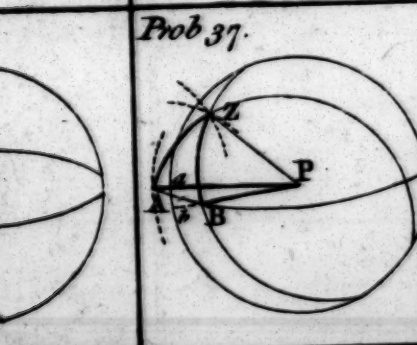
Prob: 27.



Prob 32 33.



Prob 37.



The Problems
belonging to this plate
fall between the
pages 225. 250.

The paper is of the
best quality and the
printing is of the
best quality.

There might be given a great variety of other problems to find the latitude from various circumstances; but the trouble of solving them, as well as some of the foregoing ones, is too great to render them of general use: And indeed some of them were only inserted as trigonometrical exercises for young students; it being generally allowed that the sciences are most readily learned by working many examples: And on this account it was judged, that the few following questions might not only be entertaining to those who have a love for these matters; but on some occasions might be usefully applied at sea.

185.

P R O B L E M XLI.

Given the Sun's meridian, or mid-day altitude $= 62^\circ 00'$.

And its mid-night depression, below the horizon, $= 22^\circ 00'$.

Required the latitude of the place, and the Sun's declination.

SOLUTION. Let the circle HZR be the meridian;

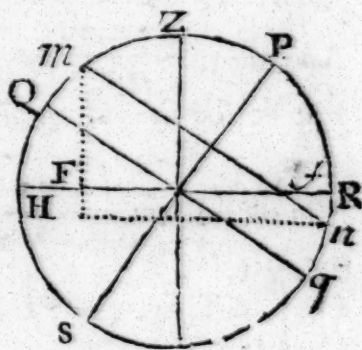
arc Hm, the meridian alt.; its sine Fm;

arc Rn, the mid-night depression; its sine nf;

mn, the parallel of declination;

Qq, parallel to mn the equator;

ps, at right angles to Qq, the axis, or 6 o'clock circle.



Now $HQ + Qm = Hm$ } For $HQ = Rq$; and $Qm = qn$.
 And $HQ - Qm = Rn$

Then $HQ = \frac{Hm + Rn}{2} = \text{co-latitude} = 42^\circ 00'$.

And $Qm = \frac{Hm - Rn}{2} = \text{declination} = 20^\circ 00'$.

In the following problems, as it was the method of computation which was chiefly intended for the information of beginners, the construction is supposed to be done: And the lines and letters, as here described, are to be understood to represent the same things in each figure.

188.

PROBLEM XLIV.

At a place in the northern hemisphere, some time in the month of May, the Sun was observed to have $14^{\circ} 43\frac{1}{2}'$ altitude at 6 h. P. M. and to set at 7 h. 35 m 24 s. : Required the latitude of that place, and day of the month.

Let m, N, v, n , be places of the Sun, at noon, 6 o'clock, setting, and at midnight. See the fig. to Problem 43.

mF, Nt , fines of the altitudes at m and N .

ov, vq , and Qv , the same as in the last problem.

Now $ov : vq :: (NV : vn ::) Nt : nf$. Then $L, nf = L, ov + L, Nt + L, vq$.
 Also $ov : Qv :: (NV : vm ::) Nt : mF$. And $L, mF = L, ov + L, Nt + L, Qv$.
 Here $ov = s, 23^{\circ} 51' = 0,40434$; and $Qv = 1,4043$.
 $qv = v, 66^{\circ} 09' = 2,55, 33^{\circ} 4\frac{1}{2}'$.

Then $ov = s, 23^{\circ} 51'$ its $L's,$	$0,39325$	And $ov = s, 23^{\circ} 51'$ its $L's,$	$0,39325$
$Nt = s, 14^{\circ} 43\frac{1}{2}'$ $Ls,$	$9,40514$	$Nt = s, 14^{\circ} 43\frac{1}{2}'$ its $Ls,$	$9,40514$
$qv = \begin{cases} ss, 33^{\circ} 4\frac{1}{2}' & 2Ls, \\ z & \end{cases}$	$9,47397$	$Qv = 1,4043$ its $L,$	$10,14746$
	$0,30103$		
$nf s, 22^{\circ} 0'$	<u>$9,57339$</u>	$mF = s, 62^{\circ} 0'$ its $Ls,$	<u>$9,94585$</u>

Then $\frac{62^{\circ} + 22^{\circ}}{2} = 42^{\circ}$, the co-latitude.

And $\frac{62 - 22}{2} = 20^{\circ}$, the declination answering to May 19th. (185)

189.

PROBLEM XLV.

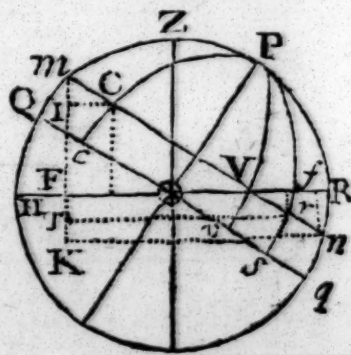
At a place in the western ocean, some time in July, the Sun's altitude was observed to be 36° at 3 h. 51 m. 49 s. P. M.; and was seen to set at 7 h. 35 m. 24 s. P. M. : Required the latitude of the place, and day of the month.

Let m, c, v, n , be the Sun's places at noon, at 3 h. 51 m. 49 s. at setting, and at midnight.

mf, IF , the fines of the altitudes at m, c ;

nf the fine of the midnight depression.

oc, ov , the fines of the times from 6 h, and cv , their sum.



Now

Then $bc =$	0,20919	0,67947	And $bc =$	0,20919	0,67947
$HI =$	0,13156	9,11912	$HI =$	0,13156	9,11912
$QC =$	$\left\{ \begin{array}{l} ss, 28^{\circ} 58' 37'' \\ 2 \end{array} \right.$	$\left\{ \begin{array}{l} 9,37050 \\ 0,30103 \end{array} \right.$	$bq =$	1,7398	10,24050
			$HK =$	1,0942	10,03909
$mi =$	29520	9,47012			

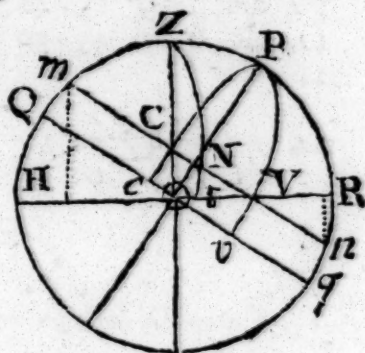
Then $mf = 0,88298$ the sine of 62° the mer. alt. } Hence lat. $= 48^{\circ}$ N. decl.
 And $nf = 0,37486$ the sine of 22 the mid. depr. } 20° on July 23.

193.

PROBLEM XLIX.

Being at sea in the western ocean, the Sun was observed to have $27^{\circ} 24'$ of altitude when due W.; and to have $14^{\circ} 43\frac{1}{2}'$ alt. at 6 h. P. M.; Required the latitude of that place, and the Sun's declination.

Let c, N , be the Sun's places at W.; and at 6 h.
 oc, nt , the sines of their altitudes.
 $cc = ON$, the arcs of declination.
 $\angle coc = \angle Not =$ latitude.



In $\triangle occ$. As $s, oc : R :: s, cc : s, \angle coc = \frac{s, cc}{s, oc} \times R$.

In $\triangle onn$. As $s, on : R :: s, nt : s, \angle not = \frac{s, nt}{s, on} \times R$.

Then $\frac{s, cc}{s, oc} = \left(\frac{s, nt}{s, on} \right) \frac{s, nt}{s, cc}$. And $ss, cc = s, oc \times s, nt$.

Or $s, \text{alt. W.} \times s, \text{alt. at 6} = ss, \text{decl.}$ Or $\frac{Ls, \text{alt. W.} + Ls, \text{alt. at 6}}{2} = Ls, \text{decl.}$
 And $Ls, \text{alt. W.} + Ls, \text{decl.} = Ls, \text{lat.}$

$oc =$	$s, 27^{\circ} 24'$ its L, s	9,66295
$nt =$	$14^{\circ} 43\frac{1}{2}'$ its L, s	9,40514
	Sum	19,06809
$cc =$	$20^{\circ} 00'$	9,53404
$oc =$	$27^{\circ} 24'$ its L	0,33705
$\angle qoz =$	$48^{\circ} 00'$	9,87109

194.

PROBLEM L.

At a place in the western ocean, the Sun at rising was observed to be $50^{\circ} 5' 40''$ from the true north point of the horizon; and at 6 h. A. M. the altitude was observed $14^{\circ} 43\frac{1}{2}'$: Required the latitude and declination.

Let v , and N , be the places of the Sun at rising and at 6 h. A. M.
 nt , the sine of the alt. at 6. on, vv are arcs of declination,
 ov , the ascensional diff. $\angle not = \text{lat.}$; $\angle vov = \text{co-latitude}$.
 $ov = \text{co-amplitude}$.

Then $\frac{1}{2}$ sum $+$ $\frac{1}{2}$ diff. $= ZB = 35^\circ 30\frac{1}{2}'$; and $\frac{1}{2}$ sum $- \frac{1}{2}$ diff. $= ZD = 20^\circ 55'$.
 Now $s, DB : s, DZB :: s, ZB : s, PDZ$; and $s, ZPD : s, PDZ :: s, ZD : s, ZP 42^\circ 1'$.
 Hence the latitude is $47^\circ 59' N$.

196.

PROBLEM LII.

By observations made at a place in the western ocean, it was found that the Sun's altitude was $36^\circ S.$, when his azimuth was $N. 100^\circ 5' E.$ and alt. was $46^\circ S.$, when his azimuth was $N. 114^\circ 28' E.$ What was the latitude of the place and the Sun's declination?

Let A, B, be the Sun's places when observed.

In the triangles PAZ, PBZ, where $PA = PB$.

By IV. 239 $\begin{cases} s, ZP \times s, ZA - s, PZA \times s, ZP \times s, ZA = s, PA. \\ s, ZP \times s, ZB - s, PZB \times s, ZP \times s, ZB = s, PA. \end{cases}$

Then $s, PZB \times s, ZB \times s, ZP - s, PZA \times s, ZA \times s, ZP = s, ZB \times s, ZP - s, ZA \times s, ZP$.

Or $s, PZB \times s, ZB - s, PZA \times s, ZA \times s, ZP = s, ZB - s, ZA \times s, ZA \times s, ZP$.

Then $\frac{s, PZB \times s, ZB - s, PZA \times s, ZA}{s, ZB - s, ZA} = \left(\frac{s, ZP}{s, ZP} = (\text{III. 33}) t, ZP = \right) s, RP$.

the latitude.

Let $Ls, PZB + Ls, ZB = L, A$: And $Ls, PZA + Ls, ZA = L, B$. Then

$$\frac{A \propto B}{s, ZB - s, ZA} = t, RP.$$

Here $A = 0,14164$; $B = 0,28770$; $s, ZB = 0,71933$; $s, ZA = 0,58779$.

Then $\frac{0,13154}{0,14606} = ,9005 = t, 41^\circ$. Hence lat. is $48^\circ N$. decl. $20^\circ N$.

197.

PROBLEM LIII.

Given two altitudes of the Sun and the time from noon when those altitudes were taken; thence to find the latitude and declination.

EXAM. At 8 h. 8 m. 11 s. A. M. the alt. was 36° ; and at 9 h. 10 m. 51 s. the alt. was 46° ; at a place in the western ocean, some time in May, 1763.

Let B, A, be the Sun's places observed;

m, n, these of noon and midnight;
 Fe, Fd, Fm, FK, represent the lines of the distances of those places from the horizon HR, to the diameter Qq; and $de = Fd - Fe$.

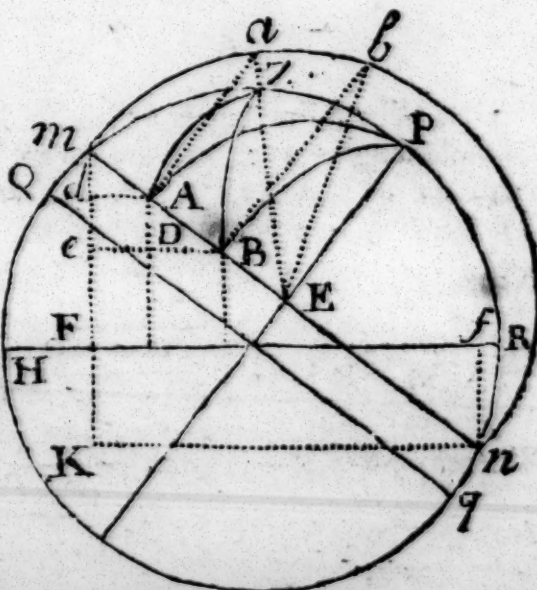
On mn describe the semicircle mbn, representing half the parallel of declination, and let Aa, Bb, be at right angles to mn: Then will the angles meB, mEa, represent the times from noon, at the observations B, A; and mB, mA, are as the versed sines of those times.

And $mB - mA = AB$.

Then $AB : de :: mB : me$; and $me + Fe = Fm$, the sine of Hm.

S 2

Also



Also $AB : de :: mn : mK$; and $Km - Fm = FK$, the sine of Rn .
Hence the latitude and declination are found. (185)

$$\text{Here } AB = (s, ma - s, mb = s, \frac{mb + ma}{2} \times 2s, \frac{mb - ma}{2} =) s, M \times 2s, N. \quad (\text{IV. 181})$$

$$\text{And } de = (s, 2d \text{ alt.} - s, 1ft \text{ alt.} = s, \frac{2d + 1ft}{2} \times 2s, \frac{2d - 1ft}{2} =) s, W \times 2s, V. \quad (\text{IV. 182})$$

$$\text{Now } L, \frac{de}{AB} \times mn = L, mK = Ls, M + Ls, N + Ls, W + Ls, V, L, 2.$$

$$\text{And } L, \frac{de}{AB} \times mB = L, me = L, mK + 2Ls, \frac{1}{2} \angle meB.$$

In this example, the $\angle meB = 3 \text{ h. } 51 \text{ } 49 = 57^\circ 57\frac{1}{4}'$; its $\frac{1}{2} = 28^\circ 58\frac{1}{8}'$.
 $\angle meA = 2 \text{ h. } 49 \text{ } 9 = 42^\circ 17\frac{1}{4}'$.

57° 57 $\frac{1}{4}'$ 42 17 $\frac{1}{4}'$	46 36	Ls, M 50° 7 $\frac{1}{4}'$ 0,11498
		Ls, N 7 50 0,86553
100 14 $\frac{1}{2}$	50° 7 $\frac{1}{4}'$ = M 82 41° 0' = W.	Ls, W 41 0 9,87778
		Ls, V 5 0 8,94030
15 40	7 50 = N 10 5 0 = V.	L, 2 0,30183
s, 36° = F = 0,58778		L, Km 1,25783 10,09962
me = 0,29521		2Ls, $\frac{1}{2}mb$ 28° 58 $\frac{1}{8}'$ 9,37050
		L, me 029521 9,47012
fm = 0,88299 = 62° 0' 19"		
km = 1,25810		
		42° 0' 55" the co-latitude.
FK = 0,37511 = 22 1 51		19 59 14 the decl. May 20th.
	84 2 10	
	39 58 28	

198. The following is another solution, on different principles.
In the triangles BPZ, APZ are given, (see the foregoing figure.)
BZ, AZ; BPZ, APZ, hour angles; and PB = PA;
To find PB, PZ; for whose sum and diff. put M and N.

$$\text{Now (IV. 239)} \begin{cases} s, PB \times s, PZ \times s, BPZ + s, PB \times s, PZ = s, BZ. \\ s, PA \times s, PZ \times s, APZ + s, PA \times s, PZ = s, AZ. \end{cases}$$

$$\text{Or (IV. 181. 174)} \begin{cases} \frac{1}{2}s, N - \frac{1}{2}s, M \times s, BPZ + \frac{1}{2}s, N + \frac{1}{2}s, M = s, BZ. \\ \frac{1}{2}s, N - \frac{1}{2}s, M \times s, APZ + \frac{1}{2}s, N + \frac{1}{2}s, M = s, AZ. \end{cases}$$

$$\text{Therefore } (\frac{1}{2}s, N - \frac{1}{2}s, M =) \frac{s, BZ - \frac{1}{2}s, N + \frac{1}{2}s, M}{s, CPZ} = \frac{s, AZ - \frac{1}{2}s, N + \frac{1}{2}s, M}{s, APZ}.$$

$$\text{And } (\frac{1}{2}s, N + \frac{1}{2}s, M =) s, BZ - \frac{1}{2}s, N - \frac{1}{2}s, M \times s, BPZ = s, AZ - \frac{1}{2}s, N - \frac{1}{2}s, M \times s, APZ.$$

$$\text{Hence } s, BZ \times s, APZ - \frac{1}{2}s, N + \frac{1}{2}s, M \times s, APZ = s, AZ \times s, BPZ - \frac{1}{2}s, N + \frac{1}{2}s, M \times s, BPZ.$$

Therefore

Therefore $s, BZ \times s, APZ - s, AZ \times s, BPZ = s, APZ - s, BPZ \times \frac{1}{2}s, N + \frac{1}{2}s, M$.

Again, $s, AZ - s, BZ = s, APZ - s, BPZ \times \frac{1}{2}s, N - \frac{1}{2}s, M$.

Therefore $\frac{1}{2}s, N + \frac{1}{2}s, M = \frac{s, BPZ \times s, AZ - s, APZ \times s, BZ}{s, APZ - s, BPZ}$ } Hence IV. 216.

And $\frac{1}{2}s, N - \frac{1}{2}s, M = \frac{s, AZ - s, BZ}{s, APZ - s, BPZ}$

$$s, M = \left(\frac{s, AZ + s, BPZ \times s, AZ - s, BZ - s, APZ \times s, BZ}{s, APZ - s, BPZ} = \right)$$

$$\frac{I + s, BPZ \times s, AZ - I + s, APZ \times s, BZ}{s, APZ - s, BPZ}.$$

$$s, N = \left(\frac{s, BPZ \times s, AZ - s, AZ - s, APZ \times s, BZ + s, BZ}{s, APZ - s, BPZ} = \right)$$

$$\frac{I - s, BPZ \times s, AZ - I - s, APZ \times s, BZ}{s, APZ - s, BPZ}.$$

$$\text{Or, } sM = \left(\frac{v, BPZ \times s, AZ - v, APZ \times s, BZ}{s, APZ - s, BPZ} = \right)$$

$$\frac{2s, \frac{1}{2}BPZ \times s, AZ - 2s, \frac{1}{2}APZ \times s, BZ}{s, APZ - s, BPZ}.$$

$$\text{And } s, N = \left(\frac{v, BPZ \times s, AZ - v, ABZ \times s, BZ}{s, APZ - s, BPZ} = \right)$$

$$\frac{2ss, \frac{1}{2}BPZ \times s, AZ - 2ss, \frac{1}{2}APZ \times s, BZ}{s, APZ - s, BPZ}.$$

Here $PBZ = 57^\circ 57\frac{1}{4}'$, its $\frac{1}{2} = 28^\circ 58\frac{1}{8}'$; $APZ = 42^\circ 17\frac{1}{4}'$, its $\frac{1}{2} = 21^\circ 8\frac{1}{8}'$.
 $BZ = 54^\circ$; $AZ = 44^\circ$; $s, BPZ = 0,53060$; $s, APZ = 0,73978$, their
 difference $= 0,20918$.

2Ls's,	28° 58 $\frac{1}{8}$ '	{ 0,30103	2Ls's,	28° 58 $\frac{1}{8}$ '	{ 0,30103
Ls,	44	9,88384	Ls,	44	9,37050
		9,85693			9,85693
L,A	1,10104	10,04180	L,a	0,33764	9,52846
2Ls's,	21° 8 $\frac{1}{8}$ '	{ 0,30103	2Ls's,	21° 8 $\frac{1}{8}$ '	{ 0,30103
Ls,	54	9,93946	Ls,	54	9,11438
		9,76922			9,76922
L,B	1,02260	10,00971	L,b	0,15298	9,18463
L,A-B	0,07844	8,89454	L,a-b	0,18466	9,26637
Ls,APZ-s,BPZ	0,20918	9,32052	Ls,APZ-s,BPZ	0,20918	9,32052
Ls,M 112° 1 $\frac{1}{2}$ '		9,57402	Ls,N 28° 0'		9,94585

Hence the latitude is $48^\circ 00'$ N. ; declination $19^\circ 59'$, which answers to the 20th of May.

199. And hence is readily derived the investigation of that method, published in the year 1759, and then used by some for finding the true latitude at sea, by knowing the latitude by account (or dead reckoning), the Sun's declination, two altitudes of the Sun, and the time between the observations. Thus. See the last figure.

Let M and N represent the half sum and half diff. of the times from noon; w and v , the half sum and half diff. of the two altitudes.

AB the diff. of the co-sines of the times from noon, to the radius Em .

AD the diff. of the sines of the altitudes.

$\angle BAD$ represents the latitude; Em the co-s. of the declination.

Now (197) $s, M \times 2s, N = AB$ reduced to the rad. $\frac{1}{2} Qq$; and $s, w \times 2s, v = AD$. But in the triangle ABD ,

$$s, \text{ lat.} : R :: AD : AB = \frac{1}{s, \text{ lat.}} \times AD, \text{ to rad. } Em.$$

And $s, \text{ decl.} : R :: AB : AB \times \frac{1}{s, \text{ decl.}} = \frac{1}{s, \text{ decl.}} \times \frac{1}{s, \text{ lat.}} \times AD = AB$ reduced, to the radius $\frac{1}{2} Qq$.

$$\text{Then } s, M \times 2s, N = \frac{1}{s, \text{ decl.}} \times \frac{1}{s, \text{ lat.}} \times s, w \times 2s, v.$$

$$\text{And } s, M = \frac{1}{s, \text{ decl.}} \times \frac{1}{s, \text{ lat.}} \times \frac{1}{s, N} \times s, w \times s, v.$$

Then $M + N = \angle meb$, the time from noon at the least altitude.

And $M - N = \angle mea$, the time from noon at the greatest altitude.

Hence the versed sines of the arcs mb or ma , are known to rad. $\frac{1}{2} Qq$.

Now $R : Em :: v, ma : ma = Em \times v, ma$; or $mb = Em \times v, mb$.

And $R : s, ma :: ma : md$ ($:: mb : me$.)

Then $Fm = Fd + dm$, or to $Fe + em$, is the sine of the mer. alt.

Hence the two operations.

1st. $Ls, \text{ decl.} + Ls, \text{ lat.} + Ls, N + Ls, w + Ls, v = Ls, M$.

Hence the times are known; viz. arcs ma, mb .

2d. $Ls, \text{ decl.} + Ls, \text{ lat.} + L, ? + 2Ls, \frac{1}{2} ma = L, md$.

Then by the merid. alt. and declination the latitude may be found. If this latitude and that assumed are the same, then the latitude by account, or dead reckoning, may be taken as the true latitude.

But if they differ, it is plain that the $\angle mad$, the co-latitude, is less or greater than was assumed.

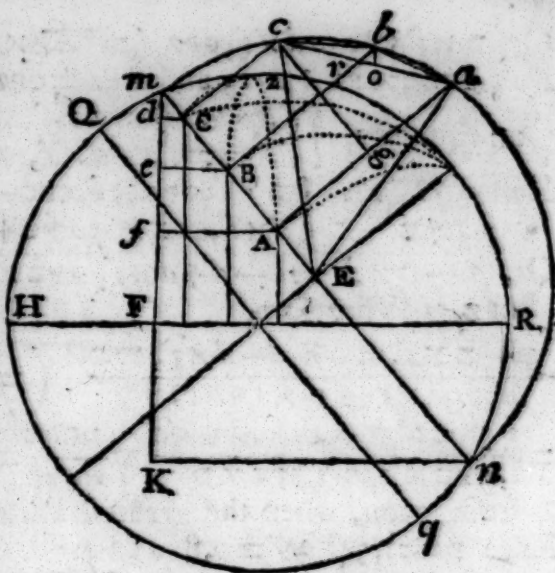
260.

PROBLEM LIV.

Given three descending (or ascending) altitudes of the Sun, taken on the same day at unequal known intervals of time; thence to find those times, the latitude of the place of observation, and the Sun's declination. Thus, suppose in July, 1763, the altitudes were $54^\circ 30'$, 46° , and 36° ; and the intervals of time 60m. 55s. and 62m. 40s.: Required the rest.

Let

Let man be the parallel of declination described on mn , and a, b, c , the places of the Sun when observed; aA, bB , and cC the sines of the times from noon to the radius Em ; m, n , the places at noon and midnight; and let fF, eE, dD, mF, Fk , represent the sines of the distances of those places from the horizon HR . Draw cg parallel to AC ; then if the $\angle gce$, which is equal to the $\angle mec$, the time from noon when the greatest altitude was taken, could be found, the times from noon when the other two were taken would be known also. Now



cb being one interval, and ba the other, bc and ba , the chords of these arcs, will be known, as also ca , which is the chord of their sum. Draw bo perpendicular to ac . Now in the right angled $\triangle cbo$, the $\angle bco = \frac{1}{2}$ the arc ba (II. 120), and the side $cb = 2 \sin \frac{1}{2} \text{ arc } ba$; consequently $R : s, bco (=s, \frac{1}{2} \text{ arc } ba) :: bc (=2 s, \frac{1}{2} \text{ arc } ba) : bo$; and $R : s, cbo (s, \frac{1}{2} \text{ arc } bc) :: bc (=2 s, \frac{1}{2} \text{ arc } bc) : co$. Now df, de , the diff. of the sines of the altitudes, are known; and the lines fd, AC, ac , are cut proportionally in e, B and r ; consequently, $df : de :: (CA : CB ::) ca : cr$, and $co - cr = ro$. In the triangle bor , (III. 46) $ro : bo :: R : t$, $\angle rbo = \angle rcg$. Now $\angle ace (= \text{compt. of } \frac{1}{2} \text{ whole interval } ac) - \angle rcg = \angle gce = \angle cem = \text{time from noon when greatest altitude was observed}$; therefore the time is known, as well as cm the versed sine of that time from noon.

Again $R : s, acg :: ac : cg = AC$.

And $AC : mc :: fd : dm$. Then $fd + dm = fm$, the sine of Hm .

And $AC : mn :: fd : mk$. Then $km - fm = FK$, the sine of Rn .

This Problem has, at times, for near a century past, exercised the talents of many ingenious persons, as well in Russia, Germany, Holland, and France, as in England; perhaps, on account of its apparent use at sea: And among the different solutions there seems none shorter or more intelligible, particularly to beginners, than that above; however, for the sake of the more inquisitive, another solution, which has been commonly given, is here subjoined.

201. In the last figure, the three triangles CPZ, BPZ, APZ , are those concerned; in which the same two sides are common in each.

$$\left. \begin{aligned} s, CPZ \times s, PC \times s, PZ + s, PC \times s, PZ &= s, ZC \\ s, BPZ \times s, PC \times s, PZ + s, PC \times s, PZ &= s, ZB \\ s, APZ \times s, PZ \times s, PC + s, PC \times s, PZ &= s, ZA \end{aligned} \right\} \text{by IV. 239.}$$

$$\text{Then } s, CPZ - s, BPZ \times s, PC \times s, PZ = (s, ZC - s, ZB =) d. \quad (\text{II. 48.})$$

$$\text{And } s, CPZ - s, APZ \times s, PC \times s, PZ = (s, ZC - s, ZA =) D.$$

$$\text{Hence } D \times s, CPZ - s, BPZ = d \times s, CPZ - s, APZ. \quad (\text{II. 147})$$

$$\text{Then } D \times s, CPZ - d \times s, CPZ = D \times s, BPZ - d \times s, APZ.$$

Or, $D \times s, CPZ - d \times s, CPZ (= \overline{D-d} \times s, CPZ) = D \times s, BPZ - d \times s, APZ$.

But, $BPZ = CPZ + BPC$, and $APZ = CPZ + APC$.

Consequently, $\left\{ \begin{array}{l} s, BPZ = s, CPZ \times s, BPC - s, CPZ \times s, BPC. \\ s, APZ = s, CPZ \times s, APC - s, CPZ \times s, APC. \end{array} \right\}$ (IV. 216.)

Hence $\overline{D-d} \times s, CPZ = \left\{ \begin{array}{l} D \times s, CPZ \times s, BPC - D \times s, CPZ \times s, BPC - \\ d \times s, CPZ \times s, APC + d \times s, CPZ \times s, APC. \end{array} \right\}$ by substitu.

Or, $D - D \times s, BPC - d + d \times s, APC \times s, CPZ = d \times s, APC - D \times s, BPC \times s, CPZ$. Wherefore,

$$\frac{D - D \times s, BPC \propto d - d \times s, APC}{d \times s, APC \propto D \times s, BPC} \left(= \frac{1 - s, BPC \times D \propto 1 - s, APC \times d}{d \times s, APC \propto D \times s, BPC} \right)$$

$$= \frac{v, BPC \times D \propto v, APC \times d}{s, APC \times d \propto s, BPC \times D} = \frac{s, CPZ}{s, CPZ} = t, CPZ, \text{ the measure of the time}$$

from noon, when the greatest altitude was observed.

Here $Fd = s, 54^\circ 30' = 0,81412$
 $Fe = s, 46^\circ 00' = 0,71934$
 $Ff = s, 36^\circ 00' = 0,58779$ } Then $df = 0,22633$; $de = 0,09478$.

The arc cb , or $\angle CPB$, = 1 h. 0 m. 55 f. = $15^\circ 13\frac{1}{4}'$; its $\frac{1}{2} = 7^\circ 36\frac{7}{8}'$.

arc ab , or $\angle BPA$, = 1 h. 2 m. 40 f. = $15^\circ 40'$; its $\frac{1}{2} = 7^\circ 50'$.

arc ca , or $\angle CPA$, = 2 h. 3 m. 35 f. = $30^\circ 53\frac{1}{4}'$; its $\frac{1}{2} = 15^\circ 26\frac{7}{8}'$.

To find the hour by the first method.

Radius	90° 00'	10,00000
Is to $s, \frac{1}{2} ba$	7 50	9,13447
As $s, \frac{1}{2} bc$	7 36 $\frac{7}{8}$	9,12224
To $\frac{1}{2} ob$	01806	8,25671
Radius	90° 00'	10,00000
Is to $s, \frac{1}{2} bc$	7 36 $\frac{7}{8}$	9,12224
As $s, \frac{1}{2} ba$	7 50	9,99593
To $\frac{1}{2} co$	013127	9,11817
As fd	22633 A.C.	0,64526
Is to de	09478	8,97672
So is $\frac{1}{2} ac$ ($s, \frac{1}{2} cea$)	15° 26 $\frac{7}{8}'$	9,42547
To $\frac{1}{2} cr$	011154	9,04745
$\frac{1}{2} co$	013127	
As $\frac{1}{2} ro$	01973	1,70480
Is to $\frac{1}{2} bo$	01806	8,25671
So is rad.	90 00	10,00000
To $i, \angle rba$	47 32 $\frac{1}{8}$	9,96151
$\angle ace$	74 33 $\frac{1}{8}$	
$\angle gec = \angle mec = 27$	1 =	1 48 4
Time of 2d observation		2 48 59
Time of 3d observation		3 51 39

To find the hour by the 2d method.

$D (=fd)$	22633	9,35474
$s, \angle BPC$	15° 13 $\frac{1}{4}'$	9,41943
$D \times s, BPC$	05945	8,77417
$d (=de)$	09478	8,97672
$s, \angle APC$	30° 53 $\frac{1}{4}'$	9,71052
$d \times s, APC$	04867	8,68724
$D (=fd)$	22633	9,35474
$v, \angle BPC$	15° 13 $\frac{1}{4}'$	8,54552
$D \times v, BPC$	00795	7,90026
$d (=de)$	09478	8,97672
v, APC	30° 53 $\frac{1}{4}'$	9,15198
$d \times v, APC$	01345	8,12870
$D \times v, BPC$	00795	
$d \times v, APC - D \times v, BPC$	00550	17,74036
$D \times s, BPC - d \times s, APC$	01078	8,03262
tang.	27° 1 $\frac{1}{4}'$	9,70774

Hence the h. fr. n. of 1st ob. is 1^h 48' 7"
 Of the second observation 2 49 2
 Of the third observation 3 51 42
 Consequently from any two of these, with their corresponding altitudes, the latitude and declination may be found by Problem 53.

Here the altitudes are supposed to be descending ones, or in the afternoon.

202. If the altitudes were taken at equal intervals of time; the two first proportions are useless: for ab and bc being equal, the point o falls in the middle of the chord ca ; and bo , the versed sine, is known.

Then $df:de::(CA:CB::)ca:cr$; and $co-cr=ro$.

And $bo:ro::Rad.:trbo=acg$; and $acE-acg=gce=cEm$.

Hence the times from noon are known; and also mc , the versed sine of cEm .

Again, in Δacg . $R:s,acg::ac:cg=AC$.

And $AC:mc::fd:dm$. Then $Fd+dm=Fm$.

Also $AC:mn::fd:mK$. Then $Km-Fm=FK$.

Here $df=0.81412$, $ef=0.71934$; $fE=0.58779$; $df=0.22633$; $de=0.09478$.

Also, arc $cb=1h. 1m. 47\frac{1}{2}s.=15^{\circ} 26\frac{1}{2}'$; $co=0.26636$; $bo=0.03613$.

Then $df:de::ca:cr=0.22309$; and $co-cr=0.04327$.

And $bo:ro::Rad.:trbo=50^{\circ} 8\frac{3}{8}'$; then $74^{\circ} 33\frac{1}{8}'-50^{\circ} 8\frac{3}{8}'=24^{\circ} 24\frac{3}{8}'$.

Then $24^{\circ} 24\frac{3}{8}'=1\ 37\ 39$ from noon at first alt.

And $24^{\circ} 24\frac{3}{8}'+15^{\circ} 26\frac{1}{2}'=39^{\circ} 51\frac{1}{8}'=2\ 39\ 26\frac{1}{2}$ from noon at 2d alt.

Also $39\ 51\frac{1}{8}'+15\ 26\frac{1}{2}'=55\ 18\frac{1}{8}'=3\ 41\ 14$ from noon at 3d alt.

Here the altitudes are supposed to be ascending ones, or in the forenoon.

Again, $rad.:s,cag::ac:AC=0.34142$.

And $AC:mc::fd:dm=0.05927$; then $mF=0.87338$, $60^{\circ} 51'$.

Also $AC:mn::fd:mK=1.32581$; then $FK=0.45243$, $26^{\circ} 54'$.

Hence the latitude is $46^{\circ} 7\frac{1}{2}'$ N. and the declin. $16^{\circ} 58\frac{1}{2}'$; or on May 7th.

203. But the operation in this case may be much contracted. Thus

since $(df:de::ca:)$ $cr=\frac{de \times ca}{df}$; thence $ro=(co-\frac{de \times 2co}{df})$

$$\frac{df-2de \times co}{df}$$

And $(bo:ro::R:)$ $t, rbo=(\frac{ro}{bo}=\frac{D \times co}{df \times 2ss, \frac{1}{2}cb})=\frac{D \times \frac{1}{2}co}{df \times ss, \frac{1}{2}cb}$

where $D=df-2de$.

But $R \times v,cb=ss, \frac{1}{2}cb=\frac{1}{2}ss,cb \times t, \frac{1}{2}cb$.

(IV. 193, 195)

Then $t, rbo=\frac{D}{df \times t, \frac{1}{2}cb}$; or $Lt, rbo=L't, \frac{1}{2}cb+L',df+L,D$.

Also $(R:s,acg::ac:)$ $AC=(s,acg \times ac=) s,acg \times 2s,cb$.

And $(AC:mn::fd:)$ $mK=(\frac{mn \times fd}{AC}=\frac{2 \times fd}{s,acg \times 2s,cb})=\frac{fd}{s,acg \times s,cb}$

And $(AC:mc::fd:)$ $md=(\frac{mc \times fd}{AC}=\frac{fd \times 2ss, \frac{1}{2}mEc}{s,acg \times 2s,cb})=\frac{fd}{s,acg \times s,cb}$

$\times ss, \frac{1}{2}mEc$.

Hence $L's,acg+L's,cb+L',fd=L,mK$.

And $L's,acg+L's,cb+L',fd+2Ls, \frac{1}{2}mEc=L,md$.

Here $df=0.22633$; $2de=0.18955$; and $D=(df-2de)=0.03678$.

Also $\frac{1}{2}cb=7^{\circ} 43' 26''$; and $acE=74^{\circ} 33\frac{1}{8}'$.

$L, p', 7^{\circ} 43' 26''$	$0,86764$	$L's, aeg = 50^{\circ} 09'$	$0,19329$
$L, df = 0,22633$	$0,64526$	$L's, bc = 15 26\frac{7}{8}$	$0,57452$
$L, D = 0,03678$	$8,56561$	$L, fd = 22633$	$9,3547$
$L, t, rbc = 50^{\circ} 09'$	$10,07851$	$L, mK, = 1,32602$	$10,12255$
$acB = 74 33\frac{1}{8}$		$2Ls, \frac{1}{2}MEC = 12^{\circ} 12\frac{1}{6}'$	$8,64998$
$MEC 24 24\frac{1}{8}$; half is $12^{\circ} 12\frac{1}{6}'$.		$L, m, d = 0,05923$	$8,77253$

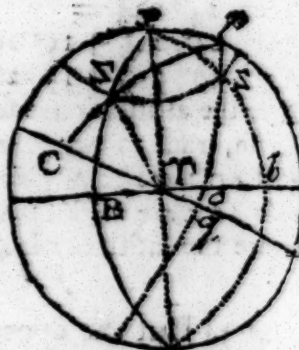
PROBLEM LV.

204. Given the obliquity of the ecliptic, the latitude of a place, and the apparent time at that place. To find the longitude of the nonagesimal degree.

CONSTRUCTION.

The given time applied to the Sun's right ascension, gives the right ascension of the mid-heaven.

Then let the primitive circle represent the solstitial colure, where P, p , represent the poles of the equator γB , and ecliptic γC , the center γ being their intersection. In the equator apply the right ascension of the mid-heaven from γ to B , and the circle PZB , being described, is that meridian, the intersection of which by a parallel of latitude described about P , gives z , the place of the zenith; through z describe a circle of longitude pzc , and the point c is the nonagesimal degree, and γc its longitude.



COMPUTATION.

In the triangle Ppz . Given Pp the obliquity of the ecliptic.
 PZ the co-latitude.

Required $\angle ZPp$ the right asc. of the mid-heaven.
 zp ———— $\{ \text{IV. 237, 238, } \}$
 $\angle Ppz$ ———— $\{ \text{and 144. } \}$

DETERMINATION.

When the right ascension of the mid-heaven falls in the first quadrant, its quantity in degrees, increased by 90, gives the angle zPp , and the acute angle Ppz is the complement of the longitude of the nonagesimal degree.

When in the second quadrant, the said right ascension in degrees taken from 270, leaves the angle zPp ; and the acute angle Ppz , increased by 90 degrees, is the longitude of the nonagesimal degree.

When the said right ascension falls in the third quadrant, its degrees, taken from 270, leaves the angle zPp , and the angle Ppz , increased by 90 degrees, is the longitude sought.

When

When in the fourth quadrant the said right ascension in degrees, lessened by 270, leaves the angle $z p p$; and the supplement of the angle, $p p z$, so long as it continues to be obtuse, being added to three right angles, or 270 degrees, gives the longitude of the nonagesimal degree; but after it becomes acute, its complement is the longitude required.

Note. In these determinations the latitude of the place is supposed to be north, and less than the distance of the tropic from the nearest pole.

EXAMPLE. At Greenwich, in latitude $51^{\circ} 28' \frac{1}{2}$ N. the obliquity of the ecliptic being $23^{\circ} 28'$: What is the longitude of the nonagesimal degree on the 14th of May 1780, at 1 h. 24 m. 24 s. P. M. at 3 h. 40 m. 2 s. P. M. at 13 h. 22 m. 26 s. P. M. and at 15 h. 38 m. 4 s. P. M.?

The several right ascensions of the the mid-heaven are thus found:

	h m s	h m s	h m s	h m s
1780. May 14th, at	1 24 24	3 40 2	13 22 26	15 38 4
The Sun's right ascen.	3 27 36	3 27 58	3 29 34	3 29 56
Right asc. mid. hea. in time	4 52 0	7 8 0	16 52 0	19 8 0
Right asc. mid. hea. in degrees	73° 00'	107° 00'	253° 00'	287° 00'
	90 00	270 00	270 00	270 00
The angle $z p p$, or $z p p$	163 00	163 00	17 00	17 00

The two following operations are wrought by IV. 237. and 238.

$p z$	$38^{\circ} 31' \frac{1}{2}$			
$p p$	$23 28$			
Sum	61 $59' \frac{1}{2}$, half is	30° $59' \frac{1}{2}$	Ar. co. s. 0,28823	Ar. co. s. 0,06691
Diff.	15 $3' \frac{1}{2}$, half is	7 $31' \frac{1}{2}$	s. 9,11729	s. 9,99624
$\angle z p p$	163 00, half is	81 30	t. 9,17450	t. 9,17450
Half diff. $\angle s z$ and p	2 $10' \frac{1}{2}$		t. 8,58002	
Half sum	9 $48' \frac{1}{2}$			t. 9,23765
The angle p	11 59			
	90 00			
The difference	78 01	is the long. nonag. in 1st. quad.		
The sum	101 59	is the long. nonag. in 2d. quad.		
Sum	61° $59' \frac{1}{2}$, half	30° $59' \frac{1}{2}$	Ar. co. s. 0,28823	Ar. co. s. 0,06691
Diff.	15 $3' \frac{1}{2}$, half	7 $31' \frac{1}{2}$	s. 9,11729	s. 9,99624
$\angle z p p$	17 00, half	8 30	t. 10,82550	t. 10,82550
Half diff. $\angle s z$ and p	59 34		t. 10,23102	
Half sum	82 38			t. 10,88865
The angle at p	142 12, its supplement is			37° 48'
	90 00			270 00
Long nonag. in 3d qd.	232 12	Long. nonag. in 4th. qd.	307 48	

* * * The altitude of the nonagesimal being equal to $z p$, the distance of the zenith from the pole of the ecliptic, it is found by the sines of opposite sides and angles in the spheric triangle $z p p$; that is, $\sin.$ long. nonag. ; $\cos.$ lat. ; ; $\sin.$ $\angle z p p$; $\sin.$ alt. nonagesimal,

SECTION

SECTION VII.

Of Practical Astronomy.

205. DESCRIPTION and USE of ASTRONOMICAL INSTRUMENTS.

By PRACTICAL ASTRONOMY is meant the knowledge of observing the celestial bodies with respect to their position, and time of the year; and of deducing from those observations, certain conclusions useful in calculating the time, when any proposed position of those bodies shall happen.

For this purpose the Astronomer, or Observer, should have an observatory properly furnished.

An OBSERVATORY is a room or place, conveniently situated, contrived, and furnished with proper astronomical instruments for observing the motions of the heavenly bodies: it should have an uninterrupted view, from the zenith, down to (or even below) the horizon, at least towards its cardinal points; and for this purpose that part of the roof which lies in the direction of the meridian, in particular, should have moveable covers, which may be easily removed and put on again: by which means an instrument may be directed to any point of the heavens between the horizon and zenith, as well to the northward as southward.

The furniture should consist of some, if not all, of the following instruments.

1st. A PENDULUM CLOCK for shewing equal time.

2d. An Achromatic REFRACTING TELESCOPE, or a REFLECTING ONE, of two feet at least in length, for observing particular phenomena.

3d. A MICROMETER for measuring small angular distances.

4th. An ASTRONOMICAL QUADRANT for observing meridian altitudes of the celestial bodies.

5th. A TRANSIT INSTRUMENT for observing objects as they pass over the meridian.

6th. An EQUATORIAL SECTOR to observe angular distances of several degrees, and the differences of right ascension and declination.

7th. An EQUAL ALTITUDE INSTRUMENT for finding when an object has the same altitude on both sides of the meridian.

It is not intended to give in this work any other than a general account of these instruments, most of which have met with considerable improvements (if they were not contrived) by the late Mr. George Graham, F. R. S. one of the most eminent Artists in mechanical contrivances that this, or any other nation has produced; those readers who are curious to see a minute description of such, and other, instruments, together with their use fully exemplified, may consult the second volume of Dr. Smith's complete Treatise of Optics, Stone's Treatise of Mathematical Instruments, the Philosophical Transactions, and the works of many writers who have treated on such subjects.

206.

Of the Pendulum Clock.

A clock which shews time in hours, minutes, and seconds, should be chosen; with which the observer, by hearing the beats of the pendulum, may count them by his ear, while his eye is employed on the motion of the celestial object he is observing.

Just before the object arrives at the position desired, the Observer should look on the clock and remark the time; suppose it $9^h 15^m 25^s$; then saying 25, 26, 27, 28, &c. responsive to the beats of the pendulum, till he sees through the instrument the object arrived at the position expected, which suppose to happen when he says 38; he then writes down $9^h 15^m 38^s$ for the time of observation, annexing the year and day of the month.

If two persons are concerned in making the observation, one may read the time audibly, while the other observes through the instrument, the Observer repeating the last second read, when the desired position happens.

207.

Of the Telescope.

THE REFRACTING TELESCOPE is an instrument with which almost every person is acquainted, especially the marine gentlemen; it will therefore be sufficient to remark here, that an *astronomical telescope* has only two convex glasses; viz. the eye-glass, or that which is used next to the eye; and one at the other end, usually called the object-glass, which has much the longer focal distance: such an instrument, although it inverts all objects, is yet as useful for viewing those in the heavens, as if it shewed them erect; the Observer knowing that the motions are in an opposite direction to those he sees through this telescope: But the Achromatic Refracting Telescope, which has been lately invented by Mr. Dollond, has its object-glass compounded of three glasses, and combined with two eye-glasses placed near each other. This instrument, which shews objects in their true position, need not exceed three feet and a half in length.

THE REFLECTING TELESCOPE, as is generally well known, shews objects in their true positions; and as it is much shorter than the old reflector, it is therefore in much greater esteem by some.

A *telescope*, used in astronomical observations, should have a metal frame fixed in the focus of its object-glass, carrying fine silver wires stretched at right angles to one another; one of them is to be vertical, and the other horizontal; the intersection of those wires ought to be exactly in the middle of the focus of the object-glass; a line passing through this intersection and the center of the object-glass, is called the line of sight, or line of *collimation*.

208.

Of the Micrometer.

A MICROMETER is an instrument used to measure small angular distances by being placed in the focus of a telescope. This is effected by turning a screw, which moves a fine wire in a position parallel to itself, and also parallel to a fixed wire; both being in a plane at right angles to the line of collimation: the distance of these parallel wires is measured by the number of turns the screw has taken to cause their recess; which number of turns is shewn on a graduated circular plate (like that of a clock) by an index,

index, or hand, which revolves by the turning of the screw: now the divisions on the plate, answering to a known angle or arc intercepted between the parallel wires, being known by experiment, any other distance, to which the wires can recede, may be known by proportion; and so a table of angles answering to every division on the circular plate may be formed, by which the observed angles will be readily known.

Thus in observing the diameter of a planet; when the wires are removed so far asunder, as to become parallel tangents at the same time to opposite points of the planet, the measure of the recess of the wires will shew the diameter of the planet in minutes and seconds.

There is another micrometer published by the late very ingenious Mr. Dollond *, an account of which was given to the Royal Society by Mr. James Short, F. R. S. and published in the Philosophical Transactions for the year 1753, which is thus:

Let a good circular object-glass be neatly cut into two semicircles; and each semicircle fitted in a metal frame, so that their diameters sliding on one another (by the means of a screw) may have their centers so brought together as to appear like one glass, and so form one image; or by their centers receding may form two images of the same object: it being a property of such glasses, for any segment to exhibit a perfect image of an object, although not so bright as the whole glass would give it.

Now proper scales being fitted to this instrument, to shew how far the centers recede, relative to the focal length of the glass, will also shew how far the two parts of the same object are asunder relative to its distance from the object-glass; and consequently give the angle under which the distance of the parts of that object are seen.

209.

Of the Astronomical Quadrant.

AN ASTRONOMICAL QUADRANT is an instrument in the form of a quarter of a circle, contained under two radii at right angles to one another, and an arch equal to one fourth part of the circumference of the circle, and consists of the following parts:

1st. *Its frame.* This is usually composed of iron or brass bars, set at right angles to one another in as strong and neat a manner as a workman can contrive, to preserve the face of the instrument in the same plane, and be as little affected by heat and cold as is possible.

2d. *Its center.* This center, which is a very fine point, should be contained in a separate piece of work screwed to the bars; and so contrived, that if the index, or telescope, by frequent motion in a length of time, should become irregular in its rotation, by the parts wearing, a new collar and socket may be fitted to the first center, and the instrument restored to its original accuracy.

* The first notion of such a micrometer was given by Roemer, a Dane, in the year 1675; Mr. Savery, an Englishman, also thought of such a contrivance, which he communicated to the Royal Society in the year 1743; Mr. Bouguer, a Frenchman, also proposed it in the year 1748; and Mr. Dollond, an Englishman, published it in the year 1753: but the public are obliged to Mr. Short for putting the theory into execution, otherwise it might still have continued only as an ingenious thought.

3d. *Its*

3d. *Its limb.* This is a brass arch of about two inches broad, well fixed to the said frame-work, and generally continued a little farther at each end than the extent of 90 degrees; the two perpendicular radii may be also covered with plates of brass screwed to them; and the whole face of the instrument is to be worked smooth, and brought into the same plane with the greatest care.

4th. *Its divisions.* The arcs of 60, 30, and 90 degrees, and also the intermediate degrees, together with such subdivisions as the size of the degrees will conveniently contain, are laid down by the accurate methods well known to good workmen.

5th. *Its index or telescope.* This, which is usually a brass tube containing the proper glasses and cross wires, is fixed near the object end to a brass plate, a little above a circular hole, or socket, in the plate: this socket goes round a collar concentric to the center, and fixed to the center-piece: so that, although the axis of the telescope does not, as a radius, pass through the center, yet it always keeps at the same distance from it in every position: to the eye-end of the tube is screwed a flat plate, which slides along the limb with the telescope; this plate, called the *VERNIER*, contains certain divisions, which, used with those on the limb, give the angle to minutes or seconds of a degree, according to the size of the instrument: the beginning of the divisions called the *index*, on the *Vernier*, is as far distant from the axis, or line of collimation, as the center is; and therefore the position of an object is given as truly, as if the line of collimation coincided with a radius.

6th. *Its pedestal.* This part, which should, by its construction, be very steady, may be either moveable or fixed: the moveable pedestal is commonly a strong pillar standing on a tripod, or three-footed stand; with holes through each foot, either to screw them to a floor, or to pin them to the ground: the fixed pedestal may be either a strong timber frame, or the wall of the observatory, or a stone shaft built from the ground through the middle of the floor of the observatory. On the top of the pillar, of either sort of pedestal, may be fixed a piece of machinery called the *arm*, which is attached by screws to the middle of the plane of the quadrant, on the under side. The arm is contrived to give to the instrument, either an horizontal, vertical, or oblique motion; which motions should be steady, and free from jerks, or shakes: but when a wall, or stone shaft, is used as the supporter, the quadrant is then fixed to the wall, or shaft (without its arm attached) and is called a *MURAL ARCH*; its plane is adjusted to that of the meridian, and this is the best method of fixing the quadrant for taking the meridian altitude of the stars, or planets.

7th. *Its plummet.* This is a sufficient ball, or weight, hanging to one end of a very fine silver wire, the upper end being fixed in the radius continued above the center. Now when the face of the quadrant is set in the plane of an azimuth circle, one of its radii is brought into a vertical position by the help of the plummet, the wire being made to bisect the center-point and the division of 90° on the arch; and to distinguish these bisections with accuracy, they are to be examined with a small prospect, or magnifying-glass: the ball should hang freely in a vessel of water to check its vibrations.

210.

Of the transit Instrument.

This instrument consists of a telescope fixed at right angles to an horizontal axis, which axis must be so supported, that the line of collimation of the telescope may move in the plane of the meridian.

The axis, to the middle of which the telescope is fixed, should gradually taper towards its ends, and terminate in cylinders well turned and smoothed: and a proper balance is to be put on the tube, so that it may stand at any elevation when its axis rests on the supporters.

Two upright posts of wood or stone, firmly fixed at a proper distance, are to sustain the supporters of this instrument: these supporters are two thick brass plates, having well smoothed angular notches in their upper ends to receive the cylindrical arms of the axis: each of the notched plates are contrived to be moveable by a screw, which slides them upon the surfaces of two other plates immoveably fixed to the two upright posts; one plate moving in a vertical, and the other in an horizontal, direction, to adjust the telescope to the planes of the horizon and meridian: to the plane of the horizon, by a spirit level hung in a parallel position to the axis, and to the plane of the meridian in the following manner:

Observe the times by the clock when a circumpolar star, seen through this instrument, transits both above and below the pole: and if the times of describing the eastern and western parts of its circuit are equal, the telescope is then in the plane of the meridian; otherwise, the notched plates must be gently moved till the time of the star's revolution is bisected by both the upper and lower transits, taking care at the same time that the axis remains perfectly horizontal.

When the telescope is thus adjusted, a mark must be set at a considerable distance (the greater the better) in the horizontal direction of the intersection of the cross-wires, and in a place where it can be illuminated in the night-time by a lanthorn hanging near it; which mark being on a fixed object, will serve at all times afterwards to examine the position of the telescope by, the axis of the instrument being first adjusted by means of the level.

211. *To adjust the Clock by the Sun's Transit over the Meridian.*

Note the times by the clock, when the preceding and following edges of the sun's limb touch the cross wires: the difference between the middle time and 12 hours, shews how much the mean, or time by the clock, is faster or slower than the apparent, or solar time for that day; to which the equation of time being applied, will shew the time of mean noon for that day, by which the clock may be adjusted.

212.

Of the Equatorial Sector.

This is an instrument contrived for finding the difference in right ascension and declination between two objects, the distance of which is too great

great to be observed by means of a micrometer. It consists of the following particulars :

1st. A brass plate called a sector, formed like a T, having the shank (as a radius) of about $2\frac{1}{2}$ feet long, and 2 inches broad, and the cross piece (as an arch) of about 6 inches long, and $1\frac{1}{2}$ inch broad ; upon which, with a radius of 30 inches, is described an arch of 10 degrees, each being subdivided into as small parts as are convenient.

2d. Round a small cylinder, containing the center of this arch, and fixed in the shank, moves a plate of brass, to which is fixed a telescope, having its line of collimation parallel to the plane of the sector, and passing through the center of the arch and the index of a *Vernier's* dividing plate, which slides on the arch, and is fixed to the eye end of the telescope. This plate, with the telescope and Vernier, are moved on the cylinder, by means of a long screw which is at the back of the arch, and communicates with the Vernier through a slit cut in the brass work, parallel to the divided arch.

3d. A circular brass plate, of 5 inches diameter, round the center of which there moves a brass cross, which has the opposite ends of one bar turned up perpendicularly about 3 inches. These serve as supporters to the sector, and are screwed to the back of its radius, so that the plane of the sector is parallel to the plane of the circular plate, and revolves round the center of that plate in this parallel position.

4th. A flat axis of 18 inches long is screwed to the back of the circular plate, along one of its diameters ; so that the axis is parallel to the plane of the sector : the whole instrument is supported on a proper pedestal, in such a manner that the said axis is parallel to the axis of the earth ; and proper contrivances are annexed for fixing it in that position.

Now the instrument, thus supported, can revolve round its axis, parallel to the earth's axis, with a motion like that of the stars ; the plane of the sector being always parallel to the plane of some hour circle, and consequently every point of the telescope describes a parallel of declination ; and if the sector be turned round the joint of the circular plate, its graduated arch may be brought parallel to an hour circle ; and consequently any two stars, between which the difference of declination is not greater than the number of degrees in that arch, may be observed by the instrument.

213. *To observe their passage.* Direct the telescope to the preceding star, and fix the plane of the sector a little to the westward of it ; move the telescope by the screw, and observe the time shewn by the clock at the transit of each star over the cross wires, and also the division shewn by the index ; then is the difference of the arches the difference of declination ; and that of the times shews the difference of right ascension of those stars.

214.

Of the Equal-Altitude Instrument.

AN EQUAL-ALTITUDE INSTRUMENT is that used to observe a celestial object, when it has the same altitude on both the east and west sides of the meridian, or in the morning and afternoon ; and consists of a telescope of about 30 inches long (with 2 vertical, and 3 or 5 horizontal, wires in its focus) supported on the end of an iron bar, or axis, of 30 inches long, and about an inch in diameter. The axis is sustained

in a vertical position by passing through a hole in the upper end of a brass box, whilst its lower end supports the lower point of the axis. The box, which is about 21 inches long, with ends about 4 inches square, has only two sides, which are fixed at right angles to each other. To one of these sides are fixed four flat arms, with a hole in each, by which the box is fixed in a vertical position to an upright post with screws. On the lower end of the box lies a brass plate, which slides in grooves, and can be moved gently backwards or forwards by means of a screw. In this plate a fine hole is punched to receive the smooth conical point, which the lower end of the axis is formed into. On the upper end of the box are two plates, which slide also in grooves; and, by the means of screws, can be moved gently sideways, till their angular notches embrace the axis; which, in this part, is made perfectly cylindrical, and very smooth.

To the upper part of the axis is fixed, by its radius, a brass sextant (or arch of 60° , to a radius of seven or eight inches) with the arch downwards, so that the center is just above the top of the axis: also a spirit-level is fixed at right angles across the axis, just under the arch, so as to be clear of the upper end of the box.

To the under part of the telescope is fixed a brass semicircle, of the same radius with the sextant, both arches having a common center-pin. In the semicircle is a groove cut through the plate parallel to its limb, to receive two screw-pins, which go into the sextantal arch near its ends; by these screw-pins the two arches may be pressed close, and the telescope fixed in any desired elevation; which might be nearly ascertained by graduating the semicircle, and putting a Vernier's scale on the sextant.

To use the Instrument. Fix the box to the post, put the axis into the box, letting the conical point drop into the punched hole, screw on the level, and annex the telescope, observing to insert the center and arch pins; then, by the help of the screw-plates at the bottom and top ends of the box, correct the vertical position of the axis, so that the same end of the air-bubble in the level may stand at the same point throughout the whole revolution of the axis, which will thereby be known to be then truly vertical, so that the telescope will describe a parallel of altitude: direct the tube to the sun, or star, and fix it at the desired elevation by pressing the two arches together with the two screw pins.

Some instruments have been contrived to answer both kinds of observation; viz. either a transit, or equal altitudes.

215

To adjust the Clock by equal altitudes of the Sun.

Having rectified the instrument by the level, and being provided with a piece of transparently coloured or smoked glass to preserve the eye; then at any convenient time from about 6 to 3 hours before noon, direct the telescope to the sun, and fix it by the arch, so that the whole body of the sun shall be above the upper wire (the ascent of the Sun appearing through the telescope as a descent): mark the times shewn by the clock, when the preceding edge of the Sun touches each of the wires; and also when the

the following edge touches those wires, writing down those times; the instrument being turned horizontally on its axis to follow the Sun, and keep his center in the middle of the telescope between the vertical wires. About the same time after noon (taking care to be early enough) turn the instrument on its vertical axis, the telescope remaining fixed at the same elevation as in the morning, and rectifying its horizontal situation by the level, observe the Sun in its descent, which through the telescope apparently ascends, and write down the times when the preceding edge touches each wire, and also when they are touched by the following edge, keeping the Sun in the middle of the telescope; and the sets of observations are made.

There are as many sets of observations, as there are horizontal wires: for the fore and afternoon contacts of the same edge of the Sun with the same wire, make one set; and the same edge which precedes in the forenoon, follows in the afternoon; and that which follows in the forenoon, precedes in the afternoon; therefore the

1st, } A. M. preceding, and the { last, } P. M. following,
 2d, } { last but one, }
 3d, &c. } { last but 2, &c. }
 make a set.

1st, } A. M. following and the { last, } P. M. preceding,
 2d, } { last but one, }
 3d, &c. } { last but 2, &c. }
 make a set.

Then to each set, or pair of observations, find the middle time, which added to the time of the morning observation, gives the time shewn by the clock when the Sun was on the meridian, if the observations were made within two or three days of the solstice, when the Sun's declination would not sensibly alter between the fore and afternoon observations: but on other days, this time must be corrected, by applying an equation to it, shewing the alteration in time, arising from the alteration in declination between the fore and afternoon observations.

The time, by the clock, of the solar or apparent noon being thus obtained, the time of the mean noon may be had by applying the proper equation of time.

When the time of noon is sought from two or more pairs of observations, if they give different times, it is best to take the medium between them, which is found by dividing the sum of all the times by their number.

216.

PROBLEM LV.

Given the latitude, the declination of the Sun, and interval of time between the Sun's having equal altitudes before and after noon, to find the distance from noon of the middle point of time between the observations.

1st. Find the change made in the Sun's declination during the interval between the observations; which will nearly bear the same proportion to the change made between the noon of the day, on which
 T 2 the

the observations are made, and the noon of the day immediately preceding or following, as the interval of time between the observations to 24 hours.

2dly. Add the co-tangent of the latitude to the co-sine of half the interval of time reduced to degrees and minutes of the equator; the sum, rejecting the radius, is the tangent of an arc to be taken less than a quadrant, when the interval of time is less than the twelve hours, and greater than a quadrant, should the interval of time exceed twelve hours.

3dly. Add together the arithmetical complements of the sine of this arc, and of the sine of the Sun's distance from the pole at noon, the logarithmic sine of the difference of these arcs, the logarithmic co-tangent of half the interval of time in degrees, and the logarithm of half the change in the Sun's declination during the interval between the observations; the sum, rejecting twice the radius, is the logarithm of an arc, which, divided by 15, gives the distance of the middle point between the observations from noon, in seconds of time.

4thly. When the Sun's distance from the elevated pole increases, this middle point of time precedes noon, otherwise it falls beyond.

DEMONSTRATION.

Let P be the pole of the equator bd , z the zenith, A the morning place, c the afternoon place, ABD a parallel of declination, ABC a parallel of altitude.

Then the afternoon hour angle, ZPC , differs from the morning hour angle, ZPA , by the hour angle BPC , the points A and B having the same declination and distance from the zenith; and the arc Po bisecting the $\angle BPC$, the $\angle ZPo$ will be half the interval of time, which being increased or diminished by half the $\angle BPC$, will give the position of the meridian to A or c ; also Po will be the Sun's distance from the pole at noon.

Now here the difference between PB , Po , PC , being but small, the $\angle BPo$ will be to the difference between Po and PB , or PC , that is, half $PB \oslash PC$, nearly as t° , PoZ to s, Po^* .

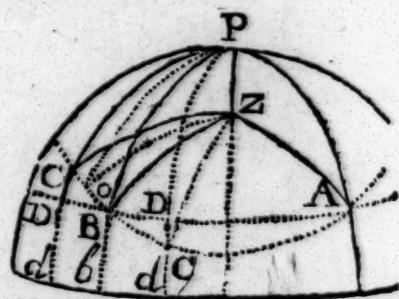
Again, in the triangle oPZ , an arc M being taken, that as $\text{rad.} : s^\circ, ZPo :: t^\circ, PZ :: t^\circ, M$ the arc M to be less than a quadrant, when the $\angle ZPo$ is acute, but greater, should the $\angle ZPo$ be obtuse; then (IV. 123)

$s, Po \oslash M : s, M :: t^\circ, PoZ : t^\circ, ZPo$, and consequently

$s, Po \oslash M \times t^\circ, ZPo = s, M \times t^\circ, PoZ$.

But $\angle BPo : \frac{PB \oslash PC}{2} :: t^\circ, PoZ : s, Po :: s, M \times t^\circ, PoZ : s, M \times s, Po$;

therefore $\angle BPo : \frac{PB \oslash PC}{2} :: s, Po \oslash M \times t^\circ, ZPo : s, M \times s, Po$, conformably to the rule above laid down.



* See Cotes's *Æstimat. Error. in Mixt. Math. Theorem 23.*

EXAMPLE.

217.

In the latitude 50° N. on the 27th of October, 1780, the Sun was observed to have equal altitudes at 9 h. 11 m. 50 s. A. M. and at 2 h. 22 m. 22 s. P. M. by a clock adjusted nearly to the true measure of time, to find what correction may be wanted to set this clock to the true hour of the day, the Sun's distance from the pole on the 27th day at noon being $103^{\circ} 6' 34''$, and on the 28th. $103^{\circ} 26' 38''$.

Here the interval of time is 5 h. 10 m. 32 s. its half is 2 h. 35 m. 16 s. in degrees and minutes of the equator $38^{\circ} 49'$, and the difference in declination in one day is $20' 4''$.

Then $24 \text{ h.} : 5 \text{ h. } 10 \text{ m. } 32 \text{ s.} :: 20' 4'' : 4' 20'' = 260''$, the alteration in declination, the half of which is $2' 10'' = 130'$.

Then

Latitude 50°	log. t , 9,92381	$33^{\circ} 10' 32''$	L's,	0,26185
$\frac{1}{2}$ time = $39^{\circ} 49'$	log. s , 9,89162	$103^{\circ} 06' 34''$	L's,	0,01147
		$69^{\circ} 56' 02''$	log. s ,	9,97280
$33^{\circ} 10' 32''$	log. t , 9,81543	$38^{\circ} 49' 0''$	log. t ,	10,09447
		$130''$	log.	2,11394
		corr. = $285''$	log.	2,45453
		or 19 sec. of time.		

Hence, for setting the clock to the true hour of the day, add the half interval of time 2 h. 35 m. 16 s. to 9 h. 11 m. 50 s.; the sum 11 h. 47 m. 6 s. is the middle time between the observations, as noted by the clock.

And 11 h. 47 m. 6 s. + 19 s. = 11 h. 47 m. 25 s. will be the time pointed out by the clock, when the Sun passes the meridian, and shews the clock to be 12 m. 35 s. behind the Sun.

Though the clock should not keep time with perfect exactness, yet if the deviation is but small, the correction computed will not differ much from the truth; and the clock being examined again within a few days, will shew whether it keeps time truly, or moves too fast or too slow, and its rate of going may be corrected accordingly.

218. The method here directed supposes the ship to be stationary: But the Abbé de la Caille proposes a method of correcting a watch at sea, even while the ship is in motion, by taking two equal altitudes of the Sun with a quadrant, one before and the other after noon.

His method is this: With the common altitude observed, together with the latitudes at the time of each observation, and the Sun's correct declination to those times, the times from noon are to be computed at both observations, which times being applied to the two times of observation, give the respective times of noon: then the mean of the two noons being taken, will give the time shewn by the watch when it was the true mid-day.

Or: Half the difference of the computed noons being applied to either of them, will also give the true time of noon.

And although the latitudes used in the computations be something erroneous, yet the altitudes being equal, the error in each of the computed distances from noon will be nearly the same, if the change in latitude and longitude between the observations be duly attended to.

219.

Of the Vernier's dividing Plate.

When the relative unit of any line is to be divided into many small equal parts, those parts may be too many to be conveniently introduced, or if introduced, they may be too close to one another to be readily estimated; and on these accounts there has been a variety of methods contrived for estimating the aliquot parts of the small divisions, into which the relative unit of a line may be commodiously divided; among those methods that is most justly preferred, which was published by PETER VERNIER (a gentleman of Franche Comté) at Brussels, in the year 1631; and which, by some strange fatality, is most unjustly, although commonly, called by the name of NONIUS: for *Nonius's* method is not only very different from that of *Vernier's*, but much less convenient.

Vernier's method is derived from the following principle.

If two equal right lines, or circular arcs A, B, are so divided, that the number of equal divisions in B is one less than the number of equal divisions of A; then will the excess of one division of B above one division of A, be compounded of the ratios of one of A to A, and one of B to B.

For let A contain 11 parts; then one of A to A, is as 1 to 11; or $\frac{1}{11}$.

Let B contain 10 parts; then one of B to B, is as 1 to 10; or $\frac{1}{10}$.

$$\text{Now } \frac{1}{10} - \frac{1}{11} = \left(\frac{1 \times 11}{10 \times 11} - \frac{1 \times 10}{11 \times 10} \right) \text{ (II. 148) } = \frac{11-10}{10 \times 11} = \frac{1}{10 \times 11} = \frac{1}{10} \times \frac{1}{11}.$$

Or: If B contains n parts, and A is of $n+1$ parts;

Then $\frac{1}{n}$ is one part of B, and $\frac{1}{n+1}$ is one part of A.

$$\text{And } \frac{1}{n} - \frac{1}{n+1} = \left(\frac{1 \times n+1}{n \times n+1} - \frac{1 \times n}{n+1 \times n} \right) \text{ (II. 148) } = \frac{n+1-n}{n \times n+1} = \frac{1}{n \times n+1} = \frac{1}{n} \times \frac{1}{n+1}.$$

Or thus: Let A and B be unequal right lines, or circular arcs; and let any part of A, considered as the relative unit, be divided into n parts; and a part of B, equal to $m+1$ parts of A, be divided into m parts; then will $\frac{1}{m}$ th of B = $\frac{1}{n}$ th of A = $\frac{1}{m}$ th of B $\times$ $\frac{1}{n}$ th of A.

But n parts of A : 1 unit of A :: $m+1$ parts of A : $\frac{m+1}{n}$ units of A.

But m parts of B = ($m+1$ parts of A =) $\frac{m+1}{n}$ units of A.

Then m parts of B : $\frac{m+1}{n}$ units of A :: 1 part of B : $\frac{m+1}{m \times n}$ units of A.

Therefore

Therefore $\frac{m+1}{m \times n} - \frac{1}{n} = \left(\frac{m+1}{m \times n} - \frac{m \times 1}{n \times m} = \frac{m+1-m}{m \times n} = \frac{1}{n \times m} = \right)$
 $\frac{1}{m} \times \frac{1}{n}$

The most commodious divisions, and their aliquot parts, into which the degrees on the circular limb of an instrument may be supposed to be divided, depend on the radius of that instrument.

Let R be the radius of a circle in inches; and a degree to be divided in n parts, each degree being $\frac{1}{p}$ th of an inch.

Now the circumference of a circle in parts of its diameter, $2R$ inches, is $3,1415926 \times 2R$ inches. (Il. 197)

Then $360^\circ : 3,1415926 \times 2R :: 1^\circ : \frac{3,1415926}{360} \times 2R$ inches.

Or, $0,01745379 \times R$ is the length of one degree in inches.

Or, $0,01745379 \times R \times p$ is the length of 1° , in p th parts of an inch.

But as every degree contains n times such parts,

Therefore $n = 0,01745379 \times R \times p$.

The most commodious perceptible division is $\frac{1}{8}$ or $\frac{1}{10}$ of an inch.

EXAM. Suppose an instrument of 30 inches radius: Into how many convenient parts may each degree be divided? How many of those parts are to go to the breadth of the Vernier, and to what parts of a degree may an observation be made by that instrument?

Now $0,01745 \times R = 0,5236$ inches, the length of each degree.

And if p be supposed about $\frac{1}{8}$ of an inch for one division,

Then $0,5236 \times p = 4,188$, shews the number of such parts in a degree.

But as this number must be an integer, let it be 4, each being $15'$.

And let the breadth of the Vernier contain 31 of those parts, or $7\frac{1}{4}^\circ$, and be divided into 30 parts.

Here $n = \frac{1}{4}$; $m = \frac{1}{30}$; then $\frac{1}{4} \times \frac{1}{30} = \frac{1}{120}$ of a degree, or $30''$.

Which is the least part of a degree that instrument can shew.

If $n = \frac{1}{5}$, and $m = \frac{1}{36}$; then $\frac{1}{5} \times \frac{1}{36} = \frac{60}{5 \times 36}$ of a minute, or $20''$.

220. The following table, taken as examples in the instruments commonly made from 3 inches to 8 feet radius, shews the divisions of the limb to nearest tenths of inches, so as to be an aliquot of 60's, and what parts of a degree may be estimated by the Vernier, it being divided into such equal parts, and containing such degrees, as their columns shew.

T 4

Rad.

Rad. inches	Parts in a deg.	Parts in Vernier.	Breadth of Ver.	Parts observed.	Rad. inches	Parts in a deg.	Parts in Vernier	Breadth of Ver.	Parts observed.
3	1	15	$15\frac{1}{4}^{\circ}$	4' 0"	30	5	30	$7\frac{1}{2}^{\circ}$	0' 20"
6	1	20	$20\frac{1}{4}^{\circ}$	3 0	36	6	30	$5\frac{1}{4}^{\circ}$	0 20
9	2	20	$10\frac{1}{4}^{\circ}$	1 30	42	8	30	$3\frac{7}{8}^{\circ}$	0 15
12	2	24	$12\frac{3}{4}^{\circ}$	1 15	48	9	40	$4\frac{5}{8}^{\circ}$	0 10
15	3	20	$6\frac{3}{4}^{\circ}$	1 0	60	10	36	$3\frac{7}{10}^{\circ}$	0 10
18	3	30	$10\frac{1}{4}^{\circ}$	0 40	72	12	30	$2\frac{7}{12}^{\circ}$	0 10
21	4	30	$7\frac{1}{4}^{\circ}$	0 30	84	15	40	$2\frac{2}{3}^{\circ}$	0 6
24	4	36	$9\frac{3}{4}^{\circ}$	0 25	96	15	60	4	0 4

By altering the number of divisions, either in the degrees or in the Vernier, or in both, an angle can be observed to a different degree of accuracy. Thus to a radius of 30 inches, if a degree be divided into 12 parts, each being five minutes, and the breadth of the Vernier be 21 such parts, or $1\frac{1}{4}^{\circ}$, and divided into 20 parts, then $\frac{1}{12} \times \frac{1}{20} = \frac{1^{\circ}}{240} = 15''$: or taking the breadth of the Vernier of $2\frac{1}{2}^{\circ}$, and divided into 30 parts; then $\frac{1}{12} \times \frac{1}{30} = \frac{1^{\circ}}{360}$, or $10''$: Or $\frac{1}{12} \times \frac{1}{50} = \frac{1^{\circ}}{600} = 6''$; where the breadth of the Vernier is $4\frac{1}{4}^{\circ}$.

SECTION VIII.

Practical Astronomy.

THE ELEMENTS of the EARTH'S MOTION.

221. By the theory of the Sun, or Earth, is meant the knowledge of all the requisites, or elements, necessary for determining its place in the ecliptic at any proposed time.

222. MEAN MOTION, or MEAN ANGULAR VELOCITY, is a motion made uniformly in the circumference of a circle, the center of motion being the center of that circle.

The mean motion of a planet is the degree and parts shewing its distance from the first point of Aries, reckoned in the order of the signs.

223. ANOMALY, or TRUE ANOMALY, is an angle made by two lines drawn from the center of motion, one to the Aphelion, or Apogee, and the other to the place of the revolving body, or planet: Or, ANOMALY is the angular distance of a planet from its Aphelion, the angular point being the center of motion.

224. MEAN ANOMALY is that made by an uniform circular motion about the center, and is the same as mean motion, beginning at the Aphelion.

225. Ex.

225. **ECCENTRIC ANOMALY** is an angular distance from the aphelion, determined in a circle on the transverse axis by a normal to that axis, passing through the planet's place in its elliptical orbit.

226. The **EQUATION OF THE CENTER**, sometimes called the *prosthaphæresis*, is the distance between the mean and true anomalies.

227. The *motion of the equinoxes* is the same as the *precession of the equinoxes*, which is backward, or contrary to the order of the signs; by which the stars appear to have advanced forward from the equinoctial point Aries: this motion is about 50 seconds of a degree in a year.

228. The *motion of the apsides* is a slow motion of the Earth's orbit around the Sun in the order of the signs; discovered by the apogee changing its place among the fixed stars: this motion is found, by comparing distant observations together, to be about 16 seconds of a degree in a year, in respect to the fixed stars; and about 66 seconds ($= 50'' + 16''$) with respect to the equinoxes.

229. A **TROPICAL or SOLAR YEAR** is the time elapsed between two successive passages of the Sun through the same equinoctial or solstitial points of the ecliptic.

230. A **SIDERAL YEAR** is the time the Sun takes between his departure from any fixed star to his next return to that star.

231. An **ANOMALASTIC YEAR** is the interval of time between two succeeding passages of the Sun through the same apsis.

232. By the annexed figure the foregoing articles may be easily comprehended.

On the line of the apsides AP describe a circle ADP, called the *excentric*; and an ellipsis AEP for the Earth's orbit, having the eccentricity cs.

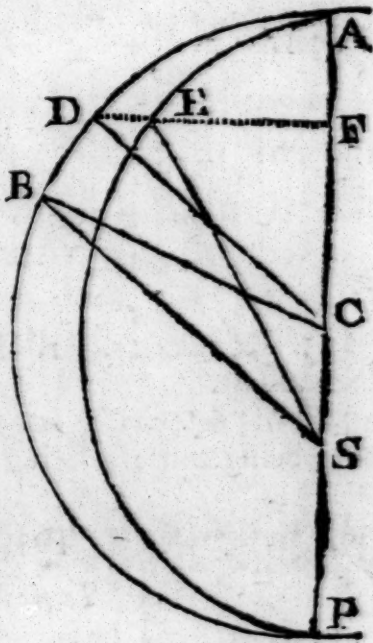
Let s be the place of the Sun, c the center of the orbit, A the aphelion, P the perihelion; SA the aphelion, or apogee distance; SP the perigee distance,

Let E be a true place of the earth in its orbit; D a corresponding place in the excentric, in FE continued, normal to AP.

Let the $\angle ACB$ represent the mean anomaly; the $\angle ACD$ is the eccentric anomaly; and the $\angle ASB$ is the true anomaly; the difference between $\angle ACB$ and $\angle ASB$ is the equation of the center.

When the Earth is in the apsides, then B and E fall together in A and P, and here is no equation of the center, the mean and true anomalies being equal; but the greatest equation of the center must be, when the Earth is at its mean distance from the Sun,

233. Observations shew, that in this age the Earth passes the apogee on the 30th of June, when its daily motion is $57' 12''$; and passes the perigee on the 30th of December, when the daily motion is $61' 12''$: and is at the mean distance about the 28th of March and 30th of September, when its daily motion is $59' 8''$.



234.

PROBLEM LVI.

To find the Latitude of a Place.

SOLUTION. Select a star, the distance of which from the pole star does not exceed 8 or 10 degrees; and observe with a quadrant the greatest and least meridional altitudes; then

If both observations are on the same side of the zenith;

Half the sum of the *alts.* is the latitude, on the same side of the zenith.

If the observations are on different sides of the zenith;

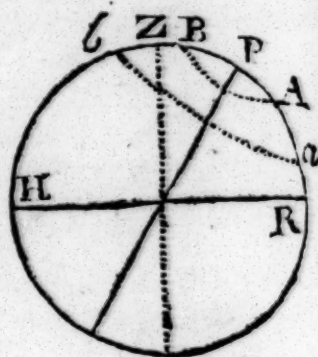
Half the difference of the altitudes is the co-latitude, on the same side of the zenith, with the lesser altitude.

For let HZR be the meridian, HR the horizon, Z the zenith, RB, RA, two altitudes on the same side of the zenith; Hb, Ra, two altitudes on contrary sides of the zenith.

Then, the arc AB, or *ab*, being bisected, will give P the position of the elevated pole.

For a star is equally distant from P in its revolution;

Therefore PA=PB; or Pa=Pb; and RP equal to the latitude.



$$\text{Hence } RP = \left(RA + PA = \frac{2RA}{2} + \frac{2PA}{2} = \frac{RA + RA + AB}{2} = \right) \frac{RA + RB}{2}$$

$$\begin{aligned} \text{And } RP &= \left(Ra + Pa = \frac{2Ra}{2} + \frac{2Pb}{2} = \frac{Ra + Ra + ab}{2} = \frac{RPb + Ra}{2} \right. \\ &= \left. \frac{180^\circ - Hb \text{ or } RA}{2} = \right) 90^\circ - \frac{Hb \text{ or } Ra}{2}. \end{aligned}$$

235. REMARKS. 1. There will be about 12 hours between the two observations.

2. This method is subject to a small error, on account of the lesser altitude being more affected by refraction, than the greater.

236.

PROBLEM LVII.

To find the Obliquity of the Ecliptic.

SOLUTION. Let the meridian altitude of the Sun's center be observed on the days of the summer and winter solstice; the difference of those altitudes will be the distance of the tropics; and half that distance will shew the obliquity of the ecliptic.

OR: The meridian altitude at the summer solstice, lessened by the co-latitude of the place, will give the obliquity of the ecliptic.

From good observations the obliquity of the ecliptic, about the time of the vernal equinox 1772, was found to be $23^\circ 28'$.

Distant observations compared together, shew that the obliquity is decreasing at the rate of about one minute in 120 years.

237. REMARK.

237. REMARK. By the second method the declination of the fixed stars, or of any other celestial phenomenon, may be found; observing that their declination is of the same name, *viz.* north or south, with the latitude of the place, when its complement is less than the altitude; otherwise, of a contrary name with the latitude.

238.

PROBLEM LVIII.

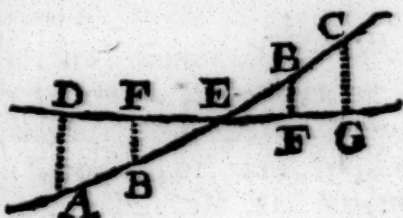
To find the Time of an Equinox.

SOLUTION. In a place the latitude of which is known, let the Sun's meridian altitude be taken on the day of the equinox, and on the day preceding, and that following it. Then the difference between those altitudes and the co-latitude will be the Sun's declinations at the times of observations. (236)

If either of the altitudes is equal to the co-latitude, that observation was made at the time of the equinox.

But if the co-latitude is unequal to either of the altitudes, proceed thus:

Let DG represent the equator; AC the ecliptic, E the equinoctial point; the points, A, B, C, the places of the Sun at the times of observation; the arcs AD, BF, CG, the corresponding declinations.



Now using either the two first, or two last observations, suppose the latter, in the right-angled spherical triangles CEG, BEF, in which there are known the obliquity of the ecliptic, and the declinations; find EC, EB; then BC, the sum or difference of EC, EB, is the ecliptic arc described in 24 hours. Then say,

As BC to BE, so 24 hours for BC, to the time corresponding to BE.

And this time shews the distance of the equinox from the time of the middle observation.

239.

PROBLEM LIX.

To find the Length of the tropical, periodical, and anomalistical Revolutions of the Earth.

SOLUTION. Let two observations be chosen, among the most authentic of those on record, of the time when the Sun had like positions, *viz.*

1st. In regard to his longitude, or place in the ecliptic.

2d. In regard to the right ascension of some noted star.

3d. In respect to the line of the apsidal.

The greater the interval (suppose 80 or 100 years) between each two observations, the more accurate will be the result: then that interval being divided by the number of revolutions made during that time, will give the time of one periodical revolution.

According to Mayer's tables the numbers are these,

A tropical year

is made in $365^d \ 5^h \ 48^m \ 42^s$.

A periodical, or siderial revolution

$365 \ 6 \ 9 \ 7$.

An anomalistic revolution

$365 \ 6 \ 15 \ 29$.

240. REMARKS.

240. REMARKS. 1st. The tropical year being shorter than the siderial by 20 m. 25 s., shews that the Sun has returned to the same point of the ecliptic, before he has made one complete revolution with regard to the stars; and consequently every point of the ecliptic must have moved in *antecedentia* during that tropical period, and so have produced what is called the precession of the equinoxes.

Now 365d. 6h. 9m. 7s. : 360° :: 20m. 25s. : $50''$,3, or nearly $50''$, for the precession in one year.

If there was no precession, the tropical and siderial years would be equal.

241. 2d. A siderial revolution being performed sooner by 6 m. 22 s. than the anomalistic, shews that the line of the apsidæ has a motion in *consequentia*: now 365d. 15h. 29m. : 360° :: 6m. 22s. : $15''$,7, the yearly quantity by which the Sun's apogee is advanced in respect to the stars: and as the equinoxes move in *antecedentia*, and the apsidæ in *consequentia*, their sum $66''$ ($=50,3 + 15,7$) shews the motion of the apsidæ from the equinoxes.

242. 3d. From the comparison of many observations it appears, that the length of the solar year, deduced from two very distant observations made at the time when the Sun was in the same point of the ecliptic near its apogee, differs by many seconds from the length of the year deduced by like observations, when the Sun was in another part of the ecliptic, near its perigee; those made near the apogee giving the revolutions less, and those made near the perigee making them greater, than the revolutions deduced from observations taken at the Sun's mean distance; this also shews, that the line of the apsidæ has a motion in *consequentia*; and that the length of a tropical revolution should be determined from very distant observations, made at the times when the Sun is at its mean distance from the Earth; or that the mean revolution should be taken between those deduced from observations made on the Sun's place, when he is in both the apogee and perigee.

243.

P R O B L E M LX.

To find the right Ascension of some noted fixed Star.

Having a good clock well regulated to mean or equal time, a large astronomical quadrant fixed in the plane of the meridian, and an equal altitude or transit instrument: then, on some day a little before or after the vernal equinox, when the daily alteration of the Sun's declination is about 18 or 20 minutes, observe the Sun's meridian altitude; and by equal altitudes find the times when both Sun and star come to the meridian; the difference of these times is their difference of right ascension.

Again: At some time a little after or before the autumnal equinox, before the Sun has passed the said declination, observe his meridian altitude; and by equal altitudes find the times of the Sun and same star's coming to the meridian, the difference of those times is also the difference of their right ascensions.

If the vernal and autumnal meridian altitudes are the same, then those observations were made, when the Sun was on the same parallel of

of declination: now the sum, or diff. of the two observed differences of right ascension, shews the equatorial arc described by the Sun between those times; which arc, being bisected, shews the distance of the nearest solstice from the Sun, at the time when the observations have equal altitudes; and that distance corrected and taken from 90° , shews the Sun's right ascension at the vernal observation, or its complement to 360 degrees.

From hence, and the first difference of right ascension between the Sun and star, the star's right ascension will be obtained.

244. If the two meridian altitudes of the Sun are not the same, their difference shews the difference of the mid-day declinations, when those observations were taken: now from some tables of right ascension and declination take the Sun's daily alteration in declination and right ascension on the day the lesser altitude was taken; then say, *As the daily change of decl. is to that of right ascen. ; so is the diff. of the altitudes, to the correction in right ascension.*

This correction being added to the vernal, or subtracted from the autumnal difference of right ascension, as either is least, reduces that difference of right ascension to what it would be when the declination is the same with the other; and then the difference between those two differences of right ascension, so reduced, gives the equatorial arc, as before recited.

245. At the Royal Observatory at Greenwich, in the year 1770, observations were made on the Sun and the star α Aquilæ.

March 15, Sun's mer. zen. dist. cleared of refraction and parallax, was $53^\circ 28' 29''$; and their diff. of rt. asc. was $60^\circ 30' 7,8''$.

Sept. 28th. Sun's mer. zen. dist. cleared of refraction and parallax, was $53^\circ 36' 26''$; and their diff. of rt. asc. was $109^\circ 59' 22,8''$.

Then $7' 57''$ is the diff. of zen. dists. or the alteration in declination.

Also $23' 40''$ and ($3^m 39^s$ or) $54' 45''$ are the diffs. of decl. and rt. asc. between the 15th and 16th days of March 1770.

Now $23' 40'' : 7' 57'' :: 54' 45'' : 18' 23,5''$ the increase of the difference of right ascension after the noon of the 15th of March.

Then $60^\circ 30' 07,8'' - 18' 23,5'' = 60^\circ 11' 44,3''$ which is the first diff. rt. asc. when the Sun had the same decl. as at the second observation.

Here, the times of the two observations fall nearest the winter solstice.

Then $60^\circ 11' 44,3'' + 109^\circ 59' 22,8'' = 170^\circ 11' 7''$; its half $85^\circ 5' 33,5''$ is the distance of the winter solstice from the Sun.

Hence $270^\circ + 85^\circ 5' 33,5'' + 18' 23,5'' = 355^\circ 23' 57''$ is the Sun's rt. asc. on March 15th.

Also $355^\circ 23' 57'' - 60^\circ 30' 7,8'' = 294^\circ 53' 49,2''$ is the rt. asc. of α Aquilæ.

246. The right ascension of one star being known, the right ascensions of all the rest are found by noting the times shewn by the clock, when those stars come to the meridian: for the differences of those times, from the transit of the chosen star, are the differences of right ascension; by which the right ascension of all the observed stars will be known; taking care to augment or diminish the right ascension of the chosen star by those differences, according as the chosen star is preceded, or followed by the other observed stars.

247. PRO-

247.

PROBLEM LXI.

To find the Sun's Place.

SOLUTION. Let the time be observed both when the Sun, and a star (the right ascension of which is known) passed the meridian, and hence the Sun's right ascension is well known.

With that right ascension, and the obliquity of the ecliptic, compute (142) the longitude, and thus his place in the ecliptic will be known.

248.

PROBLEM LXII.

To find the greatest Equation of the Center.

SOLUTION. At the times when the Sun is near his mean distance, let his longitude be found; their difference will shew the true motion for that interval of time.

Find also the Sun's mean motion for that interval of time.

Then half the difference between the true and mean motions will shew the greatest equation of the center.

Observation made at the Royal Observatory at Greenwich, shews that
1769 October 1st. at $23^h 49^m 12^s$ mean time, $\odot$ long. was $6^s 9^{\circ} 32' 0,6''$
1770 March 29th. at $0 4 50$ mean time, $\odot$ long. was $0 8 50 27,5$

The diff. of time $178d. 0 15 38$; True diff. long. $5 29 18 27$

The tropical year $= 365d. 5h. 48m. 42s. = 365,2421527$

The observed interval $= 178 0 15 38 = 178,01085648$.

Then $365,2421527 : 178,01085648 :: 360^{\circ} : 175,455948$ mean motion.

So $175^{\circ} 27' 21''$ of mean motion, answers to $179^{\circ} 18' 27''$ true motion.

Their diff. $= 3^{\circ} 51' 6''$; its half $1^{\circ} 55' 33''$ is the greatest equation of the center according to these observations.

249.

PROBLEM LXIII.

To find the Eccentricity of the Earth's Orbit.

SOLUTION. Say, As the diameter of a circle in degrees,
To the diameter in equal parts;
So the greatest equation of the center in degrees,
To the eccentricity in equal parts.

The greatest equation of the center $1^{\circ} 55' 33'' = 1,9258333$, &c.

The diam. of a circle being 1, its circumf. is 3,1415926. (II. 197)

Then $3,1415926 : 1 :: 360^{\circ} : 114,5915609$ equal to the diameter.

And $114,591609 : 1,00 \text{ \&c.} :: 1,9258333 : 0,0168061$ the eccentricity.

Hence $1,016806 (= 1,000000 + 0,016806) =$ aphelion distance.

And $0,983194 (= 1,000000 - 0,016806) =$ perihelion distance.

PROBLEM LXIV.

250.

To find the Time and Place of the Sun's Apogee.

SOLUTION. On each day of two successive apsidal let the Sun's place and the time be observed.

Then if the interval of those times and places is equal to the halves of 365 d. 6 h. 15 m. 29 s. and $360^{\circ} 1' 6''$; those observations were made when the Sun was in the apsidal.

For such intervals of time and place belong to no other points of the Earth's orbit.

But if those observed intervals of time and place differ from the said halves, take the difference between the interval of place and $180^{\circ} 0' 33''$.

Then to the daily motion of the Sun's apogee (233), the said diff. and 24 h. find the proportional time; which proportional time and difference, being applied to the time and places of the apogee observation, gives a time and place when it is $180^{\circ} 0' 33''$ distant from the observed perigee place: now if the interval of these times is equal to 182 d. 15 h. 7 m. $44\frac{1}{2}$ s. the times and places of the apsidal are known.

But if the interval of time differs from 182 d. 15 h. 7 m. $44\frac{1}{2}$ s., say, *As the diff. between the perigee and the apogee daily motions, is to the daily motion of the apogee; so is the diff. of the interval of time, to a second correction of the time of the apogee.*

This correction applied to the apogee time, corrected as above, will give the true time of the Sun's apogee.

Also, to the last correction of time find the proportional motion of the Sun's apogee; and apply it to the last corrected place of the apogee, and the true place of the apogee will be obtained.

By observations made at the Royal Observatory at Greenwich in the year 1769:

July 1st. at $0^h 3^m 20^s$ mean time $\odot$ long. $= 3^{\circ} 9' 46'' 38,5''$

December 29th. at $0 2 49$ mean time $\odot$ long. $= 9 8 10 58,1$

Interval 180 d. $23 59 29$ Interval of place $= 5 28 24 19,6$

The Sun's motion in half an anomalistic year $6 0 0 33$

The Sun's place at first observation is too forward by $1 36 13,4$

Then $57' 12'' : 1^{\circ} 36' 13,4'' :: 24 h. : 40 h. 22 m. 24 s.$ to be taken from the time of July 1st, to make the distance of the times answer to the half of $360^{\circ} 1' 6''$; and it leaves June 29d. 7 h. 40 m. 56 s.; at which time, the Sun was in $3^{\circ} 8' 10'', 25,1''$, which is distant from the December observation by $180^{\circ} 0' 33''$: But here the interval of time is 182 d. 16 h. 21 m. 53 s.; which is greater than 182 d. 15 h. 7 m. $44\frac{1}{2}$ s. the half anomalistic revolution, by 1 h. 14 m. $8\frac{1}{2}$ s.; therefore the Sun has some time to run before he comes to the apogee.

Now $4' 0'' : 57' 12'' :: 1 h. 14 m. 8\frac{1}{2} s. : 17 h. 13 m. 14 s.$ correction of time.

And $24 h. : 17 h. 13 m. 14 s. :: 57' 12'' : 42' 6,8''$ correction of place.

Then June 29d. 7 h. 4 m. 56 s. + 17 h. 13 m. 14 s. gives

June 30d. 0 h. 21 m. 10 s. for the time of the apogee.

And $3^{\circ} 8' 20'' : 25,1'' + 42' 6,8''$ gives $3^{\circ} 8' 52' 33''$ for the place of the apogee.

251. PRO-

251.

P R O B L E M LXV.

At any given time to find the Sun's mean Anomaly.

SOLUTION. Let an epocha of the Sun's passage through its aphelion be accurately determined. Then say,

As the time of a tropical revolution, or solar year,
To the interval between the aphelion and given time;
So is 360 degrees,

To the degrees shewing the mean anomaly.

Or: From the tables of mean motions find the Sun's mean motion for the given time, and this will be the mean anomaly.

252. If the Sun's motion in the ecliptic was uniform, his true place for any time could be found by the tables of his mean motion; but the Sun's longitude found by those tables, called his mean longitude, must be corrected on account of his irregular motion.

As the Earth revolves in an elliptical orbit about the Sun, placed in one of its foci, its angular motion round the Sun will differ from the angular motion it would have, were the Sun in the center of the ellipsis.

Now the table of mean motions gives the angular motions from the center of the ellipsis in a circle described on the line of the apfides, and reckoned from the first point of Aries; this motion, lessened by that of the apogee, gives the Sun's mean longitude, or mean anomaly, from the aphelion point.

But the motion of the Earth being in an elliptic orbit, its true anomaly will differ from its mean; this difference, called the equation of the center, is the correction wanted to reduce the mean motions to the true ones.

253. To find the equation of the center, or to solve (what is called) the Keplerian problem, is the most difficult operation, particularly in orbits the eccentricity of which bears a considerable proportion to the mean distance: how to do this has been shewn by Newton, Gregory, Keil, La Caille, and many others, by methods little differing from one another: it consists chiefly in finding an intermediate angle, called the eccentric anomaly, as shewn in the following problem.

254.

P R O B L E M LXVI.

The Sun's mean Anomaly being known, and the dimensions of its Orbit, to find the eccentric Anomaly.

SOLUTION.

Or $CF \times CS = (\frac{1}{2}SG =) \frac{1}{2}SE - \frac{1}{2}ES$; $CA (= \frac{1}{2}SE + \frac{1}{2}ES)$ being radius, and $= 1$.
Therefore $SE = (1 + CS \times CF \text{ (III. 47)}) = 1 + CS \times s_{ACD}$. (III. 9)

Again. In $\triangle SFE$. As $SE : R :: SF : s_{ASE} = (\frac{SF}{SE}) = \frac{SC + s_{ACD}}{1 + SC \times s_{ACD}}$.

Then $1 + s_{ASE} : 1 - s_{ASE} :: 1 + \frac{SC + s_{ACD}}{1 + SC \times s_{ACD}} : 1 - \frac{SC + s_{ACD}}{1 + SC \times s_{ACD}}$.

$$\begin{aligned} \text{Or } \frac{1 - s_{ASE}}{1 + s_{ASE}} &= \frac{(1 + SC \times s_{ACD} - SC - s_{ACD})}{(1 + SC \times s_{ACD} + SC + s_{ACD})} \\ &= \frac{1 - SC + SC \times s_{ACD} - s_{ACD}}{1 + SC + SC \times s_{ACD} + s_{ACD}} \\ &= \frac{SP + CS - 1 \times s_{ACD} \cdot SP - s_{ACD} \times SP}{SA + CS + 1 \times s_{ACD} \cdot SA + s_{ACD} \times SA} \\ &= \frac{1 - s_{ACD}}{1 + s_{ACD}} \times \frac{SP}{SA} \end{aligned}$$

But $\frac{1 - s_{ASE}}{1 + s_{ASE}} = tt, \frac{1}{2}ASE$; and $\frac{1 - s_{ACD}}{1 + s_{ACD}} \times \frac{SP}{SA} = tt, \frac{1}{2}ACD \times \frac{SP}{SA}$. (IV. 217)

Then $\frac{SP}{SA} \times tt, \frac{1}{2}ACD = tt, \frac{1}{2}ASE$. Or $\frac{SP}{SA} = \frac{tt, \frac{1}{2}ASE}{tt, \frac{1}{2}ACD}$.

Therefore $(\frac{\sqrt{SP}}{\sqrt{SA}} = \frac{t, \frac{1}{2}ASE}{t, \frac{1}{2}ACD} \text{ Or }) \sqrt{SA} : \sqrt{SP} :: t, \frac{1}{2}ACD : t, \frac{1}{2}ASE$.

256. REMARK. The $\angle CQS = \angle ACB \cup \angle ASE$ (II. 95), is the equation of the center, to be applied to the mean anomaly; and is subtractive from the aphelion to the perihelion, or in the first six signs of anomaly; and additive from the perihelion to the aphelion, or in the last six signs of anomaly.

For the lines CB, SE ; cb, se , which coincide in SA, SP , will in every other position cross one another; in Q while revolving from A to P , and in q while revolving from P to A : in the first half revolution, the mean anomaly, or the external $\angle ACB$, exceeds the $\angle ASE$, the true anomaly, by the $\angle CQS$ (II. 96): in the latter half, the true anomaly, or external $\angle PSA$, exceeds the mean anomaly, or $\angle PCB$, by the $\angle sqc$.

SECTION IX.

Practical Astronomy.

Of the EQUATION of TIME.

257. Time, which of itself flows uniformly, has its parts measured by the motion of some visible object; and the Sun being the most conspicuous moving object in the heavens, its motion has been chosen as the most proper measure of the parts of time, as well for the day, as for the year.

258. The astronomical day, at any place, begins when the Sun's center is on the meridian of that place; and is divided into 24 hours, reckoned in a numeral succession from 1 to 24: the first 12 are sometimes distinguished by the mark P. M. signifying *post meridiem*, or afternoon; and the latter 12 are marked A. M. signifying *ante meridiem*, or before noon: but astronomers generally reckon through the 24 hours, from noon to noon; and what is by the civil, or common way of reckoning, called morning hours, is by astronomers reckoned in the succession from 12, or midnight, to 24 hours.

Thus 5 o'clock, in the morning of April the 10th, is by astronomers called April the 9th, at 17 h.

259. The Sun's daily motion in longitude is the arc of the ecliptic run through in that day; and his daily motion in right ascension is the corresponding arc of the equator; and the mean daily motion in either circle is measured by $59' 8''$ nearly. For $365 \text{ d.} : 1 \text{ d.} :: 360^\circ : 59' 8''$.

260. An ASTRONOMICAL or SOLAR DAY is the interval of time between two successive transits of the Sun's center over the same meridian; and is measured by the sum of the whole equator, and an arc of it equal to the daily motion in right ascension.

For at the end of a diurnal rotation, which by observations is known to be uniform, the meridian has returned to the same star, or point of the ecliptic, which it was against at the preceding noon; but the Sun, during this rotation, has removed from that star to another, which has a greater right ascension: therefore, before the meridian can be again opposite to the Sun, so much of another rotation must be described, as is equal to the daily motion in right ascension.

261. A SIDERIAL DAY is the interval between two successive returns of the same meridian to the same fixed star, is less than the solar day, and is measured by 360° .

262. A MEAN or EQUATORIAL DAY is the time elapsed between two successive transits of the Sun over the meridian, and is measured by $360^\circ 59' 8''$ nearly.

263. MEAN or EQUAL TIME is that shewn by a clock, whose 24 hours measure the time which the Sun takes to describe an equatorial arc equal to $360^\circ 59' 8''$ nearly.

264. The difference between the measures of a mean solar day and a sidereal day, viz. $59' 8''$, reduced to time (132), gives 3 m. 56 s.; which shews, that a star which was on the meridian with the Sun on

one noon, will return to that meridian 3 m. 56 s. before the next noon; therefore a clock, which measures mean or equal days by 24 hours, will give 23 h. 56 m. 4 s. for the length of a siderial day.

265. APPARENT OR TRUE TIME, is that shewn by a *sun-dial*; where 24 hours, or a day, is measured by the sum of 360° , and that day's motion in right ascension.

266. The solar days are unequal to one another, for observations shew that the Sun's daily motion in right ascension is continually varying.

The true and mean solar days are never equal, but when the Sun's daily motion in right ascension is $59' 8''$; which happens about February 11th, May 14th, July 26th, and November 1st: at all other times the lengths of the *true* and *mean* days differ. The accumulation of these differences produces the equation of time; and sometimes the apparent noon will precede the time of the mean noon, and sometimes fall after it; their difference amounting to above 16 minutes at the beginning of November.

267. The EQUATION OF TIME is the difference between the times shewn by a *clock* and a *sun-dial*; or between the *mean* and *true* noons; or between the Sun's right ascension and his mean longitude when turned into time at the rate of 15° to an hour.

This difference arises on two accounts. First, because of the obliquity of the ecliptic the daily motions in longitude and right ascension are unequal. Secondly, because of the unequal motion of the Earth in an elliptic orbit.

In the first and third quadrants, or between the signs γ ϵ , ζ ϑ , the right ascension being less than the longitude (140°), or the mean motion taken in the equator; the point of right ascension is to the west, and therefore the apparent noon precedes, or comes in *consequentia* to the meridian before the mean noon: but in the 2d and 4th quadrants, or between the signs ϵ ζ , ϑ γ , the right ascension being greater than the longitude or mean motion, taken in the equator, the mean noon is westward, and therefore precedes, or comes in *consequentia* to the meridian before the apparent noon.

From the aphelion to the perihelion, or in the first six signs of anomaly, the mean noon precedes the apparent; and in the last six signs of anomaly the apparent noon precedes the true; their difference in either case is the equation of the center, which convert into time.

Now because the points of Aries, and of the Sun's apogee, the places where the two parts of the equation of time commence, do constantly recede from one another; therefore the whole equation of time made up of those two parts will serve only for a few years, and requires to be corrected from time to time.

268. To calculate the equation, or difference between the mean and apparent noons, for any proposed day.

Find the mean and true anomalies for that time (255); their difference, or the equation of the center, is one part.

The true anomaly gives the Sun's longitude; with which, and the obliquity of the ecliptic, compute the right ascension (139); the difference between the longitude and right ascension gives the other part.

The sum or diff. of the two parts, turned into time, gives the equation sought.

§

SECTION

SECTION X.

Practical Astronomy.

To make SOLAR TABLES.

269. I. *Tables of the mean motions of the Sun.* (301, 302, 303, 304)

Divide 360 degrees by a solar revolution, the quotient shews the mean motion for one day $0^{\circ} 59' 08''$, &c.

Take the multiples of one day's motion from 1 to 365 for every day in the year; and these properly disposed, according to the month days, will give the mean motions for every day of each month. (304)

The 24th part of one day's motion will give that for one hour, and its multiples to 24 times will shew the mean motions answering to each hour: from hence, those for the minutes of an hour, the seconds of a minute, &c. are easily obtained. (303)

The mean motion of a year of 365 days (*viz.* for the last of December) being doubled, tripled, and quadrupled, those for 1, 2, 3, and 4 years will be obtained, adding one day's mean motion to the 4th year, it being leap-year, and containing 366 days: the motion for leap-year being increased by those of 1, 2, 3, and leap-years, give those for 5, 6, 7, and 8 years: the mean motion for 8 years being increased by those for 1, 2, 3, and 4 years, give those for 9, 10, 11, and 12 years: and thus increasing the mean motion for the last leap-year by those of 1, 2, 3, and 4 years, the mean motions may be continued for any number of absolute years. (301)

270. In the following tables the numbers used were,

Length of the year	365d.	5h.	48m.	54 $\frac{1}{2}$ s.
Yearly motion of the apogee,		0°	$1'$	$5''$.
Place of the apogee, beginning the year 1760,	3°	8	47	25.
Greatest equation of the Earth's orbit,		1	55	39.

271. Now 365 d. 5 h. 48 m. 54 $\frac{1}{2}$ s. = 365,242,300,347,2 days.
Then 365,242,300,347,2 d. : 360° : : 1 d. : 0,9856470613 degrees.

Hence the mean motion for 1 day = $0^{\circ} 0' 59'' 19''' 45^{iv} 54^v 50^{vi}$

January 5th	5 days =	0	4	55	41	38	49	34	10
January 30th	30 days =	0	29	34	9	52	57	25	0
March 31st	90 days =	2	28	42	29	38	52	15	0
June 29th	180 days =	5	27	24	59	17	44	30	0
December 26th	360 days =	11	24	49	58	35	29	0	0
December 31st	365 days =	11	29	45	40	14	18	34	10

U 3

Now

Now 1 year's mean motion = $11^{\circ} 29' 45'' 14''' 18^{\text{iv}} 34^{\text{v}} 10^{\text{vi}}$.

2 years	= $11^{\circ} 29' 31'' 20' 28' 37' 8' 20''$.
3 years	= $11^{\circ} 29' 17' 0' 42' 55' 42' 30''$.
4, or leap year	= $0^{\circ} 0' 1' 49' 17' 0' 11' 30'' = 3y. + 1y. + 1d.$
5 years	= $11^{\circ} 29' 47' 29' 31' 18' 45' 40'' = 4y. + 1y.$
6 years	= $11^{\circ} 29' 33' 9' 45' 37' 19' 50'' = 4y. + 2y.$
7 years	= $11^{\circ} 29' 18' 49' 59' 55' 54' 0'' = 4y. + 3y.$
8 years B	= $0^{\circ} 0' 3' 38' 34' 0' 23' 0'' = 4y. \times 2.$
20 years B	= $0^{\circ} 0' 9' 6' 25' 0' 57' 30'' = 4y. \times 5.$
100 years B	= $0^{\circ} 0' 45' 32' 5' 4' 47' 30'' = 20y. \times 5.$
1000 years B	= $0^{\circ} 7' 35' 20' 50' 47' 55' 0'' = 100y. \times 10.$

Where B stands for biffextile, or leap-year.

272. But to find the mean motions for the years related to any particular epocha, the mean motion for some particular time in that epocha must be known. Thus,

Let the mean motion of the Sun be determined by observation (or otherwise) when the Sun is in some noted point of the ecliptic, suppose near Aries: or let the time of its entrance into the sign Aries be well ascertained. Take the difference between the time of that ingress and the 31st of December at noon, in days, hours, minutes, and seconds, (reckoning the end of the 31st of December to be the beginning of January at noon,) and find the mean motions for those days, hours, minutes, and seconds, and it will shew the motion from Aries, for the 31st of December, or the mean motion at the beginning of the year proposed; or the radix for that year with relation to the proposed epocha.

The relative mean motions for one year being known, those for any number of succeeding years belonging to that epocha, may be had, by adding such of the before found absolute years to the first relative year, as will make the number wanted: and the mean motions for any past year of that epocha will be found by lessening the radical years by such a number of the absolute years, as will produce the relative years required. (302)

And in this manner are tables constructed, by which the mean motions of the Sun for any time past or to come may be computed.

273. Suppose in the year 1760, the Sun entered Aries on the 20th of March, at 13 h. 42 m. $3\frac{1}{4}$ s. P. M.: Required the Sun's mean motion for the beginning of the year 1760.

Now 1760 being leap-year, February has 29 days, and from the equinox to the commencement of the year is 80 d. 13 h. 42 m. $3\frac{1}{4}$ s.

Then 1 d. = 0,9856470613 deg. :: 80 d. 13 h. 42 m. $3\frac{1}{4}$ s. : 79,41444 &c. degrees.

Therefore at the beginning of the year 1760, the Sun's mean longitude was $2^{\circ} 19' 24'' 52'''$ short of Aries; or his mean longitude was $9^{\circ} 10' 35'' 8'''$, which is the radical mean place for the year 1760.

274. II. Of the mean motions of the Sun's apogees. (301, 302)

The yearly motion of the apogee being determined (241); the motion for any number of absolute years will be that multiple of one year's

year's motion, and so for any part of a year : the monthly motions will be 5 seconds for some months, and 6 for others, to make 65 in the 12 months.

Let the time of the Sun's passage through the aphelion be accurately determined by observation (250), and also its place in the ecliptic ; then the distance of the place of the apogee from Aries will be known at that time : let this distance be lessened by the apogee's motion from the last day of the year preceding the proposed epocha to the time of the apogeeon passage, and the mean motion of the apogee will be known for the beginning of that year, taken as a radix.

Then that radical mean motion, increased by the multiples of the yearly motion, will give those for succeeding years : but being diminished by those multiples will give them for past years.

The Sun's mean motion for any time, lessened by that of the apogee for that time, gives the Sun's mean anomaly.

275. III. Of the equation of the Sun's center. (305)

To every degree of the first six signs of mean anomaly assumed, find the true anomaly (253, 254) : the difference between the mean and true anomalies will be the equations of the center to those degrees of mean anomaly ; which serve also for the degrees of the last six signs ; as equal anomalies are at equal distances on both sides of either apside.

Set the equations of the center orderly to their signs and degrees of anomaly, the first six being reckoned from the top of the table downwards, and signed at top with the title *subtract* ; the last six, for which the same equations serve, but taken in a contrary order, *viz.* from the bottom of the table reckoned upwards, are signed at bottom with the title *add* ; and let the difference between every adjacent two equations, called tabular differences, be set in another column.

From these equations of the center, augmented or diminished by the proportional parts of their respective tabular differences for any given minutes and seconds, are deduced equations of the center to any given mean anomaly.

276. Astronomical tables are usually computed to answer to two given denominations only ; as to signs and degrees ; degrees and minutes ; months and days ; &c. ; for if made to more names, such tables would swell into a bulk so great, as to be tedious to compute, expensive to print, and of no great advantage in the use ; but it generally happens in calculations, that numbers are wanted from tables to answer to given numbers of three or more denominations, as to signs, degrees, minutes, and seconds ; months, days, hours, minutes, and seconds ; &c. ; and to obtain from the tables numbers answering to all the given names, the tabular numbers are to be increased or diminished by a proportional part of their difference.

Thus : To find the equation of the center to $4^{\circ} 21^{\circ} 44' 36''$.

Now the equation to this number will fall between those belonging $4^{\circ} 21'$ and $4^{\circ} 22'$; which equations are $1^{\circ} 13' 59''$ and $1^{\circ} 12' 24''$ (205) Their diff. is $1' 35'' = 95''$; the pro. pt. of which is to be taken for $44' 36''$.
U 4 And

And as the diff. 1° or $60'$: diff. $95''$: : $44',6$: $70'',6 = 1' 11''$ the proportional part.

Now to $4^{\circ} 21' 0''$, $1^{\circ} 13' 59''$ is the equation of the center.
And to $0 0 44 36$, $1 11$ is the prop. part to be subtracted.

Then to $4 21 44 36$, $1 12 48$ is the equation of the center.

When the tabular numbers are increasing, the proportional part is to be added ; but when decreasing, the proportional part is to be subtracted.

277.

IV. *Tables of the Sun's true place.*

(308)

The Sun's true place at any proposed time is thus found.

Collect together the mean motions of the Sun, and also those of the apogee, for the given year, month, day, (hour, minute, and second, if given) ; and their sum will be the mean motions of the Sun and its apogee.

The Sun's mean motion, lessened by that of the apogee, gives the mean anomaly ; to which find the proper equation of the center by proportioning for the minutes and seconds.

Then the Sun's mean motion, augmented or diminished by the equation of the center, as the title of its table directs, gives the Sun's true longitude or place for that given time.

The Sun's place thus found to every day for four successive years, viz. for leap-year, and 1, 2, 3 years after ; and those places ranged under their proper years, according to their respective months and days, constitute the tables of the Sun's place.

These tables find the Sun's place at noon only ; but the place for any intermediate time is found by applying to the noon-place the proportional part of the daily difference at that time.

278. *To find the Sun's longitude, suppose on May 4, 1788, at the time of apparent noon.*

In the table of the Sun's longitude (308) for 1788, against May 4th, stands $1^{\circ} 14' 36'' 02''$, which shews that the Sun's longitude, reckoned from Aries, is $44^{\circ} 36' 02''$; or that his place is in $\varnothing 14^{\circ} 36' 02''$.

But to find the Sun's place at any other hour, suppose on May 4th, at 7 h. 24 m. 36 s. apparent time, proceed thus :

The difference between the noon places of the 4th and 5th of May is $57' 59'' = 3479''$, answering to 24 hours in time. (308)

Then $24 \text{ h.} : 7 \text{ h. } 24 \text{ m. } 36 \text{ s.} :: 3479'' : 1074'' = 17' 54''$, the proportional part.

And $1^{\circ} 14' 36'' 02'' + 17' 54'' = 1^{\circ} 14' 53' 56''$, the Sun's longitude at that time.

279. Or, the Sun's place may be found by the tables of mean motions.

From

From the apparent time $7^h 24' 36''$ take the equation of time (316) equal to $3' 35''$, and the remainder $7^h 21' 1''$, is the mean time.

1788. ☉'s m. mot. (302)	$9^s 10^s 47' 53''$	Mot. ap.	$3 9 17 45$	(302)
May 4	(305) $4 2 13 13$		22	(307)
7 hours	(303) $17 15$		$3 9 18 07$	m. apogee.
21 minutes	(303) 52	Diff.	$10 4 1 06$	m. anom.
1 second	(303) 0			

☉'s mean longitude	$1 13 19 13$
Equat. center +	$1 34 44$
Sun's true longitude	$1 14 53 57$

Now to $10^s 4^s$ the equation of the center is $1^o 34' 45''$ (305)
 And the diff. is $70''$ decreasing. $60' : 70''$
 :: $1' 06'' : 1''$ the proportional part.
 And $1^o 34' 45'' - 1'' = 1 34 44$ equation of the center.

280. *To find the Sun's longitude at any given time and place.*

Seek, in the table of the longitudes of places, at the end of Book VI, for the difference of longitude between London and the proposed place; and convert the diff. of longitude into time.

If the proposed place is to the eastward of London, take the diff. between the proposed time and diff. of longitude, and this will shew the corresponding time at London; after noon, if the proposed time is greater than the diff. of longitude; but before noon, if the proposed time is least.

If the proposed place is to the westward of London, the sum of the proposed time and diff. of longitude will be the corresponding time at London.

The Sun's place found to the corresponding time at London, will be the Sun's longitude sought for the proposed time and place.

Thus: In a place 6 h. to the east of London, when it is 8 h. P. M. at that place, it is 2 h. P. M. at London; and when it is 4 h. P. M. at that place, it is 2 h. before noon at London.

For when it is noon at London, it is 6 h. P. M. at the other place.

Also, in a place 6 h. to the west of London; when it is 8 h. P. M. at that place, it is 14 h. P. M. at London.

For when it is noon at the proposed place, it is 6 h. P. M. at London.

281. *V. Tables of the Sun's declination.* (309)

With each of the Sun's longitudes, already found, and the obliquity of the ecliptic, find the declination to each day of the four years. (139)

Or thus: To each degree of the three first signs of the ecliptic, taken as longitudes, find the declination (139): and of these declinations, regularly ranged to their sign and degree, take the difference of each adjacent two, which set against them in another column; and this auxiliary table is prepared, answering to each sign and degree of longitude (306): for to equal longitudes, taken on both sides of each equinox, belong equal declinations.

Now these auxiliary declinations augmented or diminished, according as they are increasing or decreasing, by the proportional part of their difference,

difference, for the minute and seconds in any given longitude, will give the declination for that longitude.

And this being done for every day in the four years, using the longitudes already computed, will give the declinations sought: which are to be ranged according to their year, month, and day.

282. To find the Sun's declination. Suppose on May 4, 1788, at noon. In the table of the Sun's declination (309) for 1788, against May 4, stands $16^{\circ} 14' 13''$ for the Sun's declination, which is N. as being between the vernal and autumnal equinoxes.

283. But if the declination was wanted on May 4, 1788, at 7 h. 24 m. 36 s. P. M. proceed thus:

The difference between the noons of May 4 and 5, is $16' 59''$, which answers to 24 h. Then $24 \text{ h.} : 17' 0'' :: 7 \text{ h. } 24 \text{ m. } 36 \text{ s.} : 5' 15''$, the proportional part.

And as the declination is increasing; then $16^{\circ} 14' 13'' + 5' 15''$, gives $16^{\circ} 19' 28''$ for the Sun's decl. at the proposed time.

284. But Art. 311 is a table for finding the proportional part at sight, for fitting the noon declination to any other time. Thus:

Seek in the left-hand column for a daily difference, nearest to the given one; against which, in a column marked at top with hours, nearest to those given, stands the proportional part sought.

Thus against $17' 0''$ of daily diff. and to 7 h. 24 m. time, stand $5' 15''$, the proportional part sought.

Although this table goes no farther than 8 h. yet it may be applied quite to 12 h. or 180 degrees.

285. EXAM. *What will be the Sun's declination at London, on the 25th of August, 1788, at 10 h. 35 m. P. M.?*

In 1788, the daily diff. between the noons of the 25th and the 26th of August is $20' 58''$ decreasing. (309)

Now 10 h. 35 m. is equal to 2 h. 35 m. + 8 h. 0 m.

To the diff. $20' 58''$, and to 2 h. 35 m. answers $2' 16''$.

To the diff. $20' 58''$, and to 8 h. 0 m. answers $6' 59''$. (311)

The sum $9' 15''$ is the proportional part, by which the decl. $10^{\circ} 28' 29''$ to August 25th, is to be diminished; so $10^{\circ} 19' 14''$ is the decl. sought.

Here $20' 58''$, is taken as if it was $21' 0''$.

And 2 h. 35 m. is $\frac{3}{4}$, the interval between 2 h. 20 m. and 2 h. 40 m.

Now $21'$ gives $2' 2''$ for $2^{\text{h}} 20'$, and $2' 20''$ for $2^{\text{h}} 40'$; the diff. is $18''$, three-fourths of which is $14''$; and this being added to $2' 2''$ gives $2' 16''$ for $21'$ with $2^{\text{h}} 35'$. Moreover $21'$ with 8 h. gives $7' 00''$; but I take one second less because the daily diff. in declination is $2''$ less than $21' 00''$.

286. From the table of declination, fitted to the meridian of London, or Greenwich, the declination may be found at any time, under any other meridian, at a given difference of longitude from London. Thus:

Required

Required the Sun's declination at noon under a meridian 110° to the west of London, on the 24th of February, 1788.

(311.) Now at 110° to the west of London, it is noon 7 h. 20 m. after it is noon at London; that is, when it is 7 h. 20 m. P. M. at London, it will be noon at the proposed place; so the declination found to that time at London (285) will be the declination sought.

In 1788, the diff. between the declinations of the 24th and 25th of February, is $22' 13''$ decreasing (309): and against $22' 20''$ of daily diff. and under 110° , or 7 h. 20 m. is $6' 49''$ in table, Art. 311, which taken from $9^{\circ} 27' 33''$, leaves $9^{\circ} 20' 44''$, the declination sought.

EXAM. II. *What is the Sun's declination on September 2d, 1788, at 20 h. 20 m. under a meridian 100° to the eastward of London?*

Now under a meridian 100° to the eastward of London, it is noon 6 h. 40 m. before it is noon at London (311); or when it is noon at London, it is 6 h. 40 m. after noon at the proposed place; and when 2 h. 30 m. after noon at that place, it is 13 h. 50 m. after noon at London; so the declination found at that time (285), will be the declination sought.

In 1788, the daily diff. at September 2d, is $22' 6''$ (309), against which (in tab. Art. 311), and under 8 h. and 5 h. 50 m. stand $7' 21''$ and $5' 21''$, their sum $12' 42''$ taken from the decl. to September 2, viz. $7^{\circ} 36' 30''$, leaves $7^{\circ} 23' 48''$, the declination sought.

Here 5 h. 50 m. fall in the middle between 5 h. 40 m. and 6 h. 0 m. so $5' 21''$, the middle between $5' 12''$ and $5' 30''$, is taken.

287. VI. *Tables of the Sun's right ascension.* (310)

To the obliquity of the ecliptic, and each degree in the three first signs of longitude, find the right ascensions (139), and of each take the supplement.

Range the right ascensions according to their sign and degree for the three first signs; and for the three next signs, range the supplements so that the 4th sign begins with the least supplement, and the 6th sign ends with the greatest: because the right ascensions in the 2d and 4th quadrants are the supplements of those in the first and third.

Let the differences of these right ascensions, viz. each adjacent two, through the six signs be taken, and set in other columns. (307)

Then this auxiliary table, used like that of declination, will give the right ascension to each day in the four years.

288. *To find the Sun's right ascension, suppose on June 12 at noon, in the year 1788, at London.*

In the table of the Sun's right ascension (310) for 1788, against June 12, stands 5 h. 25 m. 19 s. which is the right ascension sought, and shews how much later the Sun passed the meridian of London than the equinoctial point Aries.

EXAM.

EXAM. II. *Required the Sun's right ascension at London on the 2d of November 1788, at 9 h. 30 m. P. M.*

Between the 2d and 3d of November, 1788, the daily diff. is 3 m. 58 f. which answers to 24 h. Then $24 \text{ h.} : 3 \text{ m. } 58 \text{ f.} :: 9 \text{ h. } 30 \text{ m.} : 1 \text{ m. } 34 \text{ f.}$ the proportional part.

Then the right ascension on the 2d at noon, 14 h. 33 m. 20 f. + 1 m. 34 f. gives 14 h. 34 m. 54 f. for the right ascension at the time required.

By this table the right ascension may also be found at any time in places that are to the eastward or westward of London, the difference of longitude of those places being known; by finding the time at London corresponding to the given time at the proposed place, and seeking the right ascension to that corresponding time at London.

The table at Art. 311, pages 222, 223, may be applied to the tables of the Sun's longitude and right ascension, as well as to those of the declination, for finding the proportional parts of the difference between the noon of adjoining days, which shall answer to any intermediate hours.

Thus in the Ex. page 296. To find the pro. pts. of 58' to 7 h. 24 m. 36 f.

Now $\frac{1}{4}$ th of 58' is 14' 30''; which falls between 14' 20'' and 14' 40''.

And the time 7 h. 24 m. 36 f. falls between 7 h. 20 m. and 7 h. 40 m.

The mean of the equations under 7 h. 20 m. and 7 h. 40 m. and against 14' 20'' and 14' 40'', are 4' 26'' and 4' 38'', their diff. is 12''.

And $20 : 12 :: 4,6 : 2\frac{1}{2}$: and $4' 26'' + 2\frac{1}{2} = 4' 28\frac{1}{2}$ the pro. pts. to $\frac{1}{4}$ of 58''.

Then the proportional parts to 58', are 17' 54''.

Again. In the Exam. above. To find the parts proportional to 3 m. 57 f. as 9 h. 30 m. is to 24 h.

Here 3 m. 57 f. being taken as 4 m.; and 4 h. 40 m. as the half of 9 h. 30 m.

The equation is 47''; which doubled gives 1' 34'' for the proportional parts required.

289. VII. *Of the right ascensions and declinations of the fixed Stars.* (312)

This table, which contains 120 of the principal fixed stars, viz. 60 having north declination, and 60 with south declination, are fitted to the year 1780; and are selected partly from the catalogue which is given in the Nautical Almanac for 1773, as deduced from Dr. Bradley's Observations; and partly from that given by M. de La Caille, which, he says, "are all derived from his own observations made, during ten years attention to this business, either at Paris, or at the Cape of Good Hope; that the positions are ascertained with all the accuracy that could be derived from the modern Astronomy; and that he had all proper helps, with regard to instruments, assistants, and convenience, and neither care or pains were wanting to perfect the work."

"The right ascensions were determined by a multitude of corresponding altitudes of each, taken with a quadrant of three feet radius, to have their passage over the meridian with the greatest exactness. Almost all the stars in the northern hemisphere have been compared with the bright star in the Harp; and those in the southern hemisphere, with Sirius; that is to say, on each day that the time of the star's passing the meridian had been found by equal altitudes, that of α Lyrae and * Sirius were found in like manner; the right ascensions of these two

" Stars

“ stars having been settled by a great many observations taken when they were in the properest situation for this purpose.

“ The declinations have been deduced from a sufficient number of observations of their zenith distances, taken with an instrument of six feet radius, made with great care for this purpose.”

The table consists of nine columns; that on the left hand contains the name of the constellation; the next shews in what part of the constellation the star is; in the 3d are the names by which certain stars are distinguished; the 4th column shews the Greek characters by which the star is marked in the celestial charts, or maps of the constellations; the 5th shews the magnitude of the stars; the 6th and 7th contain the right ascension in time, reckoned from Aries, and the yearly variation in right ascension; the 8th and 9th contain the declinations and the yearly variation in declination; where those which are marked + are augmented by the yearly variation; but those which have the mark — annexed, are to be diminished by the variation: by the help of these yearly variations, the right ascensions and declinations of these stars may be fitted for any distant year.

Precepts for finding the culminating of the stars are at Articles 133, 134.

290.

VIII. *Tables of the Equation of Time.*

In page 318 are three tables, Articles 313, 314, 315: Article 313 is a table of the Sun's right ascension in degrees, to each degree of longitude in the first quadrant of the ecliptic; and also the differences between those longitudes and right ascensions. The table, Art. 314, contains the said differences turned into time (132), of minutes, seconds, and the tenth part of seconds: the numbers in this table are the differences between the mean and true noons, arising from the obliquity of the ecliptic (267); and the table, Art. 315, is nothing more than the equations of the center, table, Art. 305, converted into time; and are the differences between the times of the mean and true noons, arising from the eccentricity of the Earth's orbit: these two equations of time, properly put together, constitute another table, Art. 316, of the absolute equation of time with relation to the place of the Sun's apogee.

291. *To construct the table 316, of the absolute Equation of Time.*

1st. To the given time find the Sun's true place, or assume a place.

2d. The difference between that place, and the place of the apogee, gives the Sun's true anomaly.

3d. From the true anomaly find the mean. (294)

4th. In table I. 314, seek the equation of time to the Sun's place.

5th. In table II. 315, seek the equation of time to the mean anomaly.

6th. The sum or difference of these equations, according as their titles or signs direct, will be the absolute equation of time to the Sun's place found at first, or to the corresponding time.

The tables, Articles 314, 315, are made only to whole degrees of longitude and anomaly; the proportional parts of the differences are to be taken for minutes or seconds, above whole degrees of the Sun's longitude and anomaly.

292. EXAM.

292. EXAM. I. *What is the Equation of Time, when the Sun's longitude is $7^{\circ} 12'$?*

In table, Art. 316, against 12° in the outside column, and under m , or 7 f. stands—16 m. 12 f.; which shews that 16 m. 12 f. is to be subtracted from the apparent time; to give the mean time of apparent noon, or the time which should be shewn by a good clock, when the Sun's center is on the meridian.

293. EXAM. II. *What is the Equation of Time when the Sun's longitude is $4^{\circ} 24' 30'' 42''$?*

The difference between the equation in table, Art. 316, to $4^{\circ} 24'$ and $4^{\circ} 25'$, is $13''$ decreasing; and $30' 42'' = 30,7'$.

Then $60' : 30,7' :: 13'' : 6,65$ or $7''$, the proportional part decreasing.

And $+3$ m. 46 f.—7 f. $= +3$ m. 39 f. the equation sought.

So 24 h. the apparent time of solar noon, increased by 3 m. 39 f. will give the mean time of noon.

If the time was given, viz. the month, day, hour, &c. to find the Equation.

To the given time find the Sun's longitude.

Then to this longitude find the equation of time, as above.

(278)

294. *To find the mean anomaly from the true being given.*

SOLUTION. Say, As the square root of the perihelion distance,
To the square root of the aphelion distance;

And So the tangent of half the true anomaly,
To the tangent of half the eccentric anomaly,
As radius, to the sign of the eccentric anomaly,
So the degrees in an arc equal in length to the eccentricity,

To the degrees, &c. in the arc of correction.

The correction added to the eccentric anomaly gives the mean anomaly.

295. REMARKS. 1st. The greatest equation of the center being taken at $1^{\circ} 55' 39''$, the eccentricity (249) will be 0,01682; the aphelion distance will be 1,01682, and the perihelion 0,98318.

Hence the ratio of the square root of the perihelion distance to the square root of the aphelion distance will be expressed by the logarithm 0,00731; which constant logarithm, added to the logarithmic tangent of $\frac{1}{2}$ the true anomaly, will give the logarithmic tangent of $\frac{1}{2}$ the eccentric anomaly.

296. 2d. In the 2d proportion, the arc equal to the length of the eccentricity 1682 is a constant quantity.

Now the radius, or mean distance, is equal to the length of an arc of $57^{\circ} 29' 578$ (249); then $100000 : 1682 :: 57^{\circ} 29' 578 : 0^{\circ} 96' 375$, the length of the eccentricity in degrees; the constant logarithm of which is 9,98396, which added to the logarithmic sine of the eccentric anomaly, abating 10 in the index of the sum, gives the logarithm of an arc, the degrees, minutes, and seconds of which being added to the eccentric anomaly, give the mean anomaly.

297.

297. IX. *Table of Corrections for the middle time between equal altitudes of the Sun.* Art. 317.

This table, which is fitted to the latitudes of 30° , 40° , 50° , and 60° , will also serve, nearly, to all latitudes between 25° and 65° ; by entering the table with the nearest latitude to that given, and the given declination in degrees. It is constructed by Art. 216.

EXAM. I. In latitude 50° N. when the Sun's declination is 16° N. and the interval between the morning and afternoon observation is 5 hours: What correction must be applied to the middle time, to give the time of apparent noon?

In the table, Art. 317 against 16° of declination taken in the outside column, and under 50° latitude, and 5 hours, with N. declination, stand 12 seconds; which 12 s. applied to the middle time between the observations, give the time when the Sun was on the meridian.

The correction is applied to the middle time by the precepts at the bottom of the table.

298. EXAM. II. In latitude 50° N. on November 16th, 1761, observations at equal altitudes of the Sun were taken at the following times shewn by a clock, the equal altitude instrument having three horizontal wires.

Morning observations.		Afternoon observations.	
☉ preceding limb	☉ following limb	☉ preceding limb	☉ following limb
$9^h 28^m 55''$	$9^h 35^m 21\frac{1}{2}''$	$1^h 46^m 43\frac{1}{2}''$	$1^h 53^m 30\frac{1}{2}''$
$9 32 46\frac{1}{2}$	$9 39 23\frac{1}{2}$	$1 50 53\frac{1}{2}$	$1 57 28\frac{1}{2}$
$9 36 44\frac{1}{2}$	$9 43 30$	$1 54 56$	$2 1 20\frac{1}{2}$
Now $\frac{9^h 28^m 55'' + 2^h 1^m 20\frac{1}{2}'' + 12^h}{2} = 11^h 45^m 7\frac{3}{4}''$			
$\frac{9 32 46\frac{1}{2} + 1 57 28\frac{1}{2} + 12}{2} = 11 45 7\frac{1}{2}$			
$\frac{9 36 44\frac{1}{2} + 1 53 30\frac{1}{2} + 12}{2} = 11 45 7\frac{1}{2}$			
Again $\frac{9 35 21\frac{1}{2} + 1 54 56 + 12}{2} = 11 45 8\frac{1}{4}$			
$\frac{9 39 23\frac{1}{2} + 1 50 53\frac{1}{2} + 12}{2} = 11 45 8\frac{1}{2}$			
$\frac{9 43 30 + 1 46 43\frac{1}{2} + 12}{2} = 11 45 6\frac{3}{4}$			

the mean $= 11^h 45^m$
7,6'' by the preceding limb.

the mean $= 11^h 45^m$
8'' by the following limb.

The mean time of observation from both limbs is $11^h 45^m 7,8$ s. The declination on the day of observation is 19° S. nearly; the interval between the observations is about 4 hours; and these give + 14 seconds for the correction of the middle time.

So the Sun was on the meridian when the clock shewed $11^h 45^m 21,8$ s.

The Sun's place, at that time, was $7^\circ 24' 26'' 46''$ nearly.

In table 316, to $7^\circ 24'$ the tabular difference is 12'', decreasing.

x

Then

Then $60' : 26,75' :: 12 \text{ f. } 5,35 \text{ f. ; and } + 14 \text{ m. } 56 \text{ f. } - 5\frac{1}{3} \text{ f. } = + 14 \text{ m. } 50\frac{2}{3} \text{ f.}$ which is the equation of time: hence $11 \text{ h. } 45 \text{ m. } 21,8 \text{ f. } + 14 \text{ m. } 50,6 \text{ f. } = 12 \text{ h. } 0 \text{ m. } 12,4 \text{ f.}$

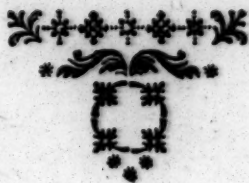
Which shews that the clock was 12 f. nearly, too fast.

299. X. *Tables of Refraction and of the Sun's parallax.* (318,319)

These tables are the result of the experience of some of the most eminent Astronomers. By the refraction of the atmosphere, objects appear more elevated than they really are, and therefore the apparent altitude is to be diminished by the refraction, which is greatest near the horizon, and gradually diminishes towards the zenith, where there is no refraction. The parallax in altitude is the difference between the altitude of an object, as seen from the center and surface of the Earth, that from the center being the true altitude, and the greatest, except at the zenith, where parallax vanishes; therefore the apparent altitude is to be augmented by the parallax.

EXAM. *The Sun's apparent altitude was observed to be $18^{\circ} 34' 48''$: What was his true altitude?*

Apparent altitude	$18^{\circ} 34' 48''$	} Refraction $2' 47'' -$ Parallax $9 +$
Correction is —	$2 \quad 38$	
Sun's true altitude	$18 \quad 32 \quad 10$	



300.

ASTRONOMICAL TABLES,

Fitted, in general, to the meridian of GREENWICH.

	Art.
I. Mean motions of the Sun and Apogee for years.	(301)
II. Mean motions of the Sun and Apogee for radical years.	(302)
III. Mean motions of the Sun for hours, minutes, and seconds.	(303)
IV. Mean motions of the Sun and Apogee for months and days.	(304)
V. Equations of the Sun's center.	(305)
VI. Sun's declination to signs and degrees.	(306)
VII. Sun's right ascension in time to signs and degrees.	(307)
VIII. Sun's declination to each day for the years 1796, 1797, 1798, and 1799.	(308)
IX. Sun's declination to each day for the years 1800, 1801, 1802, and 1803.	(309)
X. Sun's declination to each day for the years 1804, 1805, 1806, and 1807.	(310)
XI. To fit the tables VIII. IX. X. to any meridian.	(311)
XII. Right ascensions and declinations to 120 fixed stars.	(312)
XIII. Sun's right ascension in degrees, &c. to signs and degrees of longitude.	(313)
XIV. Equation of time on the obliquity of the ecliptic.	(314)
XV. Equation of time on the eccentricity of the Earth's orbit.	(315)
XVI. Absolute equation of time, to the Sun's longitude.	(316)
XVII. Correction of the middle time between equal altitudes.	(317)
XVIII. Correction of altitude for refractions.	(318)
XIX. Correction of the Sun's altitude for his parallax.	(319)

TABLES of the MEAN MOTIONS of the SUN and his APOGEE to MEAN SOLAR TIME.

301

For years absolute.		
Years.	M. lon. ☉	M. lon. ap.
1	11 29 45 40	1 5
2	11 29 31 20	2 10
3	11 29 17 1	3 15
4	0 0 1 49	4 20
5	11 29 47 29	5 25
6	11 29 33 10	6 30
7	11 29 18 50	7 35
8	0 0 3 39	8 40
9	11 29 49 19	9 45
10	11 29 34 59	10 50
11	11 29 20 39	11 55
12	0 0 5 28	13 0
13	11 29 51 8	14 5
14	11 29 36 48	15 10
15	11 29 22 29	16 15
16	0 0 7 17	17 20
17	11 29 52 57	18 25
18	11 29 38 38	19 30
19	11 29 24 18	20 35
20	0 0 9 6	21 40
30	11 29 44 5	32 30
40	0 0 18 13	43 20
50	11 29 53 11	54 10
60	0 0 27 19	1 5 0
70	0 0 2 18	1 15 50
80	0 0 36 26	1 26 40
90	0 0 11 24	1 37 30
100	0 0 45 32	1 48 20
500	0 3 47 40	9 1 40
1000	0 7 35 20	18 3 20

302

For the radical years, new file.				
Years.	M. lon. ☉	M. lon. ap.	Years.	M. lon. ☉
B 1752	9 10 31 29	3 8 38 45	1782	9 10 15 34
53	10 17 9	39 50	83	10 1 15
54	10 8 49	40 55	B 1784	10 46 3
55	9 48 30	42 0	85	10 31 43
B 1756	10 33 18	43 5	86	10 17 24
57	10 18 58	44 10	87	10 3 4
58	9 10 4 39	3 8 45 15	B 1788	9 10 47 53
59	9 50 19	46 20	89	10 33 33
B 1760	10 35 8	47 25	90	10 19 13
61	10 20 48	48 30	91	10 4 53
62	10 6 28	49 35	B 1792	10 49 42
63	9 52 8	50 40	93	10 35 22
B 1764	9 10 36 57	3 8 51 45	94	9 10 21 23
65	10 22 37	52 50	95	10 6 42
66	10 8 17	53 55	B 1796	10 51 31
67	9 53 58	55 0	97	10 37 11
B 1768	10 38 46	56 5	98	10 22 52
69	10 24 26	57 10	99	10 8 32
70	9 10 10 7	3 8 58 15	C 1800	9 54 12
71	9 55 47	59 20	B 1820	10 3 18
B 1772	10 40 35	9 0 25	B 1840	10 12 25
73	10 26 16	1 30	B 1860	10 21 31
74	10 11 56	2 35	B 1880	10 30 38
75	9 57 36	3 40	C 1900	9 40 36
B 1776	9 10 42 25	3 9 4 45	B 1920	9 49 42
77	10 28 5	5 50	B 1940	9 58 48
78	10 13 45	6 55	B 1960	10 7 55
79	9 59 25	8 0	B 1980	10 17 1
B 1780	10 44 14	9 5	B 2000	10 26 8
81	10 29 54	10 10	C 2100	10 12 31

303

For hour, minutes, and secs.				
H	Lon. ☉	M	Lon. ☉	Lon. ☉
1	2 27 51	31	1 16 23	1 16 23
2	4 55 42	32	1 18 51	1 18 51
3	7 23 32	33	1 21 19	1 21 19
4	9 51 23	34	1 23 47	1 23 47
5	12 19 14	35	1 26 15	1 26 15
6	14 47 5	36	1 28 42	1 28 42
7	17 14 56	37	1 31 10	1 31 10
8	19 42 47	38	1 33 38	1 33 38
9	22 10 37	39	1 36 6	1 36 6
10	24 38 28	40	1 38 34	1 38 34
11	27 6 19	41	1 41 2	1 41 2
12	29 34 10	42	1 43 30	1 43 30
13	32 2 1	43	1 45 57	1 45 57
14	34 29 52	44	1 48 25	1 48 25
15	36 57 42	45	1 50 53	1 50 53
16	39 25 33	46	1 53 21	1 53 21
17	41 53 24	47	1 55 49	1 55 49
18	44 21 15	48	1 58 17	1 58 17
19	46 49 6	49	2 0 44	2 0 44
20	49 16 56	50	2 3 12	2 3 12
21	51 44 47	51	2 5 40	2 5 40
22	54 12 38	52	2 8 8	2 8 8
23	56 40 29	53	2 10 36	2 10 36
24	59 8 20	54	2 13 4	2 13 4
25	61 36 11	55	2 15 32	2 15 32
26	64 4 1	56	2 17 59	2 17 59
27	66 31 52	57	2 20 27	2 20 27
28	68 59 43	58	2 22 55	2 22 55
29	71 27 34	59	2 25 23	2 25 23
30	73 55 25	60	2 27 51	2 27 51

TABLE of the MEAN MOTIONS of the SUN and APOGEE to MONTHS and DAYS.

January.	February	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Days.
☉ M. lon.	☉ M. lon.	☉ M. lon.	☉ M. lon.	☉ M. lon.	☉ M. lon.	☉ M. lon.	☉ M. lon.	☉ M. lon.	☉ M. lon.	☉ M. lon.	☉ M. lon.	☉ M. lon.
1 0 59 17	1 32 27	1 29 8 20	2 29 41 38	2 29 15 48	4 29 49 6	5 29 23 16	6 29 56 34	8 0 29 52	9 0 4 2	10 0 37 20	11 0 11 30	1
2 1 58 17	2 31 35	2 0 7 28	3 0 40 46	4 0 14 56	5 0 48 14	6 0 22 24	7 0 55 42	1 29 1	2 0 1 3	3 0 26 29	4 0 10 39	2
3 2 57 25	3 30 43	1 6 36	1 39 55	1 14 4	1 47 23	1 21 33	2 0 54 51	2 28 9	3 1 27	4 0 36	5 0 11 40	3
4 3 56 33	4 29 52	2 5 45	2 39 5	2 13 13	2 46 31	2 20 41	3 0 53 59	3 27 17	4 0 36	5 0 11 40	6 0 11 40	4
5 4 55 42	5 29 0	3 4 53	3 38 11	3 12 21	3 45 39	3 19 49	4 0 52 16	4 26 26	5 0 11 40	6 0 11 40	7 0 11 40	5
6 5 54 50	6 28 8	4 4 1	4 37 20	4 11 29	4 44 48	4 18 58	5 0 51 24	5 25 34	6 0 11 40	7 0 11 40	8 0 11 40	6
7 6 53 58	7 27 16	5 3 10	5 36 28	5 10 38	5 43 56	5 18 6	6 0 51 24	6 24 42	7 0 11 40	8 0 11 40	9 0 11 40	7
8 7 53 7	8 26 25	6 2 18	6 35 36	6 9 46	6 43 4	6 17 14	7 0 50 32	7 23 51	8 0 11 40	9 0 11 40	10 0 11 40	8
9 8 52 15	9 25 33	7 1 26	7 34 45	7 8 54	7 42 13	7 16 22	8 0 49 41	8 22 59	9 0 11 40	10 0 11 40	11 0 11 40	9
10 9 51 23	10 24 41	8 0 35	8 33 53	8 8 3	8 41 21	8 15 31	9 0 48 49	9 22 7	10 0 11 40	11 0 11 40	12 0 11 40	10
11 10 50 32	11 23 50	8 59 43	9 33 1	9 7 11	9 40 29	9 14 39	10 0 47 57	10 21 16	11 0 11 40	12 0 11 40	13 0 11 40	11
12 11 49 40	12 22 58	9 58 51	10 32 10	10 6 19	10 39 38	10 13 48	11 0 47 6	11 20 24	12 0 11 40	13 0 11 40	14 0 11 40	12
13 12 48 48	13 22 6	10 58 0	11 31 18	11 5 28	11 38 46	11 12 56	12 0 46 14	12 19 32	13 0 11 40	14 0 11 40	15 0 11 40	13
14 13 47 57	14 21 15	11 57 8	12 30 26	12 4 36	12 37 54	12 12 4	13 0 45 22	13 18 41	14 0 11 40	15 0 11 40	16 0 11 40	14
15 14 47 5	15 20 23	12 56 16	13 29 35	13 3 44	13 37 3	13 11 13	14 0 44 31	14 17 49	15 0 11 40	16 0 11 40	17 0 11 40	15
16 15 46 13	16 19 31	13 55 25	14 28 43	14 2 53	14 36 11	14 10 21	15 0 43 39	15 16 57	16 0 11 40	17 0 11 40	18 0 11 40	16
17 16 45 22	17 18 40	14 54 33	15 27 51	15 2 1	15 35 19	15 9 29	16 0 42 47	16 16 6	17 0 11 40	18 0 11 40	19 0 11 40	17
18 17 44 30	18 17 48	15 53 41	16 27 0	16 1 9	16 34 28	16 8 38	17 0 41 50	17 15 14	18 0 11 40	19 0 11 40	20 0 11 40	18
19 18 43 38	19 16 56	16 52 50	17 26 8	17 0 18	17 33 30	17 7 46	18 0 41 4	18 14 22	19 0 11 40	20 0 11 40	21 0 11 40	19
20 19 42 47	20 16 5	17 51 58	18 25 16	17 59 26	18 32 44	18 6 54	19 0 40 12	19 13 31	20 0 11 40	21 0 11 40	22 0 11 40	20
21 20 41 55	21 15 13	18 51 6	19 24 25	18 58 34	19 31 53	19 6 3	20 0 39 21	20 12 39	21 0 11 40	22 0 11 40	23 0 11 40	21
22 21 41 3	22 14 21	19 50 15	20 23 33	19 57 43	20 31 1	20 5 11	21 0 38 29	21 11 47	22 0 11 40	23 0 11 40	24 0 11 40	22
23 22 40 12	23 13 30	20 49 23	21 22 41	20 56 51	21 30 9	21 4 19	22 0 37 37	22 10 56	23 0 11 40	24 0 11 40	25 0 11 40	23
24 23 39 20	24 12 38	21 48 31	22 21 50	21 55 59	22 29 18	22 3 28	23 0 36 46	23 10 4	24 0 11 40	25 0 11 40	26 0 11 40	24
25 24 38 28	25 11 46	22 47 40	23 20 58	22 55 8	23 28 26	23 2 36	24 0 35 54	24 9 12	25 0 11 40	26 0 11 40	27 0 11 40	25
26 25 37 3	26 10 55	23 46 48	24 20 6	23 54 16	24 27 34	24 1 44	25 0 35 2	25 8 21	26 0 11 40	27 0 11 40	28 0 11 40	26
27 26 36 4	27 10 3	24 45 56	25 19 15	24 53 24	25 26 43	25 0 52	26 0 34 11	26 7 29	27 0 11 40	28 0 11 40	29 0 11 40	27
28 27 35 53	28 9 11	25 45 5	26 18 23	25 52 33	26 25 51	26 0 1	27 0 33 19	27 6 37	28 0 11 40	29 0 11 40	30 0 11 40	28
29 28 35 2	29 0 11	26 44 13	27 17 31	26 51 4	27 24 59	26 59 9	28 0 32 27	27 5 40	29 0 11 40	30 0 11 40	31 0 11 40	29
30 29 34 10	30 0 10	27 43 2	28 16 40	27 50 49	28 24 8	27 58 17	29 0 31 36	28 4 54	30 0 11 40	31 0 11 40	32 0 11 40	30
31 30 33 1	31 0 10	28 42 30	29 15 48	28 49 58	29 24 16	28 57 26	30 0 30 44	29 38 12	31 0 11 40	32 0 11 40	33 0 11 40	31
M. M. ap. C												

305. TABLE of the EQUATION of the SUN'S CENTER to each sign and degree of mean Anomaly.

Deg.	0 Subt.	Diff.	I. Subt.	Diff.	II. Subt.	Diff.	III. Subt.	Diff.	IV. Subt.	Diff.	V. Subt.	Diff.	Deg.
0	0	1 59	0 56 47	1 43	1 39 6	1	1 55 37	0 2	1 41 12	1	0 58 53	1 46	30
1	0	1 59	0 58 30	1 42	40 7	0 59	55 39	1	40 12	0	57 7	1 48	29
2	0	1 59	1 0 12	1 41	41 6	0 59	55 38	2	39 10	0	55 19	1 49	28
3	0	1 59	1 1 53	1 40	42 3	0 57	55 36	5	38 6	0	53 30	1 50	27
4	0	1 58	1 3 33	1 39	42 59	0 56	55 31	7	37 0	0	51 40	1 51	26
5	0	1 58	1 5 12	1 38	43 52	0 53	55 24	9	35 52	0	49 49	1 52	25
6	0	1 58	1 6 50	1 37	44 44	0 52	55 15	12	34 43	0	47 57	1 52	24
7	0	1 58	8 27	1 35	45 34	0 50	55 3	13	33 32	0	46 5	1 52	23
8	0	1 58	10 2	1 34	46 22	0 48	54 50	15	32 19	0	44 11	1 54	22
9	0	1 57	11 36	1 33	47 8	0 46	54 35	18	31 4	0	42 16	1 55	21
10	0	1 57	13 9	1 32	47 53	0 45	54 17	20	29 47	0	40 21	1 55	20
11	0	1 56	14 41	1 30	48 35	0 40	53 57	21	28 29	0	38 25	1 56	19
12	0	1 56	16 11	1 29	49 15	0 39	53 36	24	27 9	0	36 28	1 58	18
13	0	1 56	17 40	1 28	49 54	0 36	53 12	26	25 48	0	34 30	1 58	17
14	0	1 55	19 8	1 26	50 30	0 35	52 46	28	24 25	0	32 32	1 59	16
15	0	1 55	20 34	1 25	51 5	0 32	52 18	30	23 0	0	30 33	2 0	15
16	0	1 54	21 59	1 23	51 37	0 31	51 48	33	21 34	0	28 33	2 0	14
17	0	1 53	23 22	1 22	52 8	0 28	51 15	34	20 6	0	26 33	2 0	13
18	0	1 53	24 44	1 21	52 36	0 27	50 41	36	18 36	0	24 33	2 1	12
19	0	1 52	26 5	1 19	53 3	0 24	50 5	39	17 5	0	22 32	2 2	11
20	0	1 52	27 24	1 17	53 27	0 23	49 26	40	15 33	0	20 30	2 2	10
21	0	1 51	28 41	1 16	53 50	0 20	48 46	43	13 59	0	18 28	2 2	9
22	0	1 50	29 57	1 14	54 10	0 18	48 3	44	12 24	0	16 26	2 2	8
23	0	1 49	31 11	1 13	54 28	0 16	47 19	47	10 47	0	14 24	2 3	7
24	0	1 48	32 24	1 11	54 44	0 14	46 32	48	9 9	0	12 21	2 3	6
25	0	1 48	33 35	1 10	54 58	0 12	45 44	51	7 29	0	10 18	2 4	5
26	0	1 47	34 45	1 8	55 10	0 10	44 53	52	5 49	0	8 14	2 4	4
27	0	1 46	35 53	1 6	55 20	0 8	44 1	54	4 7	0	6 11	2 4	3
28	0	1 45	36 59	1 4	55 28	0 6	43 7	57	2 24	0	4 7	2 4	2
29	0	1 44	38 3	1 3	55 34	0 3	42 10	58	0 39	0	2 4	2 4	1
30	0	1 44	39 6	1 3	55 37	0 3	41 12	58	0 58 53	0	0 0	2 4	0

0 23

307. TABLE of the SUN'S RIGHT ASCENSION to each sign and degree of longitude.

Deg	No 6		No 7		No 8		No 9		No 10		No 11		D
	h.	m. f.	h.	m. f.	h.	m. f.	h.	m. f.	h.	m. f.	h.	m. f.	Diff.
0	0	0	1	51 37	3	50	6	0	8	45	10	8 23	3 49
1	3	40	55 27	3 50	55 25	3 50	8	22	12	54	12	12 12	3 48
2	7	20	59 17	3 51	59 36	3 51	43	43	17	3	16	0	3 48
3	11	1	3 48	3 51	4	3 48	13	5	21	11	19	48	3 47
4	14	41	3 40	3 52	8	3 51	17	26	25	19	23	35	3 47
5	18	21	10 51	3 53	12 13	3 52	21	47	27	26	27	22	3 46
6	22	2	14 44	3 53	16 26	3 53	26	9	33	32	31	8	3 46
7	25	42	2 18 37	3 53	20 40	3 53	30	30	37	37	34	54	3 46
8	29	23	22 31	3 54	24 55	3 54	34	51	41	41	38	40	3 46
9	33	4	26 25	3 55	29 10	3 54	39	11	45	45	42	25	3 44
10	36	45	30 20	3 55	33 26	3 55	43	31	49	48	46	9	3 44
11	40	26	34 16	3 56	37 42	3 56	47	51	51	51	49	53	3 44
12	44	8	38 13	3 57	41 59	3 57	52	11	57	52	53	37	3 44
13	47	50	2 42 10	3 57	46 16	3 57	56	30	1	53	57	20	3 43
14	51	32	46 8	3 58	50 34	3 58	7	0	49	53	57	11	3 43
15	55	14	50 7	3 59	54 52	3 59	5	8	0	53	58	4	3 43
16	58	57	54 7	4 0	59 11	4 0	9	26	18	52	58	28	3 42
17	1	24	58 7	4 0	5 30	4 0	13	44	17	50	58	10	3 42
18	6	23	3 2	4 1	7 49	4 1	18	1	17	47	52	15	3 42
19	10	7	3 6	4 1	9	4 1	22	18	16	44	57	34	3 42
20	13	51	10 12	4 3	16 29	4 3	26	34	16	40	56	15	3 41
21	17	36	14 15	4 4	20 49	4 4	30	50	16	35	55	23	3 41
22	21	21	18 19	4 4	25 9	4 4	35	5	15	37	54	30	3 41
23	25	6	22 23	4 5	29 30	4 5	39	20	14	41	54	37	3 41
24	28	52	26 28	4 5	33 51	4 5	43	34	14	45	53	58	3 40
25	32	38	3 30 34	4 6	38 12	4 6	47	47	13	49	53	41	3 41
26	36	25	34 41	4 7	42 34	4 7	52	0	13	53	56	45	3 40
27	40	12	38 49	4 8	46 55	4 8	56	12	12	56	52	49	3 41
28	44	0	42 57	4 9	51 17	4 9	0	24	11	10	58	52	3 40
29	47	48	47 6	4 9	55 38	4 9	8	35	10	43	58	56	3 40
30	51	37	51 15	4 9	6 0	4 9	4	45	10	43	58	0	3 40

306. TABLE of the SUN's DECLINATION to each sign and degree of longitude.

Deg.	V 0 6 Decl.	Diff.	III 7 Decl.	Diff.	II 2 8 Decl.	Diff.
0	0	0	11 29 5	21	20 10 25	12 32
1	0 23	53	11 50 6	20	20 22 57	12 10
2	0 47	47	12 10 56	20	20 35 7	11 48
3	1 11	39	12 31 34	20	20 46 55	11 25
4	1 35	30	12 51 59	20	20 58 20	11 1
5	1 59	20	13 12 12	20	21 9 21	10 38
6	2 23	8	13 32 12	19	21 19 59	10 14
7	2 46	54	13 51 58	19	21 30 13	9 50
8	3 10	37	14 11 30	19	21 40 3	9 26
9	3 34	17	14 30 48	19	21 49 29	9 1
10	3 57	54	14 49 52	18	21 58 30	8 36
11	4 21	27	15 8 40	18	22 7 6	8 11
12	4 44	57	15 27 13	18	22 15 17	7 46
13	5 8	22	15 45 30	18	22 23 3	7 21
14	5 31	42	16 3 11	17	22 30 24	6 54
15	5 54	57	16 21 16	17	22 37 18	6 29
16	6 18	6	16 38 44	17	22 43 47	6 3
17	6 41	9	16 55 55	16	22 49 50	5 37
18	7 4	6	17 12 48	16	22 55 27	5 11
19	7 26	57	17 29 23	16	23 0 38	4 44
20	7 49	41	17 45 46	15	23 5 22	4 17
21	8 12	17	18 1 38	15	23 9 39	3 50
22	8 34	45	18 17 18	15	23 13 29	3 24
23	8 57	5	18 32 38	15	23 16 53	2 57
24	9 19	17	18 47 38	14	23 19 50	2 30
25	9 41	19	19 2 18	14	23 22 20	2 2
26	10 3	12	19 16 37	13	23 24 22	1 35
27	10 24	56	19 30 35	13	23 25 57	1 8
28	10 46	30	19 44 13	13	23 27 46	0 41
29	11 7	53	19 57 39	12	23 27 5	0 14
30	11 29	5	20 10 25	12	23 28 0	
D	11 17	Diff.	10 10 4	Diff.	9 53	Diff.

A TABLE of the SUN'S DECLINATION for the Year 1796, being Leap Year.

[illegible]

308. A TABLE of the SUN'S DECLINATION for the Year 1797, being the First after Leap Year.

Days.	January.		February.		March.		April.		May.		June.		July.		August.		September.		October.		November.		December.	
	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.
1	5 57	5 57	5 56	5 57	5 56	5 57	5 55	5 56	5 54	5 55	5 53	5 54	5 52	5 53	5 51	5 52	5 50	5 51	5 49	5 50	5 48	5 49	5 47	5 48
2	5 56	5 56	5 55	5 56	5 54	5 55	5 53	5 54	5 52	5 53	5 51	5 52	5 50	5 51	5 49	5 50	5 48	5 49	5 47	5 48	5 46	5 47	5 45	5 46
3	5 55	5 55	5 54	5 55	5 53	5 54	5 52	5 53	5 51	5 52	5 50	5 51	5 49	5 50	5 48	5 49	5 47	5 48	5 46	5 47	5 45	5 46	5 44	5 45
4	5 54	5 54	5 53	5 54	5 52	5 53	5 51	5 52	5 50	5 51	5 49	5 50	5 48	5 49	5 47	5 48	5 46	5 47	5 45	5 46	5 44	5 45	5 43	5 44
5	5 53	5 53	5 52	5 53	5 51	5 52	5 50	5 51	5 49	5 50	5 48	5 49	5 47	5 48	5 46	5 47	5 45	5 46	5 44	5 45	5 43	5 44	5 42	5 43
6	5 52	5 52	5 51	5 52	5 50	5 51	5 49	5 50	5 48	5 49	5 47	5 48	5 46	5 47	5 45	5 46	5 44	5 45	5 43	5 44	5 42	5 43	5 41	5 42
7	5 51	5 51	5 50	5 51	5 49	5 50	5 48	5 49	5 47	5 48	5 46	5 47	5 45	5 46	5 44	5 45	5 43	5 44	5 42	5 43	5 41	5 42	5 40	5 41
8	5 50	5 50	5 49	5 50	5 48	5 49	5 47	5 48	5 46	5 47	5 45	5 46	5 44	5 45	5 43	5 44	5 42	5 43	5 41	5 42	5 40	5 41	5 39	5 40
9	5 49	5 49	5 48	5 49	5 47	5 48	5 46	5 47	5 45	5 46	5 44	5 45	5 43	5 44	5 42	5 43	5 41	5 42	5 40	5 41	5 39	5 40	5 38	5 39
10	5 48	5 48	5 47	5 48	5 46	5 47	5 45	5 46	5 44	5 45	5 43	5 44	5 42	5 43	5 41	5 42	5 40	5 41	5 39	5 40	5 38	5 39	5 37	5 38
11	5 47	5 47	5 46	5 47	5 45	5 46	5 44	5 45	5 43	5 44	5 42	5 43	5 41	5 42	5 40	5 41	5 39	5 40	5 38	5 39	5 37	5 38	5 36	5 37
12	5 46	5 46	5 45	5 46	5 44	5 45	5 43	5 44	5 42	5 43	5 41	5 42	5 40	5 41	5 39	5 40	5 38	5 39	5 37	5 38	5 36	5 37	5 35	5 36
13	5 45	5 45	5 44	5 45	5 43	5 44	5 42	5 43	5 41	5 42	5 40	5 41	5 39	5 40	5 38	5 39	5 37	5 38	5 36	5 37	5 35	5 36	5 34	5 35
14	5 44	5 44	5 43	5 44	5 42	5 43	5 41	5 42	5 40	5 41	5 39	5 40	5 38	5 39	5 37	5 38	5 36	5 37	5 35	5 36	5 34	5 35	5 33	5 34
15	5 43	5 43	5 42	5 43	5 41	5 42	5 40	5 41	5 39	5 40	5 38	5 39	5 37	5 38	5 36	5 37	5 35	5 36	5 34	5 35	5 33	5 34	5 32	5 33
16	5 42	5 42	5 41	5 42	5 39	5 40	5 38	5 39	5 37	5 38	5 36	5 37	5 35	5 36	5 34	5 35	5 33	5 34	5 32	5 33	5 31	5 32	5 30	5 31
17	5 41	5 41	5 40	5 41	5 38	5 39	5 37	5 38	5 36	5 37	5 35	5 36	5 34	5 35	5 33	5 34	5 32	5 33	5 31	5 32	5 30	5 31	5 29	5 30
18	5 40	5 40	5 39	5 40	5 37	5 38	5 36	5 37	5 35	5 36	5 34	5 35	5 33	5 34	5 32	5 33	5 31	5 32	5 30	5 31	5 29	5 30	5 28	5 29
19	5 39	5 39	5 38	5 39	5 36	5 37	5 35	5 36	5 34	5 35	5 33	5 34	5 32	5 33	5 31	5 32	5 30	5 31	5 29	5 30	5 28	5 29	5 27	5 28
20	5 38	5 38	5 37	5 38	5 35	5 36	5 34	5 35	5 33	5 34	5 32	5 33	5 31	5 32	5 30	5 31	5 29	5 30	5 28	5 29	5 27	5 28	5 26	5 27
21	5 37	5 37	5 36	5 37	5 34	5 35	5 33	5 34	5 32	5 33	5 31	5 32	5 30	5 31	5 29	5 30	5 28	5 29	5 27	5 28	5 26	5 27	5 25	5 26
22	5 36	5 36	5 35	5 36	5 33	5 34	5 32	5 33	5 31	5 32	5 30	5 31	5 29	5 30	5 28	5 29	5 27	5 28	5 26	5 27	5 25	5 26	5 24	5 25
23	5 35	5 35	5 34	5 35	5 32	5 33	5 31	5 32	5 30	5 31	5 29	5 30	5 28	5 29	5 27	5 28	5 26	5 27	5 25	5 26	5 24	5 25	5 23	5 24
24	5 34	5 34	5 33	5 34	5 31	5 32	5 30	5 31	5 29	5 30	5 28	5 29	5 27	5 28	5 26	5 27	5 25	5 26	5 24	5 25	5 23	5 24	5 22	5 23
25	5 33	5 33	5 32	5 33	5 30	5 31	5 29	5 30	5 28	5 29	5 27	5 28	5 26	5 27	5 25	5 26	5 24	5 25	5 23	5 24	5 22	5 23	5 21	5 22
26	5 32	5 32	5 31	5 32	5 29	5 30	5 28	5 29	5 27	5 28	5 26	5 27	5 25	5 26	5 24	5 25	5 23	5 24	5 22	5 23	5 21	5 22	5 20	5 21
27	5 31	5 31	5 30	5 31	5 28	5 29	5 27	5 28	5 26	5 27	5 25	5 26	5 24	5 25	5 23	5 24	5 22	5 23	5 21	5 22	5 20	5 21	5 19	5 20
28	5 30	5 30	5 29	5 30	5 27	5 28	5 26	5 27	5 25	5 26	5 24	5 25	5 23	5 24	5 22	5 23	5 21	5 22	5 20	5 21	5 19	5 20	5 18	5 19
29	5 29	5 29	5 28	5 29	5 26	5 27	5 25	5 26	5 24	5 25	5 23	5 24	5 22	5 23	5 21	5 22	5 20	5 21	5 19	5 20	5 18	5 19	5 17	5 18
30	5 28	5 28	5 27	5 28	5 25	5 26	5 24	5 25	5 23	5 24	5 22	5 23	5 21	5 22	5 20	5 21	5 19	5 20	5 18	5 19	5 17	5 18	5 16	5 17
31	5 27	5 27	5 26	5 27	5 24	5 25	5 23	5 24	5 22	5 23	5 21	5 22	5 20	5 21	5 19	5 20	5 18	5 19	5 17	5 18	5 16	5 17	5 15	5 16

308. A TABLE of the SUN'S DECLINATION for the Year 1798, being the Second-after Leap Year;

January.		February.		March.		April.		May.		June.		July.		August.		September.		October.		November.		December.		
Days.		Days.		Days.		Days.		Days.		Days.		Days.		Days.		Days.		Days.		Days.		Days.		
South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.		
1	22	58	22	16	56	12	7	22	5	4	44	58	15	14	22	8	5	23	6	21	17	55	2	
2	22	52	54	16	38	48	6	59	11	5	8	0	15	31	54	22	15	53	23	1	57	17	40	29
3	22	46	58	16	21	6	6	36	13	5	30	57	15	49	33	23	17	22	57	9	17	24	48	
4	22	40	36	16	3	7	6	13	8	5	53	48	16	6	55	22	30	18	22	51	17	8	50	
5	22	33	46	15	44	52	5	49	58	2	16	32	16	24	2	22	36	55	22	46	16	52	36	
6	22	26	29	15	26	21	5	26	44	2	22	43	9	22	40	21	16	36	5	18	53	16	9	47
7	22	18	46	15	7	34	5	3	25	7	1	43	16	57	26	22	33	58	16	19	17	5	54	12
8	22	10	37	14	48	32	4	40	2	7	24	17	13	43	22	54	25	22	27	11	16	2	14	
9	22	2	114	29	14	4	7	46	35	7	46	25	17	29	43	22	59	27	12	20	1	15	44	56
10	21	53	0	14	9	42	3	53	5	8	8	35	17	45	26	23	4	4	22	12	27	15	27	21
11	21	43	33	13	49	56	3	29	32	8	30	37	18	0	51	23	8	18	22	4	30	15	9	38
12	21	33	41	13	29	56	3	5	57	8	52	30	18	15	58	23	12	6	21	56	11	14	51	28
13	21	23	24	13	9	43	2	42	19	9	14	15	18	30	46	23	15	31	21	47	30	14	33	10
14	21	12	42	12	49	17	2	18	40	9	35	50	18	45	16	23	18	31	21	38	25	14	14	39
15	21	1	35	12	28	39	1	54	59	9	57	16	18	59	27	23	21	6	21	28	59	13	55	53
16	20	50	5	12	7	49	1	31	17	10	18	33	19	13	19	23	23	16	21	19	11	13	36	54
17	20	38	10	11	46	47	1	7	34	10	39	39	19	40	52	23	25	12	12	9	1	13	17	43
18	20	25	52	11	25	34	0	43	52	11	0	34	19	40	52	23	26	23	20	58	29	1	12	58
19	20	13	12	11	4	11	0	20	9	11	21	19	19	52	57	23	27	19	20	47	37	12	38	42
20	20	0	8	10	42	37	0	3	33	11	41	52	20	5	29	23	27	50	20	36	23	12	18	54
21	19	46	42	10	20	54	0	27	14	12	2	14	20	17	41	23	27	56	20	44	48	1	58	54
22	19	32	54	9	59	2	0	50	53	12	22	24	20	29	32	23	27	38	20	12	53	1	38	43
23	19	18	45	9	37	0	1	14	31	12	42	21	20	41	1	23	26	55	20	0	38	11	18	21
24	19	4	14	9	14	49	1	18	7	13	2	64	0	52	10	23	25	47	19	48	3	10	57	48
25	18	49	23	8	52	31	2	1	40	13	21	38	21	2	57	23	24	14	19	35	8	10	37	5
26	18	34	11	8	30	5	4	25	11	13	40	57	21	13	22	23	22	16	19	21	53	19	16	11
27	18	18	39	8	7	32	2	48	39	14	0	221	23	25	23	19	55	19	8	20	9	55	8	
28	18	2	47	7	44	51	3	12	3	14	18	54	21	33	6	23	17	8	54	28	9	33	55	
29	17	46	36	3	35	23	3	35	23	14	37	32	21	42	25	23	13	57	18	40	17	9	12	33
30	17	30	7	3	58	39	3	58	39	14	55	55	21	51	21	23	10	21	18	25	47	8	51	2
31	17	13	18	4	21	51	4	21	51	21	59	51	21	59	51	18	10	58	8	29	28	8	29	28

309. A TABLE of the SUN'S DECLINATION for the Year 1800, not Leap Year.

January		February		March		April		May		June		July		August		September		October		November		December	
Days.		Days.		Days.		Days.		Days.		Days.		Days.		Days.		Days.		Days.		Days.		Days.	
South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.
1 23	0 54	17 4 30	7 33 3	4 33 54	15 5 24	22 4 15	23 18 22	18 3 11	8 18 4	3 11 39	14 27 54	21 50 38	21 50 38	3 11 39	14 27 54	21 50 38	21 50 38	3 11 39	14 27 54	21 50 38	21 50 38	3 11 39	14 27 54
2 22	55 39	16 47 14	7 10 12	4 56 59	15 23 25	22 12 14	23 18 22	4 9 17	7 56 12	3 34 58	14 47 2	21 59 43	21 59 43	3 34 58	14 47 2	21 59 43	21 59 43	3 34 58	14 47 2	21 59 43	21 59 43	3 34 58	14 47 2
3 22	49 57	16 29 40	6 47 15	5 19 59	15 41 11	22 19 50	22 59 33	17 32 23	7 34 13	3 58 14	15 5 56	22 8 24	22 8 24	3 58 14	15 5 56	22 8 24	22 8 24	3 58 14	15 5 56	22 8 24	22 8 24	3 58 14	15 5 56
4 22	43 47	16 11 49	6 24 13	5 42 53	15 58 41	22 27 22	54 32 17	16 34	7 12 6	4 21 27	15 24 35	22 16 38	22 16 38	4 21 27	15 24 35	22 16 38	22 16 38	4 21 27	15 24 35	22 16 38	22 16 38	4 21 27	15 24 35
5 22	37 10	15 53 42	6 1 5	6 5 41	16 15 55	22 33 51	22 49 8	17 0 28	6 49 53	4 44 37	15 42 59	22 24 27	22 24 27	4 44 37	15 42 59	22 24 27	22 24 27	4 44 37	15 42 59	22 24 27	22 24 27	4 44 37	15 42 59
6 22	30 6	15 35 18	5 37 52	6 28 22	16 32 53	22 40 16	22 43 20	16 44 5	6 27 32	5 7 44	16 1 7	22 31 49	22 31 49	5 7 44	16 1 7	22 31 49	22 31 49	5 7 44	16 1 7	22 31 49	22 31 49	5 7 44	16 1 7
7 22	22 35	15 16 39	5 14 36	6 50 57	16 49 35	22 46 17	22 37 8	16 27 26	6 5 5	5 30 47	16 18 59	22 38 45	22 38 45	5 30 47	16 18 59	22 38 45	22 38 45	5 30 47	16 18 59	22 38 45	22 38 45	5 30 47	16 18 59
8 22	14 39	14 57 44	4 51 15	7 13 24	17 5 59	22 51 54	22 30 33	16 10 31	5 42 32	5 53 46	16 36 35	22 45 15	22 45 15	5 53 46	16 36 35	22 45 15	22 45 15	5 53 46	16 36 35	22 45 15	22 45 15	5 53 46	16 36 35
9 22	6 16	14 38 34	4 27 50	7 35 45	17 22 8	22 57 7	22 23 34	15 53 20	5 19 53	6 16 41	16 53 54	22 51 18	22 51 18	6 16 41	16 53 54	22 51 18	22 51 18	6 16 41	16 53 54	22 51 18	22 51 18	6 16 41	16 53 54
10 21	57 27	14 19 10	4 4 22	7 57 58	17 37 58	23 1 56	22 16 12	15 35 54	4 57 9	6 39 30	17 10 56	22 56 53	22 56 53	6 39 30	17 10 56	22 56 53	22 56 53	6 39 30	17 10 56	22 56 53	22 56 53	6 39 30	17 10 56
11 21	48 13	13 59 31	3 40 51	8 20 31	17 53 31	23 6 21	22 8 26	15 18 13	4 34 18	7 2 15	17 27 40	23 2 2	23 2 2	7 2 15	17 27 40	23 2 2	23 2 2	7 2 15	17 27 40	23 2 2	23 2 2	7 2 15	17 27 40
12 21	38 33	13 39 39	3 17 18	8 41 59	18 8 46	23 10 21	22 0 18	15 0 17	4 11 24	7 24 54	17 44 6	23 6 43	23 6 43	7 24 54	17 44 6	23 6 43	23 6 43	7 24 54	17 44 6	23 6 43	23 6 43	7 24 54	17 44 6
13 21	28 28	13 19 33	2 53 42	9 3 47	18 23 42	23 13 58	21 51 48	14 42 6	3 48 24	7 47 26	18 0 14	23 10 56	23 10 56	7 47 26	18 0 14	23 10 56	23 10 56	7 47 26	18 0 14	23 10 56	23 10 56	7 47 26	18 0 14
14 21	17 58	12 59 14	2 30 4	9 25 26	18 38 21	23 17 9	21 42 55	14 23 41	3 25 21	8 9 53	18 16 3	23 14 42	23 14 42	8 9 53	18 16 3	23 14 42	23 14 42	8 9 53	18 16 3	23 14 42	23 14 42	8 9 53	18 16 3
15 21	7 4	12 38 42	2 6 25	9 46 56	18 52 41	23 19 57	21 33 39	14 5 3	3 2 14	8 32 13	18 31 33	23 18 0	23 18 0	8 32 13	18 31 33	23 18 0	23 18 0	8 32 13	18 31 33	23 18 0	23 18 0	8 32 13	18 31 33
16 20	55 46	12 17 58	1 42 44	10 8 16	19 6 41	23 22 19	21 24 2	13 46 10	2 39 3	8 54 25	18 46 43	23 20 49	23 20 49	8 54 25	18 46 43	23 20 49	23 20 49	8 54 25	18 46 43	23 20 49	23 20 49	8 54 25	18 46 43
17 20	44 3	11 57 2	1 19 3	10 29 27	19 20 23	23 24 17	21 14 2	13 27 5	2 15 49	9 16 30	19 1 34	23 23 11	23 23 11	9 16 30	19 1 34	23 23 11	23 23 11	9 16 30	19 1 34	23 23 11	23 23 11	9 16 30	19 1 34
18 20	31 57	11 35 55	0 55 21	10 50 27	19 33 45	23 25 49	21 3 42	13 7 47	1 52 32	9 38 27	19 16 3	23 25 5	23 25 5	9 38 27	19 16 3	23 25 5	23 25 5	9 38 27	19 16 3	23 25 5	23 25 5	9 38 27	19 16 3
19 20	19 27	11 14 37	0 31 39	11 11 17	19 46 47	23 26 57	20 52 59	12 48 16	1 29 12	10 0 16	19 30 12	23 26 30	23 26 30	10 0 16	19 30 12	23 26 30	23 26 30	10 0 16	19 30 12	23 26 30	23 26 30	10 0 16	19 30 12
20 20	6 35	10 55 8	0 7 57	11 31 56	19 59 30	23 27 40	20 41 55	12 28 33	1 5 51	10 21 55	19 44 0	23 27 27	23 27 27	10 21 55	19 44 0	23 27 27	23 27 27	10 21 55	19 44 0	23 27 27	23 27 27	10 21 55	19 44 0
21 19	53 20	10 31 29	0 15 45	11 52 23	20 11 51	23 27 59	20 30 12	8 38	0 42 27	10 43 26	19 57 26	23 27 36	23 27 36	10 43 26	19 57 26	23 27 36	23 27 36	10 43 26	19 57 26	23 27 36	23 27 36	10 43 26	19 57 26
22 19	39 43	10 9 41	0 39 25	12 12 39	20 23 53	23 27 52	20 18 44	11 48 31	0 19 3N.	11 4 47	20 10 29	23 27 57	23 27 57	11 4 47	20 10 29	23 27 57	23 27 57	11 4 47	20 10 29	23 27 57	23 27 57	11 4 47	20 10 29
23 19	25 43	9 47 42	1 3 4	12 32 43	20 35 33	23 27 21	20 6 38	11 28 13	0 4 22S.	11 25 58	20 23 11	23 27 28	23 27 28	11 25 58	20 23 11	23 27 28	23 27 28	11 25 58	20 23 11	23 27 28	23 27 28	11 25 58	20 23 11
24 19	11 22	9 25 36	1 26 42	12 52 35	20 46 52	23 26 2	19 54 12	11 7 45	0 27 48	11 46 58	20 35 30	23 26 32	23 26 32	11 46 58	20 35 30	23 26 32	23 26 32	11 46 58	20 35 30	23 26 32	23 26 32	11 46 58	20 35 30
25 18	56 41	9 3 20	1 50 17	13 12 15	20 57 50	23 25 4	19 41 26	10 47 5	0 51 15	12 7 47	20 47 25	23 25 7	23 25 7	12 7 47	20 47 25	23 25 7	23 25 7	12 7 47	20 47 25	23 25 7	23 25 7	12 7 47	20 47 25
26 18	41 38	8 40 57	2 13 50	13 31 41	21 8 26	23 23 19	19 28 20	10 26 16	1 14 41	12 28 25	20 58 58	23 23 14	23 23 14	12 28 25	20 58 58	23 23 14	23 23 14	12 28 25	20 58 58	23 23 14	23 23 14	12 28 25	20 58 58
27 18	26 15	8 18 26	2 37 20	13 50 53	21 18 40	23 21 9	19 14 56	10 5 17	1 38 7	12 48 52	21 10 6	23 20 53	23 20 53	12 48 52	21 10 6	23 20 53	23 20 53	12 48 52	21 10 6	23 20 53	23 20 53	12 48 52	21 10 6
28 18	10 32	7 55 48	3 0 47	14 9 53	21 28 32	23 18 34	19 1 12	9 44 8	2 1 32	13 9 6	21 20 51	23 18 3	23 18 3	13 9 6	21 20 51	23 18 3	23 18 3	13 9 6	21 20 51	23 18 3	23 18 3	13 9 6	21 20 51
29 17	54 30	7 32 17	3 24 10	14 28 37	21 38 21	23 15 34	18 47 9	9 22 50	2 24 56	13 20 8	21 31 11	23 14 46	23 14 46	13 20 8	21 31 11	23 14 46	23 14 46	13 20 8	21 31 11	23 14 46	23 14 46	13 20 8	21 31 11
30 17	38 8	7 17 38	3 47 29	14 47 8	21 47 9	23 12 10	18 32 48	9 1 23	2 48 18	13 48 57	21 41 7	23 11 1	23 11 1	13 48 57	21 41 7	23 11 1	23 11 1	13 48 57	21 41 7	23 11 1	23 11 1	13 48 57	21 41 7
31 17	21 29	7 1 21	4 10 44	15 6 14	21 55 54	23 11 18	18 18 8	8 39 47		14 8 3	21 41 7	23 6 48	23 6 48	14 8 3	21 41 7	23 6 48	23 6 48	14 8 3	21 41 7	23 6 48	23 6 48	14 8 3	21 41 7

Table of the Sun's Declination for the Year 1801, the Third before Leap Year.

309. A TABLE of the SUN'S DECLINATION for the Year 1801, the Third before Leap Year.

January.		February.		March.		April.		May.		June.		July.		August.		September.		October.		November.		December.	
Days.	South.	South.	South.	South.	South.	North.	North.	North.	North.	North.	North.	North.	North.	North.	North.	North.	North.	South.	South.	South.	South.	South.	
1	23 2	7 17	8 41	7 38	36	4 28	15 15	15 0	57	22 2	15 23	9 21	18 6	53	8 23	23	3 6	1 14	23 17	21 48	24	1	
2	22 56	59 16	51 29	7 15	48	4 51	21 15	15 19	1 12	10 19	23 5	14 17	51 42	8 1	32	3 29	21 14	42 30	21 57	37	2		
3	22 51	23 16	34 0	6 52	53	5 14	22 15	36 51	22 18	0 23	0 44	17 36	14 7	39	33	3 52	39 15	1 28	22 6	23	3		
4	22 45	20 16	16 14	6 29	52	5 37	17 15	54 25	22 25	18 22	55 49	17 20	28 7	17	27	4 15	54 15	20 11	22 14	44	4		
5	22 38	50 15	58 10	6 6	45	6 0	7 16	11 43	22 32	13 22	50 30	17 4	25	6	55	4 39	6 15	38 40	22 22	40	5		
6	22 31	53 15	39 50	5 43	34	6 22	50 16	28 45	22 38	44 22	44 47	16 48	5 6	32	54	5 2	15 15	56 52	22 30	9	6		
7	22 24	29 15	21 14	5 20	18	6 45	27 16	45 31	22 44	51 22	38 40	16 31	29 6	10	27	5 25	20 16	14 49	22 37	11	7		
8	22 16	38 15	2 22	4 56	57	7 7	57 17	2 12	50 35	22 32	10 16	14 37	5 47	54	5 49	21 16	32 29	22 43	47	8			
9	22 8	22 14	43 15	4 33	33	7 30	21 17	18 14	22 55	54 22	25 17	15 57	29 5	25	15	6 14	18 16	49 53	22 49	56	9		
10	21 59	39 14	23 53	4 10	4	7 52	36 17	34 10	23 0	49 22	18 0	15 40	5 5	2	31	6 34	10 17	6 59	22 55	38	10		
11	21 50	30 14	4 17	3 46	33	8 14	44 17	49 47	23 5	20 22	10 20	15 22	27 4	39	42	6 56	56 17	23 48	23 0	53	11		
12	21 40	56 13	44 27	3 22	59	8 36	44 18	5 8	23 9	27 22	2 17	15 4	34	4	16	7 19	36 17	40 18	23 5	41	12		
13	21 30	57 13	24 23	2 59	22	8 58	35 18	20 9	23 23	23 9	21 53	14 46	26 3	53	49	7 42	11 17	56 30	23 10	0	13		
14	21 20	33 13	4 6	2 35	44	9 20	17 18	34 53	23 16	26 21	45 4	14 28	5 3	30	47	8 4	39 18	12 23	23 13	52	14		
15	21 9	44 12	43 36	2 12	4	9 41	50 18	49 18	23 19	19 21	35 54	14 9	29	3	7	8 26	59 18	27 57	23 17	17	15		
16	20 58	31 12	22 55	1 48	22	10 3	14 19	3 24	23 21	48 21	26 21	13 50	41 2	44	31	8 49	13 18	43 11	23 20	13	16		
17	20 46	54 12	2 2	1 24	40	10 24	27 19	17 10	23 23	51 21	16 27	13 31	39 2	21	18	9 11	19 18	58 5	23 22	41	17		
18	20 34	53 11	40 57	1 0	58	10 45	30 19	30 37	23 25	30 21	6 11	13 12	24 1	58	3	9 33	17 19	12 39	23 24	41	18		
19	20 22	29 11	19 41	0 37	15	11 6	22 19	43 44	23 26	44 20	55 34	12 52	57 1	34	45	9 55	7 19	26 52	23 26	13	19		
20	20 9	42 10	58 15	0 13	33	11 27	3 19	56 30	23 27	33 20	44 35	12 33	17 1	11	25	10 16	47 19	40 44	23 27	17	20		
21	19 56	32 10	36 39	0 10	8N.	11 47	33 20	8 57	23 27	57 20	33 16	12 13	26 0	48	4	10 38	19 19	54 14	23 27	52	21		
22	19 42	59 10	14 53	0 33	49	12 7	51 20	21 2	23 27	57 20	21 36	11 53	23 0	24	41	10 59	41 20	7 23	23 27	59	22		
23	19 29	6 9	52 58	0 57	27	12 27	57 20	32 47	23 27	32 20	9 36	11 33	9 0	1	16	11 20	54 20	20 9	23 27	38	23		
24	19 14	51 9	30 54	1 21	5	12 47	51 20	44 11	23 26	42 19	57 16	11 12	43 0	22	9S.	11 41	56 20	32 33	23 26	49	24		
25	19 0	14 9	8 42	1 44	40	13 7	32 20	55 13	23 25	27 19	44 35	10 52	8 0	45	34	12 2	47 20	44 34	23 25	31	25		
26	18 45	17 8	46 22	2 8	12	13 27	0 21	5 54	23 23	47 19	31 34	10 31	21 1	9	0	12 23	28 20	56 12	23 23	45	26		
27	18 30	0 8	23 54	2 31	41	13 46	15 21	16 13	23 21	43 19	18 15	10 10	25 1	32	26	12 43	57 21	7 27	23 21	30	27		
28	18 14	22 8	1 19	2 55	8	14 5	17 21	26 10	23 19	15 19	4 36	9 49	19 2	55	51	13 4	14 21	18 17	23 18	48	28		
29	17 58	25 3	18 31	3 18	31	14 24	4 21	35 45	21 16	21 18	50 38	9 28	3 2	19	16	13 24	19 21	28 44	23 15	37	29		
30	17 42	9 14	1 50	3 41	50	14 42	38 21	44 57	23 13	4 18	16 21	9 6	39 2	42	39	13 44	12 21	38 46	23 11	58	30		
31	17 25	34 4	5 5	4 5	5			53 47		18 21	46 8	45 5				14 3	51		23 7	52	31		

309. A TABLE of the SUN'S DECLINATION for the Year 1803, the Second before Leap Year.

January.		February.		March.		April.		May.		June.		July.		August.		September.		October.		November.		December.	
Days.	South.	South.	South.	South.	South.	North.	North.	North.	North.	North.	North.	North.	North.	North.	North.	North.	North.	South.	South.	South.	South.	South.	
1	23 3	16 17	12 44	7 44	4 4	4 22	4 1	14 56	34 22	0 14	23 10	15 18	10 27	8 28	33 8	28 3	3 0	28 3	14 18	41 14	18 41	21 46	8 1
2	22 58	14 16	55 36	7 21	16 4	4 45	49 15	14 42	22 8	24 23	6 15	17 55	21 17	6 44	8 6	44 3	23 48	14 37	57 14	37 57	21 55	26 2	
3	22 52	14 16	38 10	6 58	22 5	5 8	52 15	32 36	22 16	11 23	1 56	17 39	56 17	44 47	7 44	47 3	47 6	14 56	58 22	4 18	3 3		
4	22 46	14 16	20 27	6 35	21 6	5 31	50 15	50 14	22 23	35 22	57 0	17 24	14 17	22 43	7 22	43 7	40 22	15 15	44 22	12 45	4 4		
5	22 40	14 16	2 27	6 12	15 6	5 54	42 16	7 37	22 30	36 22	51 47	17 8	15 17	20 33	7 0	33 7	43 34	15 34	16 15	20 46	5 5		
6	22 33	15 44	10 10	5 49	4 4	6 17	27 16	24 44	22 37	12 22	46 10	16 52	0 16	38 15	6 38	15 4	43 56	15 52	31 15	28 21	6 6		
7	22 26	15 44	25 37	5 25	48 5	6 40	6 16	41 34	22 43	25 22	40 9	16 35	28 16	15 50	6 15	50 6	48 19	16 10	31 16	35 30	7 7		
8	22 18	15 44	40 49	5 2	28 5	7 2	38 16	58 7	22 49	14 22	33 46	16 18	40 16	53 20	5 53	20 5	42 49	16 28	14 16	42 12	8 8		
9	22 10	14 47	46 4	4 39	4 4	7 25	3 17	14 24	22 54	39 22	26 58	16 1	37 15	30 44	5 30	44 6	46 5	16 45	41 16	48 28	9 9		
10	22 01	14 28	28 4	4 15	37 4	7 47	20 17	30 23	22 59	39 22	19 46	15 44	18 15	8 2	5 8	2 6	28 37	17 2	51 17	54 16	10 10		
11	21 52	14 08	56 7	3 52	7 7	8 9	29 17	46 05	23 4	16 22	12 12	15 26	44 15	15 4	4 45	15 4	51 24	17 19	43 17	59 37	11 11		
12	21 43	13 49	10 10	3 28	34 3	8 31	30 18	1 29	23 8	28 22	4 15	15 8	56 15	22 23	4 22	23 7	40 17	17 36	17 23	4 31	12 12		
13	21 33	13 29	10 3	3 4	59 3	8 53	22 18	16 34	23 12	16 21	55 56	14 50	53 14	59 27	3 59	27 7	36 40	17 52	33 23	8 57	13 13		
14	21 23	13 8	57 8	2 41	22 9	9 15	5 18	31 21	23 15	39 21	47 14	14 32	35 14	36 26	3 36	26 3	59 8	18 8	31 18	12 56	14 14		
15	21 12	12 48	32 2	2 17	43 2	9 36	39 18	45 50	23 18	38 21	38 9	14 14	4 14	13 22	3 13	22 3	21 31	18 24	9 18	16 27	15 15		
16	21 1	12 27	54 1	1 54	3 1	9 58	4 18	59 59	23 21	12 21	28 43	13 55	19 13	50 13	2 50	13 2	43 46	18 39	28 23	19 30	16 16		
17	20 49	12 07	05 1	1 30	23 1	10 19	19 13	50 23	23 22	18 54	18 54	13 36	21 13	27 01	2 27	01 9	54 9	18 54	28 23	22 6	17 17		
18	20 37	11 46	4 1	1 06	42 1	10 40	23 19	27 20	23 25	7 21	8 45	13 17	9 9	3 47	2 3	47 9	54 9	19 9	7 23	24 13	18 18		
19	20 25	11 24	51 0	0 43	0 43	11 1	17 19	40 32	23 26	27 20	58 12	12 57	45 12	40 29	1 40	29 9	46 9	19 23	25 23	25 52	19 19		
20	20 12	11 03	28 0	0 19	19 0	11 22	0 19	53 23	23 27	22 20	47 19	12 38	8 12	17 9	1 17	9 9	29 10	19 37	23 23	27 32	20 20		
21	19 59	10 41	55 0	0 4	22N. 0	11 42	32 20	5 54	23 27	52 20	36 5	12 18	19 12	53 47	0 53	47 0	33 4	19 50	59 23	27 45	21 21		
22	19 46	10 20	12 0	0 28	2 0	12 2	53 20	18 5	23 27	57 20	24 30	11 58	19 11	30 23	0 30	23 0	54 30	20 4	14 23	27 59	22 22		
23	19 32	9 58	20 0	0 51	41 0	12 23	22 20	29 55	23 27	38 20	12 34	11 38	7 0	6 58N. 0	6 58	N. 0	46 11	20 17	7 23	27 44	23 23		
24	19 18	9 36	18 1	1 15	18 1	12 42	59 20	41 24	23 26	54 20	0 18	11 17	43 0	16 28S. 0	16 28	S. 0	51 11	20 29	37 23	27 12	24 24		
25	19 3	9 14	7 1	1 38	53 1	13 2	44 20	52 32	23 25	46 19	47 42	10 57	9 10	39 55	0 39	55 0	46 11	20 41	43 23	25 50	25 25		
26	18 48	8 51	48 2	2 2	27 2	13 22	16 21	3 16	23 24	12 19	34 45	10 36	24 10	3 22	1 3	22 3	18 12	20 53	28 23	24 11	26 26		
27	18 33	8 29	21 2	2 25	58 2	13 41	35 21	13 43	23 22	14 19	21 30	10 15	29 10	26 49	1 26	49 1	39 12	21 4	48 23	22 32	27 27		
28	18 18	8 06	46 4	2 49	26 4	14 0	41 21	23 46	23 19	51 19	7 55	9 54	24 9	50 15	1 50	15 4	59 24	21 15	45 23	19 27	28 28		
29	18 2	7 45	2 17	3 12	50 3	14 19	33 21	33 27	23 17	4 18	54 01	9 33	10 41	23 41	2 23	41 2	19 33	21 26	17 23	16 23	29 29		
30	17 46	7 24	5 3	3 36	11 3	14 38	11 21	42 45	23 13	51 18	39 48	9 11	46 9	23 7	2 37	7 5	39 29	21 36	25 23	12 51	30 30		
31	17 29	7 34	34 3	3 59	28 3	21 51	41 21	51 41	23 13	51 18	25 17	8 50	14 8	51 31	3 51	31 13	59 13	23 8	51 23	51 31	31 31		

309. A TABLE of the SUN'S DECLINATION for the Year 1803, being the last before Leap Year.

Days.	January.		February.		March.		April.		May.		June.		July.		August.		September.		October.		November.		December.	
	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.
1	23	4	23	17	16	46	7	49	29	4	17	12	4	17	12	4	17	12	4	17	12	4	17	12
2	22	5	27	16	59	43	7	26	43	4	40	20	15	10	26	22	6	30	23	7	12	17	58	57
3	22	5	4	16	42	22	7	3	50	5	3	23	15	28	23	22	14	22	23	2	53	17	43	37
4	22	4	8	16	24	44	6	40	52	5	26	21	15	46	4	23	21	51	22	58	9	17	28	0
5	22	4	1	16	6	49	6	17	48	5	49	13	16	3	30	22	28	56	22	53	2	17	12	6
6	22	3	5	15	48	37	5	54	40	6	12	0	16	20	39	22	35	38	22	47	31	16	55	56
7	22	2	8	15	30	9	5	31	26	6	34	38	16	37	32	22	41	56	21	41	36	16	39	28
8	22	2	11	15	11	26	5	8	8	6	57	11	16	54	9	22	47	51	22	35	17	16	22	44
9	22	1	14	14	52	27	4	44	46	7	19	37	17	10	30	22	53	22	28	35	16	5	45	5
10	22	1	17	14	33	13	4	21	21	7	41	55	17	26	33	22	58	28	21	30	15	48	29	5
11	21	5	4	14	13	44	3	57	52	8	4	6	17	42	18	23	3	11	22	14	1	15	30	59
12	21	4	7	13	54	2	3	34	20	8	26	9	17	57	47	23	7	29	22	16	10	15	13	13
13	21	3	10	13	34	5	3	10	46	8	48	4	18	12	57	23	11	23	21	57	56	14	55	13
14	21	2	13	13	13	55	2	47	9	9	50	5	18	27	49	23	14	52	21	49	19	14	36	59
15	21	1	16	12	53	32	2	23	30	9	31	27	18	42	22	23	17	57	21	40	19	14	18	30
16	21	3	19	12	32	57	1	59	50	9	52	54	18	56	37	23	20	37	21	30	57	13	59	47
17	20	5	22	12	12	9	1	36	9	10	14	13	19	10	33	23	22	52	21	21	13	13	40	51
18	20	4	25	11	51	10	1	12	27	10	35	21	19	24	9	23	24	43	21	11	8	13	21	42
19	20	3	28	11	30	0	0	48	44	10	56	18	19	37	26	23	26	9	21	0	40	13	2	20
20	20	2	31	11	8	38	0	25	2	11	17	5	19	50	23	23	27	10	20	49	52	12	42	45
21	20	2	34	10	47	6	0	1	19	12	37	41	20	2	59	23	27	46	20	38	42	12	22	59
22	19	4	37	10	25	25	0	22	22	11	58	6	20	15	15	23	27	58	20	27	12	12	3	1
23	19	3	40	10	3	33	0	46	2	12	18	18	20	27	10	43	27	44	20	15	21	11	42	51
24	19	2	43	9	41	33	1	9	41	12	38	19	20	38	45	23	27	6	20	3	9	11	22	31
25	19	1	46	9	19	24	1	33	18	12	58	7	20	49	58	23	26	3	19	50	38	11	1	59
26	18	5	49	8	57	6	1	56	53	13	17	42	21	0	49	23	24	35	19	37	47	10	41	18
27	18	4	52	8	34	41	2	20	25	13	37	4	21	11	19	23	22	43	19	24	36	10	20	26
28	18	3	55	8	12	9	2	43	54	13	56	13	21	21	27	23	20	26	19	11	6	9	59	25
29	18	2	58	7	19	5	3	7	19	14	15	7	21	31	13	23	17	44	18	57	17	9	38	14
30	17	4	61	7	7	57	3	30	41	14	33	48	21	40	36	23	14	38	18	43	9	9	16	54
31	17	3	64	6	5	31	3	53	58	14	33	48	21	49	37	18	28	43	8	55	25	8	55	25

310. A TABLE of the SUN'S DECLINATION for the Year 1804, being Leap Year.

Days.	January.		February.		March.		April.		May.		June.		July.		August.		September.		October.		November.		December.	
	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	South.	North.	South.	North.	
1	23	5 30	17	20 53	7	32 17	4	34 40	15	5 59	22	4 28	23	8 9	18	2 37	8	17 21	3	12 24	14	28 31	21	50 52
2	23	0 41	17	3 55	7	9 27	4	57 45	15	24	0 22	12 26	23	3 56	17	47 21	7	55 29	3	35 43	14	47 40	21	59 58
3	22	55 25	16	46 38	6	46 30	5	20 45	15	41 45	22	20	122	59	18	31 48	7	33 30	3	59 1	15	6 34	22	8 37 3
4	22	49 42	16	29 4	6	23 27	5	43 39	15	59 15	22	27	132	54	16	17 15	7	11 23	4	22 15	15	25 13	22	16 52 4
5	22	43 31	16	11 12	6	0 19	6	27	16	16 29	22	34	122	48	51	16 59	6	49 9	4	45 26	15	43 37	22	24 40 5
6	22	36 53	15	53 4	5	37 6	6	29 9	16	33 27	22	40	252	43	116	43 25	6	26 48	5	8 34	16	1 45	22	32 1 6
7	22	29 48	15	34 39	5	13 48	6	51 45	16	50 9	22	46	262	36	48	16 26	6	4 20	5	31 38	16	19 37	22	38 56 7
8	22	22 17	15	15 58	4	50 26	7	14 14	17	6 34	22	52	222	30	116	9 48	5	41 45	5	54 38	16	37 13	22	45 25 8
9	22	14 19	14	57 2	4	27 0	7	36 35	17	22 42	22	57	152	23	1215	52 30	5	19 4	6	17 32	16	54 31	22	51 26 9
10	22	5 55	14	37 51	4	3 30	7	58 49	17	38 33	23	2	322	15	48	15 35	9	4 56 18	6	40 22	17	11 32	22	57 1 10
11	21	57 5	14	18 25	3	39 58	8	20 55	17	54 6	23	6	2722	8	215	17 27	4	33 27	7	3 6	17	28 16	23	2 8 11
12	21	47 49	13	58 44	3	16 23	8	42 52	18	9 21	23	10	2721	59	53	14 59	30	4 10 31	7	25 45	17	44 41	23	6 48 12
13	21	38 8	13	38 50	2	52 46	9	4 40	18	24 17	23	14	221	51	21	14 41	19	3 47 29	7	48 17	18	0 47	23	11 0 13
14	21	28 2	13	18 43	2	29 7	9	26 20	18	38 55	23	17	1221	42	27	14 22	54	3 24 25	8	10 42	18	16 35	23	14 44 14
15	21	17 31	12	58 23	2	5 27	9	47 50	18	53 15	23	19	5821	33	11	4 15	3	1 17	8	33 1	18	32 3	23	18 1 15
16	21	6 35	12	37 50	1	41 45	10	9 10	19	7 15	23	22	1921	23	33	13 45	23	2 38 6	8	55 12	18	47 12	23	20 49 16
17	20	55 15	12	17 6	1	18 3	10	30 20	19	20 55	23	24	1621	13	32	13 26	18	2 14 52	9	17 16	19	2 0	23	23 10 17
18	20	43 32	11	56 10	0	54 21	10	51 20	19	34 16	23	25	4821	3	11	7 1	1	51 35	9	39 11	19	16 28	23	25 2 18
19	20	31 25	11	35 2	0	30 39	11	12 8	19	47 17	23	26	5520	52	28	12 47	31	1 28 16	10	0 58	19	30 36	23	26 26 19
20	20	18 54	11	13 44	0	6 58 S.	11	32 46	19	59 58	23	27	3720	41	24	12 27	48	1 4 55	10	22 36	19	44 22	23	27 22 20
21	20	6 1	10	52 16	0	16 43 N.	11	53 12	20	12 18	23	27	5420	29	59	12 7	54	0 41 32	10	44 5	19	57 47	23	27 50 21
22	19	52 46	10	30 37	0	40 22	12	13 27	20	24 18	23	27	4720	18	14	11 47	48	0 18 9	11	5 25	20	10 49	23	27 50 22
23	19	39 8	10	8 49	1	4 1	12	33 29	20	35 57	23	27	1520	6	8	11 27	31	0 5 15	11	26 35	20	23 30	23	27 20 23
24	19	25 8	9	46 52	1	27 37	12	53 19	20	47 14	23	26	1819	53	42	11 7	3	0 28 39	11	47 34	20	35 48	23	26 23 24
25	19	10 48	9	24 46	1	51 11	13	12 57	20	58 11	23	24	5619	40	56	10 46	24	0 52 5	12	8 23	20	47 43	23	24 57 25
26	18	56 6	9	3 32	2	14 42	13	32 22	21	8 45	23	23	1019	27	50	10 25	35	1 15 29	12	29 1	20	59 15	23	23 326
27	18	41 3	8	40 10	2	38 11	13	51 33	21	18 58	23	20	5919	14	25	10 4	36	1 38 54	12	49 27	21	10 23	23	20 41 27
28	18	25 40	8	17 40	3	1 36	14	10 31	21	28 49	23	18	2319	0	41	9 43	27	2 2 18	13	9 41	21	21 7	23	17 51 28
29	18	9 58	7	55 2	3	24 58	14	29 14	21	38 18	23	15	2318	46	38	9 22	9	2 25 41	13	29 43	21	31 27	23	14 32 29
30	17	53 55			3	48 16	14	47 44	21	47 24	23	11	5818	32	16	9 0	41	2 49 3	13	49 33	21	41 22	23	10 46 30
31	17	37 34			4	11 30			21	56 8				18	17	36	8	39 5	14	9 9			23	6 32 31

310. A TABLE of the SUN'S DECLINATION for the Year 1805, being the First after Leap Year.

Days	January		February		March		April		May		June		July		August		September		October		November		December	
	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.
1	23 1	51 17	8 08	7 37	45	4 29	14 1	15 1	40 2	22 2	31 23	9 08	18 6	16 8	22 37	3 6	42 3	30 1	14 23	44 21	48 31	1	2	
2	22 56	42 16	50 56	7 14	56	4 52	21 15	19 42	22 10	34 23	5 00	17 51	05 8	0 46	15 14	42 55	3 30	14 42	55 21	57 43	2	3		
3	22 51	05 16	33 25	6 52	00	5 15	23 15	37 34	22 18	13 23	0 29	17 35	37 7	38 49	15 1	52 22	3 53	18 15	1 52	22 6	28 3	4	5	
4	22 45	01 16	15 35	28 58		5 38	17 15	54 14	22 25	30 22	55 33	17 19	51 7	16 44	15 20	33 4	4 16	30 15	20 33	22 14	48 4	5	6	
5	22 38	30 15	57 36	6 5	50	6 1	06 6	12 22	22 32	24 22	50 13	17 3	48 6	54 32	15 39	00 4	4 39	40 15	39 00	22 22	43 5	6	7	
6	22 31	32 15	39 15	5 42	38	6 23	47 16	29 22	22 38	54 22	44 29	16 47	28 6	32 13	16 47	28 6	32 13	5 2	48 15	57 11	22 30	12 6	8	9
7	22 24	09 15	30 38	5 19	27	6 46	23 16	46 07	22 45	00 22	38 22	16 30	52 6	9 49	16 30	52 6	9 49	5 25	52 16	15 07	22 37	13 7	9	10
8	22 16	18 15	1 47	4 56	01	7 8	51 17	2 36	22 50	42 22	31 52	16 14	01 5	47 17	16 14	01 5	47 17	5 48	51 16	32 46	22 43	48 8	10	11
9	22 8	01 14	42 40	4 32	38	7 31	13 17	18 47	22 56	00 22	25 00	15 56	55 5	24 39	15 56	55 5	24 39	6 11	47 16	50 09	22 49	56 9	11	12
10	21 59	18 14	23 17	4 9	10	7 53	27 17	34 41	23 0	54 22	17 45	15 39	31 5	1 55	15 39	31 5	1 55	6 34	38 17	7 14	22 55	38 10	12	13
11	21 50	09 14	3 42	3 45	40	8 15	34 17	50 17	23 5	24 22	10 04	15 21	54 4	39 7	15 21	54 4	39 7	6 57	24 17	24 01	23 0	51 11	13	14
12	21 40	34 13	43 52	3 22	07	8 37	32 18	5 35	23 9	30 22	2 00	15 4	01 4	16 13	15 4	01 4	16 13	7 20	4 17	40 31	23 5	39 12	14	15
13	21 30	35 13	23 49	2 58	31	8 59	23 18	20 36	23 13	11 21	53 34	14 45	53 3	53 15	14 45	53 3	53 15	7 42	37 17	56 43	23 09	58 13	15	16
14	21 20	12 13	3 32	2 34	54	9 21	03 18	35 18	23 16	28 21	44 46	14 27	31 3	30 12	14 27	31 3	30 12	8 5	04 18	12 36	23 13	49 14	16	17
15	21 9	23 12	43 04	2 11	15	9 42	35 18	49 43	23 19	19 21	35 35	14 8	55 3	7 7	14 8	55 3	7 7	8 27	26 18	28 10	23 17	14 15	17	18
16	20 58	10 12	22 23	1 47	32	10 3	57 19	3 45	23 21	47 23	21 26	13 50	07 2	43 56	13 50	07 2	43 56	8 49	40 18	43 25	23 20	09 16	18	19
17	20 46	33 12	1 27	1 23	50	10 25	10 19	17 33	23 23	50 21	16 07	13 31	05 2	20 42	13 31	05 2	20 42	9 11	46 18	58 20	23 22	36 17	19	20
18	20 34	32 11	40 21	1 0	9	10 46	12 19	30 50	23 25	28 21	5 50	13 11	49 1	57 25	13 11	49 1	57 25	9 33	45 19	12 54	23 24	35 18	20	21
19	20 22	7 11	19 05	0 36	27	11 7	03 19	44 06	23 26	41 20	55 12	12 51	21 1	34 7	12 51	21 1	34 7	9 55	36 19	27 07	23 26	07 19	21	22
20	20 9	20 10	57 37	0 12	44	11 27	43 19	56 53	23 27	28 20	44 12	12 32	40 1	10 46	12 32	40 1	10 46	10 17	17 19	40 59	23 27	09 20	22	23
21	19 56	10 10	36 01	0 10	58	11 48	13 20	9 17	23 27	51 20	32 52	12 12	48 0	47 24	12 12	48 0	47 24	10 38	49 19	54 28	23 27	43 21	23	24
22	19 42	37 10	14 14	0 34	39	12 8	31 20	21 21	23 27	50 20	21 11	11 52	44 0	23 59	11 52	44 0	23 59	11 0	12 20	7 37	23 27	49 22	24	25
23	19 28	43 9	52 17	0 58	18	12 28	33 20	33 06	23 27	24 20	9 09	11 32	28 0	33	11 32	28 0	33	11 21	26 20	20 24	23 37	27 23	25	26
24	19 14	28 9	30 12	1 21	57	12 48	32 20	44 31	23 26	33 19	56 48	11 12	1 0	22 53	11 12	1 0	22 53	11 42	29 20	32 47	23 26	37 24	26	27
25	18 59	50 9	7 58	1 45	34	13 8	14 20	55 32	23 25	17 19	44 06	10 51	24 0	46 18	10 51	24 0	46 18	12 3	19 20	44 47	23 25	18 25	27	28
26	18 44	51 8	45 36	2 9	06	13 27	42 21	6 13	23 23	36 19	31 04	10 30	37 1	9 45	10 30	37 1	9 45	12 24	0 20	56 24	23 23	31 26	28	29
27	18 29	33 8	23 06	2 32	36	13 46	58 21	16 33	23 21	31 19	17 43	9 39	1 33	11	9 39	1 33	11	12 44	28 21	7 38	23 21	16 27	29	30
28	18 13	53 8	0 29	2 56	04	14 5	59 21	26 29	23 19	02 19	4 03	9 48	39 2	56 35	9 48	39 2	56 35	13 4	44 21	18 28	23 18	33 28	30	31
29	17 57	54 3	19 28	3 19	28	14 24	47 21	36 03	23 16	08 18	50 04	9 27	15 2	19 59	9 27	15 2	19 59	13 24	49 21	28 54	23 15	21 29	31	32
30	17 41	37 3	42 47	3 42	47	14 43	20 21	45 14	23 12	51 18	35 46	9 5	51 2	43 20	9 5	51 2	43 20	13 44	41 21	38 55	23 11	42 30	32	33
31	17 25	02 4	6 04	4 6	04	21 54	4 21	54 4	21 54	4 21	54 4	8 44	18 21	10 8	8 44	18 21	10 8	14 4	19 23	7 36	23 7	36 31	33	34

310. A TABLE of the SUN'S DECLINATION for the Year 1806, being the Second after Leap Year.

Days.	January.		February.		March.		April.		May.		June.		July.		August.		September.		October.		November.		December.	
	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.
1	3 02	17 12	16 55	08 20	7 43	18 30	4 23	33 14	57 11	22 08	26 23	10 02	18 09	54 18	8 27	50 8	3 1	6 14	19 07	21 46	15 1	0 1	21 46	15 1
2	22 57	39 16	55 08	7 20	30 46	15 17	4 46	40 15	55 17	22 08	35 23	6 01	17 54	48 17	8 6	2	3 24	26 14	38 23	21 55	33 2	0 1	21 55	33 2
3	22 52	27 16	37 42	6 57	38 50	15 33	5 09	42 15	57 33	22 08	35 23	1 36	17 39	22 1	7 44	5	3 47	44 14	57 24	22 4	24 3	0 1	22 4	24 3
4	22 46	30 16	20 06	6 34	38 50	15 33	5 32	39 15	50 46	22 08	35 23	56 45	17 23	39 2	7 22	0	4 11	0 15	16 10	22 12	50 4	0 1	22 12	50 4
5	22 40	06 16	2 01	6 11	32 55	29 16	5 55	29 16	08 08	22 30	44 22	51 31	17 7	40 17	6 59	50	4 34	13 15	34 41	22 20	51 5	0 1	22 20	51 5
6	22 33	15 15	43 44	5 48	21 64	12 16	6 18	12 16	25 13	22 37	20 22	45 53	16 51	24 16	6 37	32	4 57	23 15	52 56	22 28	26 6	0 1	22 28	26 6
7	22 25	58 15	25 10	5 25	05 64	02 16	6 40	52 16	42 22	43 32	22 39	52 16	34 51	16 34	6 15	6	5 20	29 16	10 57	22 35	34 7	0 1	22 35	34 7
8	22 18	12 15	6 22	5 14	5 70	03 24	7 03	24 16	58 33	22 49	20 22	33 28	16 18	02 16	5 52	34	5 43	30 16	28 41	22 42	16 8	0 1	22 42	16 8
9	22 10	01 14	47 18	4 38	21 72	01 14	7 25	49 17	14 51	22 54	44 22	26 39	16 0	57 16	5 29	56	6 6	28 16	46 08	22 48	32 9	0 1	22 48	32 9
10	22 1	14 27	59 4	14 53	7 48	06 17	7 48	06 17	30 51	23 59	43 22	19 27	15 43	37 15	5 7	13	6 29	20 17	3 19	22 54	19 10	0 1	22 54	19 10
11	21 52	21 14	8 25	3 51	23 81	15 17	8 10	15 17	46 32	23 4 19	22 11	51 15	26 01	15 26	4 44	26	6 52	08 17	20 10	22 59	38 11	0 1	22 59	38 11
12	21 42	52 13	48 37	3 27	49 83	17 18	8 32	17 18	55 23	8 31	22 3 53	15 8	11 4	21 32	4 21	32	7 14	49 17	36 44	23 4	31 12	0 1	23 4	31 12
13	21 32	59 13	28 36	3 4 12	54 101	18 17	8 54	10 18	17 12	12 18	21 55	33 14	50 07	14 50	3 58	34	7 37	25 17	52 59	23 8	56 13	0 1	23 8	56 13
14	21 22	40 13	8 21	2 40 34	53 15	18 31	9 15	53 18	31 48	23 15	41 21	46 49	14 31	48 14	3 35	32	7 59	53 18	8 57	23 12	55 14	0 1	23 12	55 14
15	21 11	56 12	47 55	2 16 53	53 9	37 28	9 37	28 18	46 17	23 18	38 21	37 43	14 13	15 14	3 12	28	8 22	16 18	24 34	23 16	25 15	0 1	23 16	25 15
16	21 0	48 12	27 16	1 53 11	58 9	54 19	9 58	54 19	0 26	23 21	12 21	28 15	13 54	30 13	2 49	19	8 44	30 18	39 52	23 19	27 16	0 1	23 19	27 16
17	20 49	16 12	6 24	1 29 29	58 10	20 10	10 20	10 19	14 18	23 21	21 18	25 13	35 31	13 35	2 26	7	9 6	38 18	54 52	23 22	02 17	0 1	23 22	02 17
18	20 37	21 11	45 21	1 5 47	58 10	41 13	10 41	13 19	27 49	23 25	05 21	8 14	13 16	18 13	2 2	53	9 28	38 19	9 29	23 24	08 18	0 1	23 24	08 18
19	20 25	02 11	24 07	0 42 03	58 11	2 9	11 2	9 19	41 03	23 26	24 20	57 41	12 56	53 12	1 39	36	9 50	29 19	23 46	23 25	46 19	0 1	23 25	46 19
20	20 12	20 11	2 42	0 18 21	58 11	22 51	11 22	51 19	53 49	23 27	17 20	46 47	12 37	16 12	1 16	17	10 12	12 19	37 43	23 26	56 20	0 1	23 26	56 20
21	19 59	15 10	41 08	0 5 21	58 11	43 23	11 43	23 20	6 20	23 27	45 20	35 32	12 17	28 12	0 52	56	10 33	44 19	51 17	23 27	37 21	0 1	23 27	37 21
22	19 45	48 10	19 25	0 29 02	58 12	3 44	12 3 44	20 18	31 23	27 49	20 23	57 11	57 29	11 57	0 29	33	10 55	08 20	4 31	23 27	50 22	0 1	23 27	50 22
23	19 31	58 9	57 33	0 52 41	58 12	23 52	12 23	52 30	20 30	23 27	29 20	12 01	11 37	18 11	0 6	9	11 16	22 20	17 23	23 27	35 23	0 1	23 27	35 23
24	19 17	48 8	35 19	1 16 18	58 12	43 47	12 43	47 20	41 48	23 26	45 19	59 46	11 16	54 11	0 17	15	11 37	26 20	29 51	23 26	51 24	0 1	23 26	51 24
25	19 3	17 8	13 18	1 39 52	58 13	3 32	13 3 32	20 52	56 23	25 36	19 47	09 10	56 21	10 56	0 40	41	11 58	19 20	41 56	23 25	39 25	0 1	23 25	39 25
26	18 48	26 8	51 00	2 3 26	58 13	23 02	13 23	02 21	3 39	23 24	02 19	34 13	10 35	38 10	1 4	6	12 19	0 20	53 39	23 23	59 26	0 1	23 23	59 26
27	18 33	13 8	28 34	2 26 57	58 13	42 20	13 42	20 21	14 03	23 22	03 19	20 57	10 14	44 10	1 27	31	12 39	33 21	4 58	23 21	50 27	0 1	23 21	50 27
28	18 17	39 8	5 59	2 50 24	58 14	1 24	1 24	21 24	4 23	19 39	19 7	22 9	53 39	9 32	26	2 14	12 59	53 21	15 51	23 19	13 28	0 1	23 19	13 28
29	18 1	46 8	13 46	3 13 46	58 14	20 15	14 20	15 21	33 44	23 16	51 18	53 28	9 32	26 9	2 14	22	13 20	01 21	26 26	23 16	9 29	0 1	23 16	9 29
30	17 45	35 8	37 06	3 37 06	58 14	38 50	14 38	50 21	43 12	23 13	38 18	39 15	9 11	3 8 49	2 37	44	13 39	56 21	36 33	23 12	36 30	0 1	23 12	36 30
31	17 29	05 8	4 0 22	4 0 22	58 14	51 55	21 51	55 21	51 55	18 24	43 18	24 43	8 49	31 8	2 3	44	13 59	38 23	59 38	23 8	35 31	0 1	23 8	35 31

310. A TABLE of the SUN'S DECLINATION for the Year 1807, being the Third after Leap Year.

Days.	January.		February.		March.		April.		May.		June.		July.		August.		September.		October.		November.		December.	
	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	North.	South.	South.	North.	South.	South.	North.	South.	
1	23 4 07	17 16 17	0 1 11	17 16 17	0 1 11	17 16 17	0 1 11	17 16 17	0 1 11	17 16 17	0 1 11	17 16 17	0 1 11	17 16 17	0 1 11	17 16 17	0 1 11	17 16 17	0 1 11	17 16 17	0 1 11	17 16 17	0 1 11	
2	22 59 10	16 59 13	0 1 11	16 59 13	0 1 11	16 59 13	0 1 11	16 59 13	0 1 11	16 59 13	0 1 11	16 59 13	0 1 11	16 59 13	0 1 11	16 59 13	0 1 11	16 59 13	0 1 11	16 59 13	0 1 11	16 59 13	0 1 11	
3	22 53 46	16 41 50	0 1 11	16 41 50	0 1 11	16 41 50	0 1 11	16 41 50	0 1 11	16 41 50	0 1 11	16 41 50	0 1 11	16 41 50	0 1 11	16 41 50	0 1 11	16 41 50	0 1 11	16 41 50	0 1 11	16 41 50	0 1 11	
4	22 47 55	16 24 12	0 1 11	16 24 12	0 1 11	16 24 12	0 1 11	16 24 12	0 1 11	16 24 12	0 1 11	16 24 12	0 1 11	16 24 12	0 1 11	16 24 12	0 1 11	16 24 12	0 1 11	16 24 12	0 1 11	16 24 12	0 1 11	
5	22 41 37	16 06 15	0 1 11	16 06 15	0 1 11	16 06 15	0 1 11	16 06 15	0 1 11	16 06 15	0 1 11	16 06 15	0 1 11	16 06 15	0 1 11	16 06 15	0 1 11	16 06 15	0 1 11	16 06 15	0 1 11	16 06 15	0 1 11	
6	22 34 52	15 48 02	0 1 11	15 48 02	0 1 11	15 48 02	0 1 11	15 48 02	0 1 11	15 48 02	0 1 11	15 48 02	0 1 11	15 48 02	0 1 11	15 48 02	0 1 11	15 48 02	0 1 11	15 48 02	0 1 11	15 48 02	0 1 11	
7	22 27 41	15 29 32	0 1 11	15 29 32	0 1 11	15 29 32	0 1 11	15 29 32	0 1 11	15 29 32	0 1 11	15 29 32	0 1 11	15 29 32	0 1 11	15 29 32	0 1 11	15 29 32	0 1 11	15 29 32	0 1 11	15 29 32	0 1 11	
8	22 20 02	15 10 47	0 1 11	15 10 47	0 1 11	15 10 47	0 1 11	15 10 47	0 1 11	15 10 47	0 1 11	15 10 47	0 1 11	15 10 47	0 1 11	15 10 47	0 1 11	15 10 47	0 1 11	15 10 47	0 1 11	15 10 47	0 1 11	
9	22 11 56	14 51 47	0 1 11	14 51 47	0 1 11	14 51 47	0 1 11	14 51 47	0 1 11	14 51 47	0 1 11	14 51 47	0 1 11	14 51 47	0 1 11	14 51 47	0 1 11	14 51 47	0 1 11	14 51 47	0 1 11	14 51 47	0 1 11	
10	22 3 26	14 32 31	0 1 11	14 32 31	0 1 11	14 32 31	0 1 11	14 32 31	0 1 11	14 32 31	0 1 11	14 32 31	0 1 11	14 32 31	0 1 11	14 32 31	0 1 11	14 32 31	0 1 11	14 32 31	0 1 11	14 32 31	0 1 11	
11	21 54 30	14 13 01	0 1 11	14 13 01	0 1 11	14 13 01	0 1 11	14 13 01	0 1 11	14 13 01	0 1 11	14 13 01	0 1 11	14 13 01	0 1 11	14 13 01	0 1 11	14 13 01	0 1 11	14 13 01	0 1 11	14 13 01	0 1 11	
12	21 45 08	13 53 17	0 1 11	13 53 17	0 1 11	13 53 17	0 1 11	13 53 17	0 1 11	13 53 17	0 1 11	13 53 17	0 1 11	13 53 17	0 1 11	13 53 17	0 1 11	13 53 17	0 1 11	13 53 17	0 1 11	13 53 17	0 1 11	
13	21 35 22	13 33 22	0 1 11	13 33 22	0 1 11	13 33 22	0 1 11	13 33 22	0 1 11	13 33 22	0 1 11	13 33 22	0 1 11	13 33 22	0 1 11	13 33 22	0 1 11	13 33 22	0 1 11	13 33 22	0 1 11	13 33 22	0 1 11	
14	21 25 09	13 13 11	0 1 11	13 13 11	0 1 11	13 13 11	0 1 11	13 13 11	0 1 11	13 13 11	0 1 11	13 13 11	0 1 11	13 13 11	0 1 11	13 13 11	0 1 11	13 13 11	0 1 11	13 13 11	0 1 11	13 13 11	0 1 11	
15	21 14 31	12 52 35	0 1 11	12 52 35	0 1 11	12 52 35	0 1 11	12 52 35	0 1 11	12 52 35	0 1 11	12 52 35	0 1 11	12 52 35	0 1 11	12 52 35	0 1 11	12 52 35	0 1 11	12 52 35	0 1 11	12 52 35	0 1 11	
16	21 3 29	12 32 14	0 1 11	12 32 14	0 1 11	12 32 14	0 1 11	12 32 14	0 1 11	12 32 14	0 1 11	12 32 14	0 1 11	12 32 14	0 1 11	12 32 14	0 1 11	12 32 14	0 1 11	12 32 14	0 1 11	12 32 14	0 1 11	
17	20 52 02	12 11 26	0 1 11	12 11 26	0 1 11	12 11 26	0 1 11	12 11 26	0 1 11	12 11 26	0 1 11	12 11 26	0 1 11	12 11 26	0 1 11	12 11 26	0 1 11	12 11 26	0 1 11	12 11 26	0 1 11	12 11 26	0 1 11	
18	20 40 15	11 50 27	0 1 11	11 50 27	0 1 11	11 50 27	0 1 11	11 50 27	0 1 11	11 50 27	0 1 11	11 50 27	0 1 11	11 50 27	0 1 11	11 50 27	0 1 11	11 50 27	0 1 11	11 50 27	0 1 11	11 50 27	0 1 11	
19	20 28 03	11 29 18	0 1 11	11 29 18	0 1 11	11 29 18	0 1 11	11 29 18	0 1 11	11 29 18	0 1 11	11 29 18	0 1 11	11 29 18	0 1 11	11 29 18	0 1 11	11 29 18	0 1 11	11 29 18	0 1 11	11 29 18	0 1 11	
20	20 15 26	11 7 57	0 1 11	11 7 57	0 1 11	11 7 57	0 1 11	11 7 57	0 1 11	11 7 57	0 1 11	11 7 57	0 1 11	11 7 57	0 1 11	11 7 57	0 1 11	11 7 57	0 1 11	11 7 57	0 1 11	11 7 57	0 1 11	
21	20 2 26	10 46 26	0 1 11	10 46 26	0 1 11	10 46 26	0 1 11	10 46 26	0 1 11	10 46 26	0 1 11	10 46 26	0 1 11	10 46 26	0 1 11	10 46 26	0 1 11	10 46 26	0 1 11	10 46 26	0 1 11	10 46 26	0 1 11	
22	19 49 06	10 24 45	0 1 11	10 24 45	0 1 11	10 24 45	0 1 11	10 24 45	0 1 11	10 24 45	0 1 11	10 24 45	0 1 11	10 24 45	0 1 11	10 24 45	0 1 11	10 24 45	0 1 11	10 24 45	0 1 11	10 24 45	0 1 11	
23	19 35 22	10 2 53	0 1 11	10 2 53	0 1 11	10 2 53	0 1 11	10 2 53	0 1 11	10 2 53	0 1 11	10 2 53	0 1 11	10 2 53	0 1 11	10 2 53	0 1 11	10 2 53	0 1 11	10 2 53	0 1 11	10 2 53	0 1 11	
24	19 21 17	9 40 53	0 1 11	9 40 53	0 1 11	9 40 53	0 1 11	9 40 53	0 1 11	9 40 53	0 1 11	9 40 53	0 1 11	9 40 53	0 1 11	9 40 53	0 1 11	9 40 53	0 1 11	9 40 53	0 1 11	9 40 53	0 1 11	
25	19 6 50	9 18 44	0 1 11	9 18 44	0 1 11	9 18 44	0 1 11	9 18 44	0 1 11	9 18 44	0 1 11	9 18 44	0 1 11	9 18 44	0 1 11	9 18 44	0 1 11	9 18 44	0 1 11	9 18 44	0 1 11	9 18 44	0 1 11	
26	18 52 02	8 56 27	0 1 11	8 56 27	0 1 11	8 56 27	0 1 11	8 56 27	0 1 11	8 56 27	0 1 11	8 56 27	0 1 11	8 56 27	0 1 11	8 56 27	0 1 11	8 56 27	0 1 11	8 56 27	0 1 11	8 56 27	0 1 11	
27	18 36 54	8 34 05	0 1 11	8 34 05	0 1 11	8 34 05	0 1 11	8 34 05	0 1 11	8 34 05	0 1 11	8 34 05	0 1 11	8 34 05	0 1 11	8 34 05	0 1 11	8 34 05	0 1 11	8 34 05	0 1 11	8 34 05	0 1 11	
28	18 21 26	8 11 27	0 1 11	8 11 27	0 1 11	8 11 27	0 1 11	8 11 27	0 1 11	8 11 27	0 1 11	8 11 27	0 1 11	8 11 27	0 1 11	8 11 27	0 1 11	8 11 27	0 1 11	8 11 27	0 1 11	8 11 27	0 1 11	
29	18 5 37		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11	
30	17 49 29		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11	
31	17 33 03		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11		0 1 11	

311 A TABLE for fitting the TABLES of the SUN'S LONG. DECL. or Rt. ASCEN. to any meridian: Or, for finding either quantity at any hour.
Degrees of longitude from the Meridian of London: Or, time before and after noon.

5 Deg.	10 D.	15 D.	20 D.	25 D.	30 D.	35 D.	40 D.	45 D.	50 D.	55 D.	60 D.	65 D.	70 D.	75 D.	80 D.	85 D.	90 D.	95 D.	100 D.	105 D.	110 D.	115 D.	120 D.	Daily diff.		
h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.		
0 20	0 40	1 0	1 20	1 40	2 0	2 20	2 40	3 0	3 20	3 40	4 0	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	0 20		
0 40	1 0	1 20	1 40	2 0	2 20	2 40	3 0	3 20	3 40	4 0	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	1 0		
1 0	1 20	1 40	2 0	2 20	2 40	3 0	3 20	3 40	4 0	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	2 0		
1 20	1 40	2 0	2 20	2 40	3 0	3 20	3 40	4 0	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	3 0		
1 40	2 0	2 20	2 40	3 0	3 20	3 40	4 0	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	4 0		
2 0	2 20	2 40	3 0	3 20	3 40	4 0	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	5 0		
2 20	2 40	3 0	3 20	3 40	4 0	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	6 0		
2 40	3 0	3 20	3 40	4 0	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	7 0		
2 60	3 20	3 40	4 0	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	8 0		
2 80	3 40	4 0	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	9 0		
3 0	3 60	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	10 0		
3 20	3 80	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	11 0		
3 40	4 0	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	
3 60	4 20	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	1 0	
3 80	4 40	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	2 0	
4 0	4 60	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	3 0	
4 20	5 0	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	4 0	
4 40	5 20	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	5 0	
4 60	5 40	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	6 0	
4 80	6 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	7 0	
5 0	6 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	8 0	
5 20	6 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	9 0	
5 40	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	10 0	
5 60	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	11 0	
5 80	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	12 0	
6 0	7 60	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	13 0	
6 20	8 0	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	14 0	
6 40	8 20	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	15 0	
6 60	8 40	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	16 0	
6 80	8 60	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	17 0	
7 0	9 0	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	18 0	
7 20	9 20	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	19 0	
7 40	9 40	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	20 0	
7 60	9 60	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	21 0	
7 80	9 80	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	22 0	
8 0	10 0	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 20	23 0
8 20	10 20	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 40	24 0
8 40	10 40	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 20	4 60	25 0
8 60	10 60	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 20	4 40	4 80	26 0
8 80	10 80	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 20	4 40	4 60	5 0	27 0
9 0	11 0	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 20	4 40	5 20	28 0
9 20	11 20	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 20	4 40	4 60	5 40	29 0
9 40	11 40	12 0	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 20	4 40	4 60	4 80	6 0	30 0
9 60	11 60	12 20	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 20	4 40	4 60	4 80	5 0	6 20	31 0
9 80	11 80	12 40	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 20	4 40	4 60	4 80	5 0	5 20	6 40	32 0
10 0	12 0	1 0	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 20	4 40	4 60	4 80	5 0	5 20	5 40	7 0	33 0
10 20	12 20	1 20	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 20	4 40	4 60	4 80	5 0	5 20	5 40	5 60	7 20	34 0
10 40	12 40	1 40	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 20	4 40	4 60	4 80	5 0	5 20	5 40	5 60	5 80	7 40	35 0
10 60	12 60	1 60	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 20	4 40	4 60	4 80	5 0	5 20	5 40	5 60	5 80	6 0	8 0	36 0
10 80	12 80	1 80	2 0	2 20	2 40	2 60	2 80	3 0	3 20	3 40	3 60	3 80	4 0	4 20												

311. TABLE for fitting the SUN'S LONGITUDE, DECLINATION, or Right ASCENSION, to any meridian, continued.

Daily diff.	5 Degs.		10 D.		15 D.		20 D.		25 D.		30 D.		35 D.		40 D.		45 D.		50 D.		55 D.		60 D.		65 D.		70 D.		75 D.		80 D.		85 D.		90 D.		95 D.		100 D.		105 D.		110 D.		115 D.		120 D.		Daily diff.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																				
	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
12	0	10	20	30	40	50	1	0	1	10	20	30	40	50	1	0	1	10	20	30	40	50	1	0	1	10	20	30	40	50	1	0	1	10	20	30	40	50	1	0	1	10	20	30	40	50	1	0	12																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																				
13	0	10	20	31	41	52	1	1	12	23	34	45	56	1	13	24	35	46	57	1	14	25	36	47	58	1	15	26	37	48	59	1	16	27	38	49	50	1	17	28	39	50	1	18	29	40	51	1	19	30	41	52	1	20																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																															
14	0	11	21	31	42	53	1	2	13	24	35	46	57	1	14	25	36	47	58	1	15	26	37	48	59	1	16	27	38	49	50	1	17	28	39	50	1	18	29	40	51	1	19	30	41	52	1	20																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																					
15	0	11	22	32	43	54	1	3	14	25	36	47	58	1	15	26	37	48	59	1	16	27	38	49	50	1	17	28	39	50	1	18	29	40	51	1	19	30	41	52	1	20																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																											
16	0	11	22	33	44	55	1	4	15	26	37	48	59	1	16	27	38	49	50	1	17	28	39	50	1	18	29	40	51	1	19	30	41	52	1	20	1	21	32	43	54	1	22	1	23	2	24	1	25	1	26	1	27	1	28	1	29	1	30	1	31	1	32	1	33	1	34	1	35	1	36	1	37	1	38	1	39	1	40	1	41	1	42	1	43	1	44	1	45	1	46	1	47	1	48	1	49	1	50	1	51	1	52	1	53	1	54	1	55	1	56	1	57	1	58	1	59	1	60	1	61	1	62	1	63	1	64	1	65	1	66	1	67	1	68	1	69	1	70	1	71	1	72	1	73	1	74	1	75	1	76	1	77	1	78	1	79	1	80	1	81	1	82	1	83	1	84	1	85	1	86	1	87	1	88	1	89	1	90	1	91	1	92	1	93	1	94	1	95	1	96	1	97	1	98	1	99	1	100	1	101	1	102	1	103	1	104	1	105	1	106	1	107	1	108	1	109	1	110	1	111	1	112	1	113	1	114	1	115	1	116	1	117	1	118	1	119	1	120	1	121	1	122	1	123	1	124	1	125	1	126	1	127	1	128	1	129	1	130	1	131	1	132	1	133	1	134	1	135	1	136	1	137	1	138	1	139	1	140	1	141	1	142	1	143	1	144	1	145	1	146	1	147	1	148	1	149	1	150	1	151	1	152	1	153	1	154	1	155	1	156	1	157	1	158	1	159	1	160	1	161	1	162	1	163	1	164	1	165	1	166	1	167	1	168	1	169	1	170	1	171	1	172	1	173	1	174	1	175	1	176	1	177	1	178	1	179	1	180	1	181	1	182	1	183	1	184	1	185	1	186	1	187	1	188	1	189	1	190	1	191	1	192	1	193	1	194	1	195	1	196	1	197	1	198	1	199	1	200	1	201	1	202	1	203	1	204	1	205	1	206	1	207	1	208	1	209	1	210	1	211	1	212	1	213	1	214	1	215	1	216	1	217	1	218	1	219	1	220	1	221	1	222	1	223	1	224	1	225	1	226	1	227	1	228	1	229	1	230	1	231	1	232	1	233	1	234	1	235	1	236	1	237	1	238	1	239	1	240	1	241	1	242	1	243	1	244	1	245	1	246	1	247	1	248	1	249	1	250	1	251	1	252	1	253	1	254	1	255	1	256	1	257	1	258	1	259	1	260	1	261	1	262	1	263	1	264	1	265	1	266	1	267	1	268	1	269	1	270	1	271	1	272	1	273	1	274	1	275	1	276	1	277	1	278	1	279	1	280	1	281	1	282	1	283	1	284	1	285	1	286	1	287	1	288	1	289	1	290	1	291	1	292	1	293	1	294	1	295	1	296	1	297	1	298	1	299	1	300	1	301	1	302	1	303	1	304	1	305	1	306	1	307	1	308	1	309	1	310	1	311	1	312	1	313	1	314	1	315	1	316	1	317	1	318	1	319	1	320	1	321	1	322	1	323	1	324	1	325	1	326	1	327	1	328	1	329	1	330	1	331	1	332	1	333	1	334	1	335	1	336	1	337	1	338	1	339	1	340	1	341	1	342	1	343	1	344	1	345	1	346	1	347	1	348	1	349	1	350	1	351	1	352	1	353	1	354	1	355	1	356	1	357	1	358	1	359	1	360	1	361	1	362	1	363	1	364	1	365	1	366	1	367	1	368	1	369	1	370	1	371	1	372	1	373	1	374	1	375	1	376	1	377	1	378	1	379	1	380	1	381	1	382	1	383	1	384	1	385	1	386	1	387	1	388	1	389	1	390	1	391	1	392	1	393	1	394	1	395	1	396	1	397	1	398	1	399	1	400	1	401	1	402	1	403	1	404	1	405	1	406	1	407	1	408	1	409	1	410	1	411	1	412	1	413	1	414	1	415	1	416	1	417	1	418	1	419	1	420	1	421	1	422	1	423	1	424	1	425	1	426	1	427	1	428	1	429	1	430	1	431	1	432	1	433	1	434	1	435	1	436	1	437	1	438	1	439	1	440	1	441	1	442	1	443	1	444	1	445	1	446	1	447	1	448	1	449	1	450	1	451	1	452	1	453	1	454	1	455	1	456	1	457	1	458	1	459	1	460	1	461	1	462	1	463	1	464	1	465	1	466	1	467	1	468	1	469	1	470	1	471	1	472	1	473	1	474	1	475	1	476	1	477	1	478	1	479	1	480	1	481	1	482	1	483	1	484	1	485	1	486	1	487	1	488	1	489	1	490	1	491	1	492	1	493	1	494	1	495	1	496	1	497	1	498	1	499	1	500	1	501	1	502	1	503	1	504	1	505	1	506	1	507	1	508	1	509	1	510	1	511	1	512	1	513	1	514	1	515	1	516	1	517	1	518	1	519	1	520	1	521	1	522	1	523	1	524	1	525	1	526	1	527	1	528	1	529	1	530	1	531	1	532	1	533	1	534	1	535	1	536	1	537	1	538	1	539	1	540	1	541	1	542	1	543	1	544	1	545	1	546	1	547	1	548	1	549	1	550	1	551	1	552	1	553	1	554	1	555	1	556	1	557	1	558	1	559	1	560	1	561	1	562	1	563	1	564	1	565	1	566	1	567	1	568	1	569	1	570	1	571	1	572	1	573	1	574	1	575	1	576	1	577	1	578	1	579	1	580	1	581	1	582	1	583	1	584	1	585	1	586	1	587	1	588	1	589	1	590	1	591	1	592	1	593	1	594	1	595	1	596	1	597	1	598	1	599	1	600	1	601	1	602	1	603	1	604	1	605	1	606	1	607	1	608	1	609	1	610	1	611	1	612	1	613	1	614	1	615	1	616	1	617	1	618	1	619	1	620	1	621	1	622	1	623	1	624	1	625	1	626	1	627	1	628	1	629	1	630	1	631	1	632	1	633	1	634	1	635	1	636	1	637	1	638	1	639	1	640	1	641	1	642	1	643	1	644	1	645	1	646	1	647	1	648	1	649	1	650	1	651	1	652	1	653	1	654	1	655	1	656	1	657	1	658	1	659	1	660	1	661	1	662	1	663	1	664	1	665	1	666	1	667	1	668	1	669	1	670	1	671	1	672	1	673	1	674	1	675	1	676	1	677	1	678	1	679	1	680	1	681	1	682	1	683	1	684	1	685	1	686	1	687	1	688	1	689	1	690	1	691	1	692	1	693	1	694	1	695	1	696	1	697	1	698	1	699	1	700	1	701	1	702	1	703	1	704	1	705	1	706	1	707	1	708	1	709	1	710	1	711	1	712	1	713	1	714	1	715	1	716	1	717	1	718	1	719	1	720	1	721	1	722	1	723	1	724	1	725	1	726	1	727	1	728	1	729	1	730	1	731	1	732	1	733	1	734	1	735	1	736	1	737	1	738	1	739	1	740	1	741	1	742	1	743	1	744	1	745	1	746	1	747	1	748	1	749	1	750	1	751	1	752	1	753	1	754	1	75

312. TABLE of the RIGHT ASCENSIONS and DECLINATIONS of sixty STARS in the Northern Hemisphere; for the Year 1780.

Constellations.	Places of the Stars in the Constellations.	Names.	Marks.	Magn.	Rt. Ascen. in time. h. m. s.	Year. Var. f.	Declinat. North. ° ' "	Yearly Var.
Pegasus	Ending of the Wing	Schedar	γ	2	0 1 56	3,08	13 57 36	20,04+
Cassiopea	Breast	Alruccabah	α	3	0 28 8	3,31	55 19 46	19,91+
Pole Star		Mirach	α	2	0 47 43	10,05	88 7 57	19,69+
Andromeda	Girdle		β	2	0 57 29	3,30	34 26 56	19,45+
Aries	Preceding Horn	Almaach	β	3	1 42 32	3,28	19 43 32	18,10+
Andromeda	Foot		γ	2	1 50 29	3,62	41 15 54	17,80+
Aries	Following Horn		α	3	1 54 49	3,34	22 24 52	17,64+
Whale	Following in the Cheek	Menkar	γ	3	2 31 56	3,12	2 17 59	15,86+
	Jaw	Algol	α	2	2 50 48	3,13	3 12 56	14,80+
Medusa	Head	Algenib	β	2	2 53 56	3,85	40 5 37	14,63+
Perseus	Brightest	Pleiades	α	2	3 8 43	4,20	49 3 44	13,72+
Taurus	Brightest in		η	3	3 34 27	3,55	23 24 38	12,00+
Perseus	Knee	Hyades	ε	3	3 43 08	3,94	38 21 24	11,41+
Taurus	First of the		γ	3	4 7 18	3,39	15 4 51	9,60+
	Northern Eye	Aldebaran	ε	3	4 15 48	3,48	18 40 32	8,90+
	Southern Eye	Capella	α	1	4 23 19	3,43	16 3 6	8,32+
Auriga	In the Goat		α	1	5 0 28	4,40	45 44 48	5,30+
Taurus	Northern Horn		β	2	5 12 24	3,79	28 24 13	4,26+
Orion	West Shoulder	Bellatrix	γ	2	5 13 21	3,22	6 8 2	4,15+
	East Shoulder	Betelgeuse	α	1	5 43 16	3,25	7 20 56	1,58+
Auriga	In the Hand		θ	3	5 44 43	4,09	37 10 36	1,40+
Gemini	Foot of Pollux		γ	3	6 25 00	3,47	16 34 9	2,10-
	Knee of Pollux	Castor	ζ	3	6 51 3	3,58	20 52 31	4,33-
	Brightest in the Head		α	2	7 20 33	3,88	32 21 8	6,78-
Little Dog	Brightest	Procyon	α	1	7 27 48	3,20	5 46 56	7,40-
Gemini	Head of Pollux	Pollux	β	2	7 31 51	3,75	28 32 29	7,70-
Great Bear	North Paw		ι	3	8 44 4	4,25	48 53 29	13,00-
Cancer	In the Claw	Acutens	α	3	8 46 27	3,24	12 41 50	13,30-
Great Bear	Preceding Knee		θ	3	9 18 5	4,23	52 40 21	15,18-
Leo	North in the Head		μ	3	9 40 13	3,47	27 1 58	16,33-
Great Bear	In the Heart	Regulus	α	1	9 56 39	3,24	13 2 6	17,16-
	Lower Pointer	Dubhe	β	2	10 48 27	3,74	57 33 25	19,05-
	Upper Pointer		α	2	10 50 0	3,88	62 56 05	19,09-
Leo	In the Tail		β	2	11 37 50	3,10	15 48 8	19,95-
Great Bear	S following in the Square		γ	2	11 42 10	3,24	54 55 4	19,99-
	Last in the Square		δ	2	12 4 27	3,05	58 15 25	20,05-
	First in the Tail	Alioth	ε	2	12 44 17	2,69	57 9 25	19,69-
	Middle of the Tail		ζ	2	13 15 2	2,44	56 4 46	19,01-
	Last in the Tail	Benetnach	η	2	13 38 52	2,41	50 25 03	18,24-
Dragon	In the Tail		α	2	13 58 27	1,63	65 25 54	17,46-
Bootes	Skirt of the Coat	Arcturus	α	1	14 5 40	2,82	20 20 39	17,16-
	In the following Thigh	Mirach	ε	3	14 35 28	2,63	28 0 37	15,67-
Crown	The brightest	Alphacca	α	2	15 25 23	2,54	27 28 00	12,60-
Serpent	In the Neck		α	2	15 33 27	2,94	7 7 48	12,03-
Hercules	Preceding Shoulder		β	3	16 20 48	2,59	11 58 52	8,51-
	Following at the Side		ε	3	16 51 52	2,29	31 15 43	5,96-
	The Head	Raf. Algethi	α	2	17 4 38	2,74	14 39 18	4,87-
Ophiucus	The Head	Raf. Alhague	α	2	17 24 43	2,75	12 44 8	3,15-
Dragon	In the Head	Raf. taben	γ	2	17 51 31	1,37	51 31 21	0,78-
Harp	The brightest	Vega	α	1	18 29 29	2,02	38 35 14	2,52+
	Following in Lozange		δ	3	18 46 49	2,11	36 37 48	3,97+
Eagle	Preceding Wing		δ	3	19 14 24	3,02	2 41 22	6,31+
	The brightest	Atair	α	2	19 40 02	2,90	8 17 53	8,40+
Swan	The Breast		γ	3	20 14 20	2,16	39 33 43	11,00+
	The Tail	Deneb	α	2	20 33 56	2,05	44 30 7	12,44+
Cepheus	Preceding Shoulder	Alderaimin	α	3	21 13 19	1,44	61 39 32	14,95+
Pegasus	The Neck		ζ	3	22 30 29	2,99	9 41 18	18,46+
	The Thigh	Scheat	β	2	22 53 8	2,88	26 53 32	19,18+
	The Wing	Markab	α	2	22 53 49	2,98	14 1 30	19,20+
Andromeda	The Head		α	2	23 57 3	3,07	27 52 22	20,05+

312. TABLE of the RIGHT ASCENSIONS and DECLINATIONS of sixty STARS in the Southern Hemisphere; for the Year 1780.

Constellations.	Places of the Stars in the Constellations.	Names.	Marks.	Magn.	Rt. Ascen. in time. h. m. s.	Year. Var. f.	Declinat. South. ° ' "	Yearly Var.
Phenix	The Head		α	2	0 15 22	3,01	43 29 43	20,00—
Whale	Brightest in the Tail		β	2	0 32 32	3,01	19 11 50	19,86—
Phenix	Thigh		β	3	0 56 15	2,73	47 54 58	19,46—
Eridanus	Following Wing	Achernar	γ	3	1 18 49	2,67	44 26 52	18,90—
Whale	Source of the River		α	1	1 29 31	2,25	58 21 35	18,56—
	Preceding Jaw		δ	3	2 28 13	3,07	0 37 50	16,00—
Eridanus	Near the Whale		γ	3	3 5 10	2,91	9 38 55	13,92—
	The following		δ	3	3 32 44	2,88	10 31 30	12,08—
	The fourth Bend		γ	3	3 47 46	2,80	14 8 47	11,01—
Goldfish	In the Tail		α	3	4 29 15	1,28	55 30 20	7,76—
Orion	Bright Foot	Rigel	β	1	5 3 58	2,89	8 28 11	4,94—
	Preceding in Belt		δ	2	5 20 47	3,07	0 28 40	3,50—
	Middle of Belt		ϵ	2	5 25 4	3,05	1 21 30	3,13—
	Last in the Belt		ζ	2	5 29 41	3,04	2 4 27	2,77—
Dove	Preceding of the brightest		α	2	5 31 43	2,20	34 12 6	2,56—
Orion	In the Knee		κ	3	5 37 20	2,85	9 45 35	2,10—
Dove	Following of brightest		β	3	5 43 13	2,11	35 51 51	1,65—
Argo	The brightest	Canopus	α	1	6 19 5	1,34	52 34 57	1,60+
Great Dog	The brightest	Syrius	α	1	6 35 28	2,69	16 25 8	3,10+
	In the Back		δ	2	6 59 27	2,45	26 3 37	5,08+
	In the Tail		η	2	7 15 24	2,38	28 53 5	6,42+
Argo	In the Poop		ζ	2	7 55 52	2,12	39 23 28	9,62+
	Preceding in the Hull		γ	2	8 2 46	1,85	46 41 40	10,16+
	Brightest in the Middle		δ	2	8 38 36	1,61	53 54 24	12,73+
Female Hydra	Bright among the Oars		β	1	9 10 44	0,75	68 48 50	4,79+
Argo	The Heart	Alphard	α	2	9 16 47	2,96	7 42 51	15,13+
Centaur	No thorn in Section		η	2	10 36 34	2,27	58 31 59	18,68+
Crofs	Preceding in the Crupper		δ	3	11 57 3	3,06	49 29 40	20,04+
	Preceding Arm		δ	3	12 3 35	3,10	57 31 31	20,04+
	The Foot		α	1	12 14 33	3,22	61 52 48	20,01+
Centaur	The Head		γ	2	12 19 4	3,24	55 52 42	19,98+
Crofs	Top of the Crupper		γ	2	12 29 30	3,27	47 44 50	19,89+
Female Hydra	Following Arm		β	2	12 35 2	3,42	58 29 3	19,83+
Virgo	The Tail		γ	3	13 7 00	3,22	22 0 22	19,22+
Centaur	The Sheaf	Virgins Spike	α	1	13 13 38	3,15	10 0 24	19,00+
	Preceding Leg		β	2	13 48 30	4,09	59 17 59	17,91+
Libra	South in the Shield		η	3	14 21 37	3,75	41 10 42	16,43+
Centaur	Bright in the Foot	Zubenesh	α	1	14 25 2	4,41	59 55 17	16,26+
Libra	Southern Scale		α	2	14 38 45	3,31	15 6 55	15,50+
Southern Δ	Following in the Head		κ	3	14 44 56	3,84	41 12 21	15,17+
	Northern Scale	Zubenelg.	β	2	15 5 12	3,22	8 33 31	13,93+
	The Vertex		β	3	15 35 56	5,12	62 43 31	11,88+
Scorpio	Middle of the Forehead		δ	3	15 47 21	3,53	21 58 42	11,00+
Ophiucus	N. in the Forehead		β	2	15 52 41	3,47	19 11 14	10,70+
Scorpio	Preceding Hand		δ	3	16 2 50	3,14	3 6 45	9,89+
	The Heart	Antares.	α	1	16 15 57	3,66	25 55 32	8,90+
Ophiucus	First Joint in the Tail		ϵ	3	16 35 58	3,90	33 52 21	7,34+
	Following Knee		η	2	16 57 47	3,44	15 26 14	5,52+
Altar	In the Middle		α	3	17 14 52	4,61	49 40 36	4,12+
Scorpio	Bright at Tail's End		λ	3	17 18 42	4,08	36 55 22	3,77+
Sagittarius	Sixth Knot in the Tail		ι	3	17 32 13	4,18	40 1 7	2,60+
	S. End of the Bow		ϵ	2	18 9 35	4,00	34 27 58	0,72—
	Preceding Shoulder		σ	3	18 41 37	3,73	26 32 57	3,54—
	Following in the Head		π	3	18 56 41	3,58	21 21 17	4,80—
Capricorn	In the following Horn		α	3	20 5 54	3,35	13 12 45	10,40—
Peacock	The Eye		α	2	20 8 8	4,89	57 25 14	10,44—
Capricorn	In the Forehead		β	3	20 8 37	3,35	15 27 43	10,60—
Crane	Preceding Wing		α	2	21 54 18	3,92	48 0 51	17,00—
Aquarius	Following Arm		γ	3	22 10 17	3,11	2 29 23	17,76—
Southern Fish	The Brightest	Fomalhaut	α	1	22 45 27	3,33	30 46 51	18,97—

313. TABLE of the Sun's Right Ascension, in Degrees, &c. to each Deg. of Long. And of the Diff. between Long. and Rt. Ascen.

0 S. or 6 S.			1 S. or 7 S.			2 S. or 8 S.		
Add to	Sub. from		Add to	Sub. from		Add to	Sub. from	
180°	360°	°	180°	360°	°	180°	360°	°
0	0	0	27 54	20	2	57 48	48	2 11 12 30
1	0 55	2	28 51	43	2	58 51	21	2 8 39 29
2	1 50	5	29 49	15	2	59 54	4	2 5 56 28
3	2 45	8	30 46	56	2	60 56	57	2 3 3 27
4	3 40	12	31 44	46	2	62	0	2 0 0 26
5	4 35	18	32 42	45	2	63	3 12	1 56 48 25
6	5 30	25	33 40	54	2	64	6 34	1 53 26 24
7	6 25	34	34 39	13	2	65	10 4	1 49 56 23
8	7 20	45	35 37	41	2	66	13 43	1 46 17 22
9	8 15	59	36 36	19	2	67	17 31	1 42 29 21
10	9 11	16	37 35	8	2	68	21 27	1 38 33 20
11	10 6	35	38 34	7	2	69	25 31	1 34 29 19
12	11 1	58	39 33	16	2	70	29 42	1 30 18 18
13	11 57	25	40 32	36	2	71	34	1 25 59 17
14	12 52	56	41 32	7	2	72	38 27	1 21 33 16
15	13 48	32	42 31	48	2	73	42 59	1 17 1 15
16	14 44	12	43 31	40	2	74	47 38	1 12 22 14
17	15 39	57	44 31	43	2	75	52 23	1 7 37 13
18	16 35	47	45 31	56	2	76	57 13	1 2 47 12
19	17 31	43	46 32	21	2	78	2 9	0 57 51 11
20	18 27	45	47 32	57	2	79	7 9	0 52 51 10
21	19 23	53	48 33	43	2	80	12 13	0 47 47 9
22	20 20	7	49 34	41	2	81	17 21	0 42 39 8
23	21 16	28	50 35	49	2	82	22 33	0 37 27 7
24	22 12	55	51 37	8	2	83	27 49	0 32 11 6
25	23 9	30	52 38	38	2	84	33 7	0 26 53 5
26	24 6	12	53 40	19	2	85	38 27	0 21 33 4
27	25 3	2	54 42	11	2	86	43 48	0 16 12 3
28	26 0	0	55 44	13	2	87	49 11	0 10 49 2
29	26 57	6	56 46	25	2	88	54 35	0 5 25 1
30	27 54	20	57 48	48	2	90	0 0	0 0 0 0

314. TABLE I. of the Equat.
of Time on the Diff. betw.
the Sun's Lon. and Rt. Asc.

Deg.	♂	♀	Deg.
0	0 0,0	8 22,7	0
1	0 19,9	8 33,1	1
2	0 39,7	8 43,0	2
3	0 59,5	8 52,3	3
4	1 19,2	9 0,9	4
5	1 38,8	9 9,0	5
6	1 58,3	9 16,4	6
7	2 17,7	9 23,1	7
8	2 37,0	9 29,3	8
9	2 56,1	9 34,7	9
10	3 14,6	9 39,5	10
11	3 33,7	9 43,5	11
12	3 52,1	9 46,9	12
13	4 10,3	9 49,6	13
14	4 28,3	9 51,5	14
15	4 45,9	9 52,8	15
16	5 3,2	9 53,3	16
17	5 20,2	9 53,1	17
18	5 36,9	9 52,3	18
19	5 53,1	9 50,6	19
20	6 9,0	9 48,2	20
21	6 24,5	9 45,1	21
22	6 39,5	9 41,3	22
23	6 54,1	9 36,7	23
24	7 8,3	9 31,5	24
25	7 22,0	9 25,5	25
26	7 35,2	9 18,7	26
27	7 47,7	9 11,3	27
28	8 0,0	9 3,1	28
29	8 11,6	8 54,3	29
30	8 22,7	8 44,8	30
Deg.	♂	♀	Deg.

315. TABLE II. of the Equation of Time on the Sun's Anomaly; or the Equation of the Center turned into Time.

☉	Sign	Sign	Sign	Sign	☉
Mca.	m. f.	m. f.	m. f.	m. f.	Mca.
An.	I	2	3	4	5
0	0	6 35	7 42	6 45	3 56
1	0 8	6 40	7 43	6 41	3 48
2	0 16	6 44	7 43	6 37	3 41
3	0 24	6 48	7 42	6 32	3 34
4	0 32	6 52	7 42	6 28	3 27
5	0 39	6 55	7 42	6 23	3 19
6	0 47	6 59	7 41	6 19	3 12
7	0 55	7 2	7 40	6 14	3 4
8	1 3	7 5	7 39	6 9	2 57
9	1 11	7 9	7 38	6 4	2 49
10	1 23	7 12	7 37	5 59	2 41
11	1 26	7 14	7 36	5 54	2 32
12	1 34	7 17	7 34	5 49	2 26
13	1 43	7 20	7 33	5 43	2 18
14	1 50	7 22	7 31	5 38	2 10
15	1 57	7 24	7 29	5 32	2 2
16	2 5	7 26	7 27	5 26	1 54
17	2 13	7 28	7 25	5 20	1 46
18	2 20	7 30	7 23	5 14	1 38
19	2 28	7 32	7 20	5 8	1 30
20	2 35	7 34	7 18	5 2	1 22
21	2 43	7 35	7 15	4 56	1 14
22	2 50	7 37	7 12	4 50	1 6
23	2 57	7 38	7 9	4 43	0 58
24	3 5	7 39	7 6	4 37	0 49
25	3 12	7 40	7 3	4 31	0 41
26	3 19	7 41	7 0	4 23	0 33
27	3 26	7 41	6 56	4 16	0 25
28	3 33	7 42	6 52	4 10	0 16
29	3 40	7 42	6 49	4 3	0 8
30	3 47	7 42	6 45	3 56	0 0
	11	9	8	7	6
	Sign	Sign	Sign	Sign	Sign

Place of the Apogee $\odot$ 9° . Obliquity of Ecliptic $23^{\circ} 28'$.

Deg.	0	1	2	3	4	5	6	7	8	9	10	11	Deg.
	+	-	+	-	+	-	+	-	+	-	+	-	
	m.	f.	m.	f.	m.	f.	m.	f.	m.	f.	m.	f.	
0	7 36	1 9	3 51	1 13	5 57	2 20	7 38	15 31	13 33	1 11	11 28	14 19	0
1	7 17	1 23	3 47	1 26	5 59	2 4	7 58	15 39	13 17	0 42	11 45	14 13	1
2	6 58	1 36	3 41	1 40	6 0	1 48	8 19	15 40	13 0	- 12	12 1	14 6	2
3	6 39	1 48	3 37	1 53	6 1	1 31	8 40	15 52	12 42	+ 17	12 17	13 59	3
4	6 20	2 0	3 32	2 7	6 1	1 14	9 1	15 57	12 23	0 46	12 32	13 51	4
5	6 1	2 11	3 26	2 20	6 0	0 56	9 21	16 2	12 4	1 16	12 46	13 43	5
6	5 42	2 22	3 19	2 33	5 59	0 38	9 41	16 6	11 44	1 45	12 59	13 34	6
7	5 24	2 32	3 12	2 45	5 57	0 20	10 1	16 9	11 23	2 14	13 12	13 24	7
8	5 5	2 42	3 4	2 58	5 54	+ 1	10 20	16 11	11 1	2 43	13 24	13 14	8
9	4 47	2 51	2 56	3 11	5 51	- 18	10 39	16 13	10 39	3 11	13 35	13 3	9
10	4 28	3 0	2 47	3 23	5 47	0 37	10 57	16 13	10 16	3 39	13 45	12 51	10
11	4 9	3 8	2 38	3 35	5 42	0 57	11 15	16 13	9 53	4 7	13 54	12 39	11
12	3 50	3 16	2 29	3 46	5 37	1 17	11 33	16 12	9 29	4 35	14 2	12 27	12
13	3 32	3 23	2 19	3 58	5 31	1 38	11 51	16 10	9 5	5 2	14 9	12 14	13
14	3 13	3 30	2 8	4 9	5 24	1 58	12 8	16 7	8 40	5 29	14 16	12 0	14
15	2 55	3 36	1 57	4 19	5 17	2 19	12 25	16 4	8 14	5 56	14 22	11 46	15
16	2 37	3 41	1 46	4 29	5 9	2 40	12 41	16 0	7 48	6 22	14 27	11 31	16
17	2 19	3 46	1 35	4 39	5 1	3 1	12 57	15 55	7 22	6 48	14 31	11 16	17
18	2 1	3 50	1 23	4 48	4 52	3 22	13 12	15 49	6 55	7 13	14 35	11 1	18
19	1 43	3 53	1 11	4 57	4 43	3 44	13 27	15 42	6 28	7 37	14 38	10 46	19
20	1 26	3 56	0 59	5 5	4 33	4 5	13 42	15 35	6 0	8 1	14 40	10 30	20
21	1 9	3 58	0 46	5 13	4 22	4 26	13 56	15 26	5 32	8 24	14 41	10 14	21
22	0 52	4 0	0 34	5 20	4 11	4 47	14 9	15 17	5 4	8 47	14 42	9 58	22
23	0 36	4 1	0 21	5 27	3 59	5 9	14 21	15 7	4 36	9 9	14 41	9 41	23
24	0 20	4 1	- 8	5 33	3 46	5 30	14 33	14 56	4 8	9 31	14 40	9 24	24
25	+ 4	4 1	+ 5	5 39	3 33	5 52	14 44	14 44	3 39	9 53	14 39	9 6	25
26	- 11	4 0	0 19	5 44	3 19	6 13	14 53	14 31	3 10	10 14	14 37	8 48	26
27	0 26	3 59	0 31	5 48	3 4	6 35	15 5	14 17	2 41	10 34	14 34	8 30	27
28	0 40	3 57	0 46	5 52	2 50	6 56	15 14	14 3	2 11	10 53	14 30	8 12	28
29	0 53	3 54	0 59	5 55	2 35	7 17	15 23	13 48	1 41	11 11	14 25	7 54	29
30	1 9	3 51	1 13	5 57	2 20	7 3	15 31	13 33	1 11	11 28	14 19	7 36	30

The preceding mark, whether + or —, at the head of any column, belongs to all equations in that column until the sign changes; and those columns having two signs at the head, shew that the preceding sign changes to the following somewhere in that column.

317. TABLE of Corrections for the Middle Time between the Equal Altitudes of the Sun.

Deg. of Declin.	Latitude 30 D.N.						Latitude 40 D.N.						Latitude 50 D.N.						Latitude 60 D.N.						Deg. of Declin.					
	Hours bet. Obser.						Hours bet. Obser.						Hours bet. Obser.						Hours bet. Obser.											
	6	5	4	6	5	4	6	5	4	6	5	4	6	5	4	6	5	4	6	5	4	6	5	4						
	N. decl.			S. decl.			N. decl.			S. decl.			N. decl.			S. decl.			N. decl.			S. decl.				N. decl.			S. decl.	
	sec.	dec.	sec.	dec.	sec.	dec.	sec.	dec.	sec.	dec.	sec.	dec.	sec.	dec.	sec.	dec.	sec.	dec.	sec.	dec.	sec.	dec.	sec.	dec.	sec.	dec.	sec.	dec.	sec.	dec.
0	9	9	8	9	9	8	14	13	13	14	13	13	20	19	18	20	19	18	28	28	27	28	28	27	0					
1	9	9	8	9	9	9	14	13	12	14	13	13	19	19	18	20	19	18	28	27	27	29	28	27	1					
2	9	8	8	9	9	9	13	13	12	14	14	13	19	18	18	20	19	18	28	27	26	29	28	27	2					
3	8	8	7	10	9	9	13	12	12	14	14	13	18	18	17	20	19	18	28	27	26	29	28	27	3					
4	8	8	7	10	10	9	13	12	12	14	14	13	18	17	17	20	19	19	27	26	25	29	29	28	4					
5	8	7	7	10	10	10	12	12	11	15	14	14	18	17	17	20	19	19	27	26	25	29	29	28	5					
6	8	7	6	10	10	10	12	11	11	15	14	14	17	17	16	20	20	19	27	25	24	29	29	28	6					
7	7	7	6	10	10	10	12	11	11	15	14	14	17	16	16	20	20	19	26	25	24	29	28	27	7					
8	7	7	6	10	10	10	11	11	10	15	14	14	17	16	16	20	20	19	26	24	23	29	28	27	8					
9	7	6	6	10	10	10	11	10	10	15	15	14	16	15	15	20	20	19	25	23	23	28	28	27	9					
10	7	6	5	10	10	10	11	10	10	15	15	14	16	15	15	20	19	19	24	23	22	28	28	27	10					
11	6	6	5	10	10	10	10	10	9	14	14	14	15	15	14	19	19	18	23	22	22		27	27	11					
12	6	6	5	10	10	10	10	9	9	14	14	14	15	14	14	19	19	18	22	21	21		27	26	12					
13	6	5	5	10	10	9	9	9	8	14	14	14	14	14	13	19	19	18	21	21	20		26	26	13					
14	5	5	4	10	9	9	9	8	8	14	14	14	13	13	12	18	18	17	20	20	19	25	25	14						
15	5	5	4	9	9	9	8	8	7	14	14	13	13	12	12	18	18	17	19	19	18		24	24	15					
16	5	4	4	9	9	9	8	7	7	13	13	13	12	12	11	17	17	17	18	18	17		23	23	16					
17	4	4	3	9	9	8	7	7	6	12	12	12	11	11	10	16	16	16	17	17	16		22	22	17					
18	4	3	3	9	8	8	6	6	5	12	12	12	10	10	9	15	15	15	16	15	15				18					
19	3	3	2	8	8	8	6	5	5	11	11	11	9	9	8	14	14	14	15	14	13				19					
20	3	2	2	8	7	7	5	5	4	10	10	10	8	8	7	13	13	13	14	13	12				17					
21	2	2	1	7	7	7	4	3	3	9	9	9	7	6	6		11	11	12	11	10				14					
22	1	1	0	6	6	5	3	2	2	7	7	7	5	5	5		9	9	8	8	8				11					
23	0	0	0	3	3	3	2	1	0	4	4	4	3	2	2		5	5	5	5	4				7					

The correction is subtractive from December 22 to June 21, or in ascending signs; and additive from June 21 to December 22, or in descending signs.

318.

A TABLE of REFRACTIONS.

Alt. Deg.	Refr. min.	Refr. sec.	Alt. Deg.	Refr. min.	Refr. sec.	Alt. Deg.	Refr. min.	Refr. sec.	Alt. Deg.	Refr. min.	Refr. sec.	Alt. Deg.	Refr. min.	Refr. sec.	Alt. Deg.	Refr. min.	Refr. sec.	Alt. Deg.	Refr. min.	Refr. sec.	Alt. Deg.	Refr. min.	Refr. sec.
0	33	00	4	11	51	14	3	45	26	1	56	38	1	13	50	0	48	62	0	30	1	14	55
1	30	35	4 1/2	10	48	15	3	30	27	1	51	39	1	10	51	0	46	63	0	29	2	11	52
2	28	22	5	9	54	16	3	17	28	1	47	40	1	8	52	0	44	64	0	28	3	10	53
3	26	21	5 1/2	9	8	17	3	4	29	1	42	41	1	5	53	0	43	65	0	26	4	9	54
4	24	29	6	8	28	18	2	54	30	1	38	42	1	3	54	0	41	66	0	25	5	8	55
5	22	48	7	7	20	19	2	44	31	1	35	43	1	1	55	0	40	67	0	24	6	7	56
6	21	15	8	6	29	20	2	35	32	1	31	44	0	55	56	0	38	68	0	23	7	6	57
7	19	51	9	5	48	21	2	27	33	1	28	45	0	57	57	0	37	69	0	22	8	5	58
8	18	35	10	5	15	22	2	20	34	1	24	46	0	55	58	0	35	70	0	21	9	4	59
9	16	24	11	4	47	23	2	14	35	1	21	47	0	53	59	0	34	71	0	19	10	3	60
10	14	36	12	4	23	24	2	7	36	1	18	48	0	51	60	0	33	72	0	18	11	2	61
11	13	6	13	4	5	25	2	2	37	1	16	49	0	49	61	0	32	73	0	17	12	1	62

319.

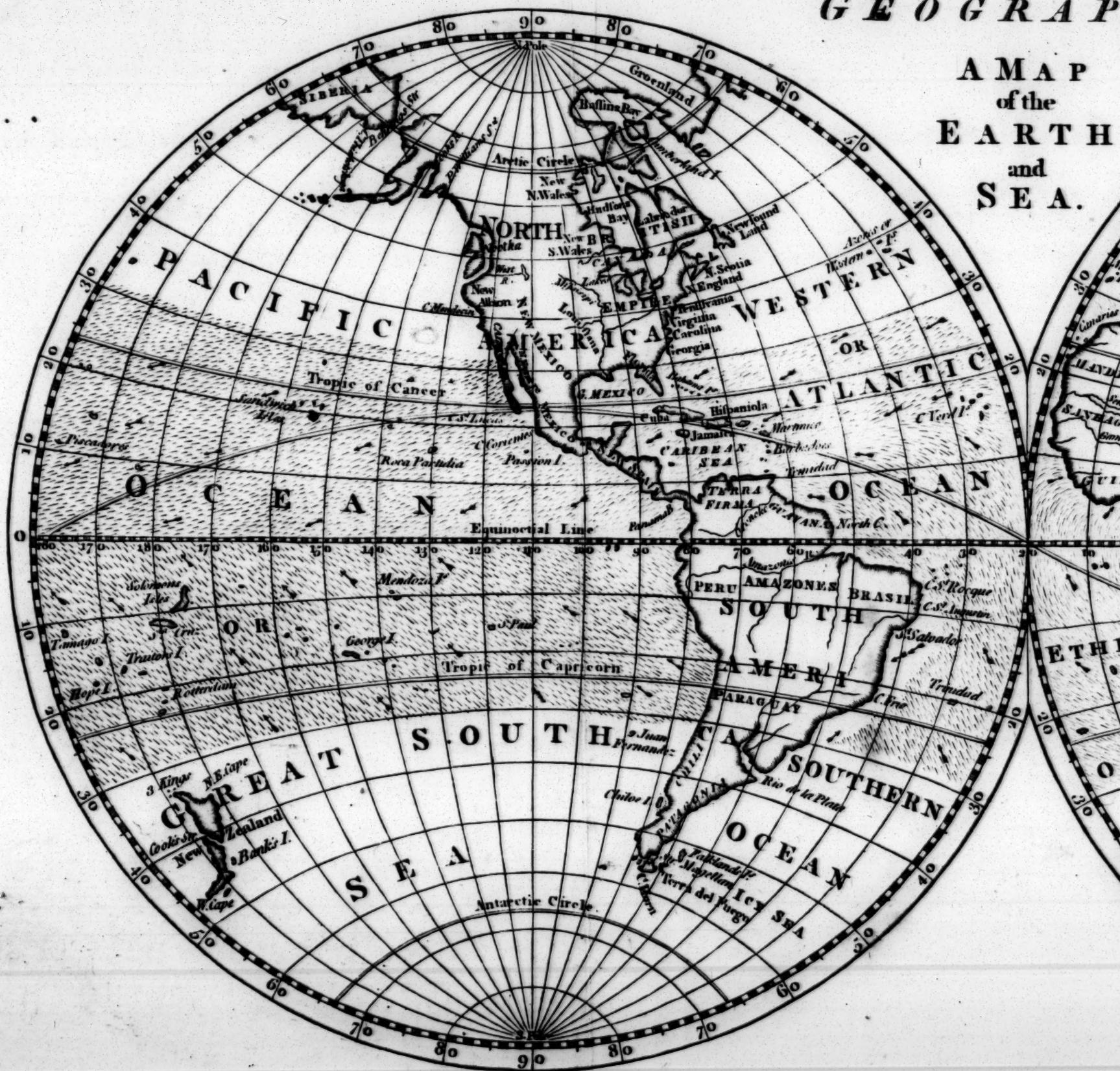
Of the Sun's Parallax.

Altitudes 0° 10° 20° 30° 40° 50° 60° 70° 80° 90°.
Parallax 9" 9" 8" 8" 6" 5" 3" 2" 1" 0".

• END OF BOOK V.

GEOGRAPH

A MAP of the EARTH and SEA.

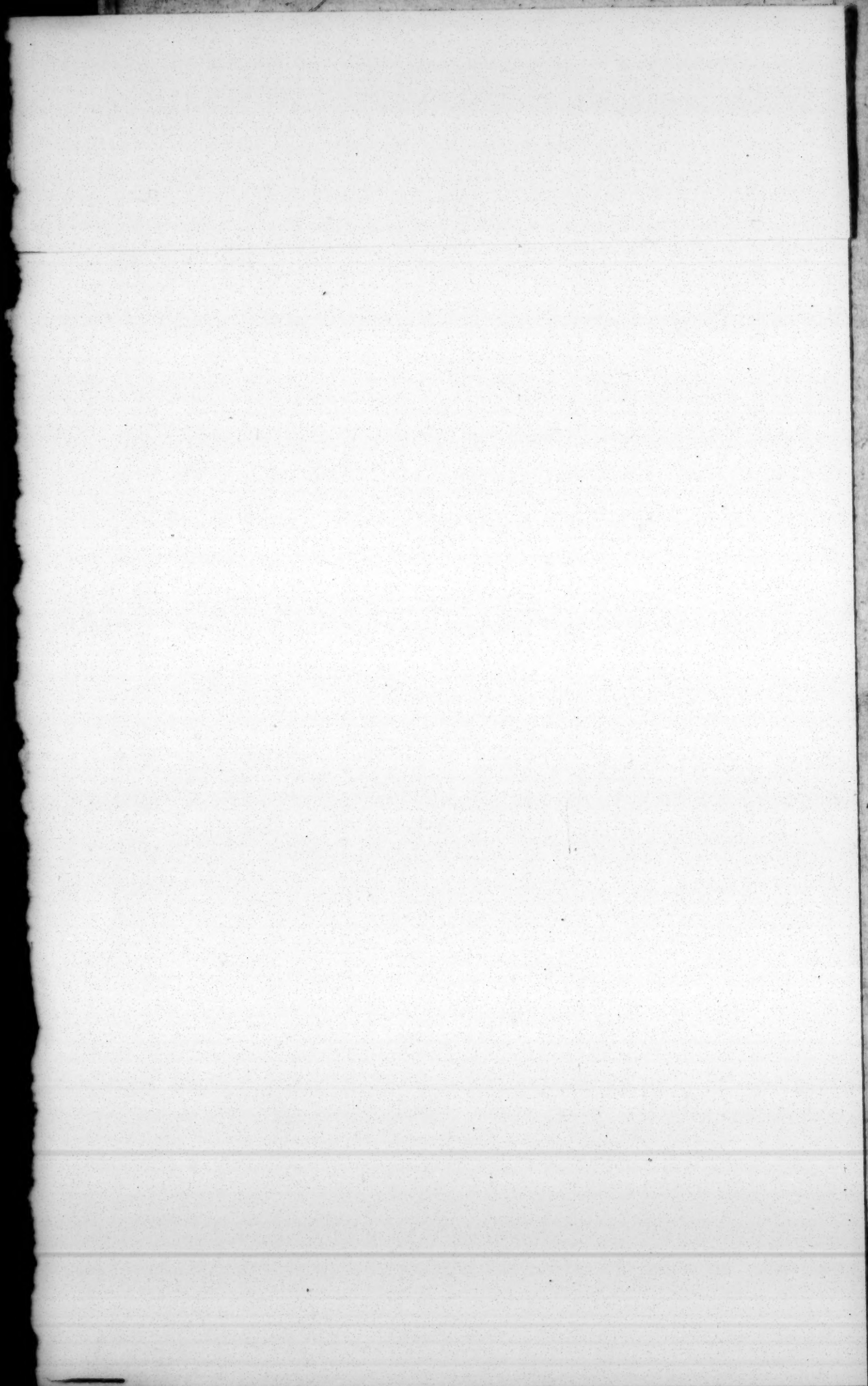


PHY.

H



J. Basire Sculp.





THE
ELEMENTS
OF
NAVIGATION.

BOOK VI.
OF GEOGRAPHY.

SECTION I.

Definitions and Principles.

1. **G**EOGRAPHY is the art of describing the figure, magnitude, and positions of the several parts of the surface of the Earth.
2. The **EARTH** is a spherical or globular figure *, and is usually called the *terraqueous globe*.
3. There are two points on the surface of the *terraqueous globe*, called the **POLES OF THE EARTH**, which are diametrically opposite to one another : one is called the *north pole*, and the other the *south pole*.

* For in ships at sea the first parts of them that become visible are the upper sails ; and as they approach nearer, the lower sails appear ; and so on until they shew their hulls.

Also ships in sailing from high capes, or head lands, lose sight of those eminences gradually from the lower parts, until the top vanishes.

Now as these appearances are the objects of our senses in all parts of the Earth,

Therefore the surface of the Earth must be convex.

And this convexity is, at sea, observed to be every where uniform.

But a body, the surface of which is every where uniformly convex, is a globe ;

Therefore the figure of the Earth is globular.

In order to describe the positions of places, Geographers have found it necessary to imagine certain circles drawn on the surface of the Earth, to which they have given the names of Equator, Meridian, Horizon, Parallels of latitude, &c.

4. The EQUATOR is a great circle on the Earth, equally distant from each pole, dividing the terraqueous globe into two equal parts; one called the *northern hemisphere*, in which is the north pole; and the other, containing the south pole, is called the *southern hemisphere*.

5. MERIDIANS are imaginary circles on the Earth passing through both the poles, and cutting the equator at right angles.

Every point on the surface of the Earth has its proper meridian.

6. LATITUDE is the distance of a place from the equator, reckoned in degrees and parts of degrees on a meridian.

On the north side of the equator it is north latitude; and on the south side it is south latitude.

As latitude begins at the equator, where it is nothing; so it ends at the poles, where the latitude is greatest, or 90 degrees.

7. PARALLELS OF LATITUDE are circles parallel to the equator.

Every place on the Earth has its parallel of latitude.

DIFFERENCE OF LATITUDE is an arc of a meridian, or the least distance of the parallels of latitude of two places; shewing how far one of them is to the northward or southward of the other.

The difference of latitude can never exceed 180 degrees.

8. In north latitudes, if about the middle of the months of March and September a person looks toward the Sun at noon, the south is before him, the north behind, the west on the right hand, and the east on the left: and in south latitudes, if the face is turned toward the Sun at the same times, the north is before, the south behind, the east to the right, and the west on the left.

In latitudes greater than $23\frac{1}{2}$ degrees, these positions, found at noon, will hold good on any day of the year.

9. LONGITUDE of any place on the Earth is expressed by an arc of the equator, shewing the east or west distance of the meridian of that place from some fixed meridian, where longitude is reckoned to begin.

10. DIFFERENCE OF LONGITUDE is an arc of the equator, intercepted between the meridians of two places, shewing how far one of them is to the eastward or westward of the other.

As longitude begins at the meridian of some place, and is counted from thence both eastward and westward, till they meet at the same meridian on the opposite point of the equator; therefore the difference of longitude can never exceed 180° degrees.

11. When two places have latitudes both north or both south; or have longitudes both east or both west, they are said to be of the same or of like name: but when one has north latitude and the other south; or if one has east longitude and the other west, then they are said to have contrary, or different, or unlike names.

12. The HORIZON is that apparent circle which limits or bounds the view of a spectator on the sea, or on an extended plain; the eye of the spectator being always supposed in the center of his horizon.

When

When the Planets or Stars come above the eastern part of the horizon, they are said to rise; and when they descend below the western part, they are said to set.

When a ship is on the equator, both the poles appear in the horizon; and in proportion as she sails toward either, or increases her latitude, that pole is seen proportionally higher above the horizon, and the other disappears as much: but when a ship is sailing toward the equator, or decreases her latitude, she depresses the elevated pole; that is, its distance from the horizon decreases.

Of the division of the Earth into Zones.

13. A **ZONE** is a broad space on the Earth, included between two parallels of latitude.

There are five zones: namely, one *Torrid*, two *Frigid*, and two *Temperate*; these names arise from the degree of heat or cold, to which their situations are liable.

14. The **TORRID ZONE** is that portion of the Earth, over every part of which the Sun is perpendicular at one time of the year or other.

This zone is about 47 degrees in breadth, extending to about $23\frac{1}{2}$ degrees on each side of the equator; the parallel of latitude, terminating the limits in the northern hemisphere, is called the *Tropic of Cancer*; and in the southern hemisphere, the limiting parallel is called the *Tropic of Capricorn*.

15. The **FRIGID ZONES** are those regions about the poles, where the Sun does not rise for some days, nor set for some days, of the year.

These zones extend round the poles to the distance of about $23\frac{1}{2}$ degrees: that in the northern hemisphere is called the north frigid zone, and is bounded by a parallel of latitude, called the *Arctic polar circle*: and the other, in the southern hemisphere, the south frigid zone; the parallel of latitude bounding it, being called the *Antarctic polar circle*.

16. The **TEMPERATE ZONES** are the spaces between the *Torrid* and the *Frigid* zones.

Of the division of the Earth by Climates.

17. A **CLIMATE**, in a geographical sense, is that space of the Earth contained between two parallels of latitude, when the difference between the longest day in each parallel is half an hour.

These climates are narrower the farther they are from the equator; therefore, supposing the equator to be the beginning of the first climate, the polar circle will be the end of the 24th climate; for afterward the longest day does not increase by half hours, but by days and months.

SECTION II.

Of the Natural division of the Earth.

18. By the natural division of the Earth is meant the parts on its surface formed by nature ; such as *Continents, Oceans, Islands, Seas, Rivers, Mountains, &c.*

The surface of the Earth is naturally divided into Land and Water.

Land is divided into	1. Continents.	4. Isthmuses.
	2. Islands.	5. Promontaries.
	3. Peninsulas.	6. Mountains.
Water is divided into	1. Oceans.	4. Straits.
	2. Seas.	5. Lakes.
	3. Gulphs.	6. Rivers.

19. A **CONTINENT**, or, as it is frequently called, the *main land*, is a very large track, comprehending several contiguous Countries, Kingdoms, and States.

20. An **OCEAN** is a vast collection of salt water, separating the continents from one another.

21. An **ISLAND** is a part of dry land, surrounded with water.

22. A **SEA** is a branch of the Ocean, flowing between some parts of the Continent, or separating an Island from the Continent.

23. A **PENINSULA** is a part of dry land encompassed by water, except a narrow neck which joins it to some other land.

24. An **ISTHMUS** is the neck joining the peninsula to the adjacent land, and forms the passage between them.

25. A **MOUNTAIN** is a part of the land more elevated than the adjacent country, and to be seen at a greater distance than the neighbouring lower lands.

26. A **PROMONTORY** is a mountain stretching itself into the sea ; the extremity of which is called a *Cape*, or *Head-land*.

27. A **HILL** is a small kind of mountain : A *Cliff* is a steep shore, hill, or mountain : And *Rocks* are great stones, rising like hills above the dry land, or above the bottom of the sea.

28. A **GULF**, or **BAY**, is a part of the Ocean, or Sea, contained between two shores : and is every where environed with land, except at its entrance, where it communicates with other Bays, Seas, or Oceans.

29. A **STRAIT** is a narrow passage, by which there is a communication between a Gulf and its neighbouring sea, or which joins one part of the sea, or ocean, with another.

30. A **LAKE** is a collection of Waters contained in some hollow or cavity, in an inland place, of a large extent, and every where surrounded with land, having no visible communication with the Ocean.

31. **RIVERS** are streams of Water, flowing chiefly from the Mountains, and running in long narrow channels or cavities through the land, till they fall into the sea, or into other rivers, which at last run into the sea.

32. There

32. There are generally reckoned four Continents, namely, EUROPE, ASIA, AFRICA, and AMERICA.

To these may be added the *Terra arctica*, or northern continent, and the *Terra antarctica*, or lands detached from *Asia*, towards the south.

The continent of *America* is usually divided into two parts, called *North* and *South America*; they are joined together by the *Isthmus* of *Darien*. Also the continents of *Asia* and *Africa* are joined together by the *Isthmus* of *Sues*.

The *Terra arctica*, *Europe*, and *Asia*, lie all within the northern hemisphere; and also part of *Africa* and *America*: The other parts of these two continents, together with the *Terra antarctica*, lie in the southern hemisphere.

33. There are five Oceans, namely, the NORTHERN, the ATLANTIC, the PACIFIC, the INDIAN, and the SOUTHERN.

The *Atlantic ocean* is usually divided into two parts, one called the *north Atlantic ocean*, and the other the *south Atlantic*, or *Ethiopic ocean*.

The *Northern ocean* stretches to the northward of *Europe*, *Asia*, and *America*, toward the north pole.

The *Atlantic ocean* lies between the continents of *Europe* and *Africa* on the east, and *America* on the west.

That part of the north Atlantic ocean, lying between *Europe* and *America*, is frequently called the *Western ocean*.

The *Pacific ocean*, or, as it is sometimes called, the *South Sea*, is bounded by the western and north-west shores of *America*, and by the eastern and north-east shores of *Asia*.

The *Indian ocean* washes the shores of the eastern coasts of *Africa*, and the south of *Asia*; and is bounded on the east by the Indian islands, New Holland, and New Zealand.

The *Southern ocean* extends to the southward of *Africa* and *America* toward the south pole.

The northern and southern continents not being sufficiently known to Geographers, all that need be said of them is, that the *Terra Arctica*, or land to the northward of *Hudson's Bay* and *Greenland*, is in general too cold for the residence of mankind; and that the lands formerly supposed to be parts of the southern continent, are found to be very large islands: viz. *New Zealand* is much larger than *Great Britain*, and has a strait dividing it into two islands. *New Holland* is an island as large as *Europe*. *New Guinea* is a very large island; and *New Britain* is a cluster of large and small islands, and are thought by some to be the islands hitherto called the *Solomon's islands*.

SECTION III.

Of the Political division of the Earth.

34. By the political division of the Earth is meant the different Countries, Empires, Kingdoms, States, and other denominations established by men, either from the ambition of tyrants, or for the sake of good government.

OF EUROPE.

Europe is bounded on the north by the northern, or frozen, ocean; on the east by Asia; on the south by the Mediterranean Sea, separating Europe from Africa; and by the north Atlantic, or western ocean, on the west. It lies between the latitudes of 36 and 72 degrees of north latitude; and between the longitudes of 10 degrees west, and 65 degrees east from London; is about 3000 miles long, reckoning from the N. E. to the S. W. and about 2500 miles broad.

35. The countries, their position with regard to the middle parts of Europe, the chief cities, principal rivers, with their courses, and the most noted mountains, and what quarter of the country they are in, are exhibited in the following table; where E stands for empire, K. for kingdom, R. for republic, Nd. for northward, &c.

Countries.	Position.	Chief Cities.	Rivers.	Course.	Mountains.
E. Turkey	S. E.	Constantinople	Danube	E.	Argemont Nd.
K. Poland	Mid.	Warsaw	Vistula	N. N. W.	Carpathian Sd.
E. Muscovy	N. E.	{ Moscow	Volga	E. to S.	Boglowy Sd.
E. Russia		{ Peterburg	Neiper	S.	Riphean Wd.
K. Sweden	N.	Stockholm	Dalecarlia	E.	Dofrine Wd.
K. Norway	N. N. W.	Bergen	Glama	S.	Dofrine Ed.
K. Denmark	N. W.	Copenhagen	Eyder	W.	
K. Hungary	Mid.	Presburg	Danube	S. E.	Carpathian Nd.
E. Germany	Mid.	Vienna	Danube	E.	Alps Sd.
Italy	S.	Rome	{ Po	E.	Alps Nd.
			{ Tyber	S.	Apennine Mid.
R. Switzerland	Mid.	Bern	Rhine	W.	Alps Sd.
Netherlands	W.	Brussels	Maese	N.	
R. Holland	W.	Amsterdam	Rhine	N. N. W.	
K. France	W.	Paris	{ Loire	N. to W.	Pyrenees S. W.
			{ Rhone	S.	Alps Ed.
K. Spain	S. W.	Madrid	Tagus	W.	Pyrenees N. E.
K. Portugal	S. W.	Lisbon	Tagus	W.	C. Rocca W.
K. England	W.	London	Thames	E.	Malvern N. W.
K. Scotland	W.	Edinburgh	Forth	E.	Grampian Nd.
K. Ireland	W.	Dublin	Shannon	S. W.	Knockpatric W.

There are in Europe four Kingdoms, beside those enumerated above; but they are contained in the forenamed Countries.

The Kingdom of Prussia, which is part of Poland; the King's residence is at Berlin, a city in Germany.

The Kingdom of Bohemia, a part of Germany; the chief city is Prague.

The Kingdom of Sardinia, an Italian island; the King resides at Turin, a city in Italy.

The Kingdom of the Sicilies, appending to Italy; the King resides at Naples, a city in Italy.

In some of the forenamed countries are several dominions independent one of the other; particularly in Germany and Italy.

The principal States in Germany are the following 12; where D. stands for duchy, El. for electorate, P. for principality:

States Ch.cities	D. Austria Vienna	K. Bohemia Prague	El. Bavaria Munich	El. Brandenburg Berlin
States Ch.cities	El. Saxony Dresden	El. Hanover Hanover	El. Palatine Manheim	El. Mentz Mentz
States Ch.cities	El. Triers Triers	El. Cologne Cologne	P. Hesse Cassel Cassel	D. Wurtemberg Stutgard

The principal States in Italy are the following 12:

States Ch.cities	D. Savoy Chambery	P. Piedmont Turin	D. Milanese Milan	D. Parmesan Parma
States Ch.cities	D. Modenese Modena	D. Mantuan Mantua	R. Venice Venice	R. Genoa Genoa
States Ch.cities	D. Tuscany Florence	Patriarchate Rome	R. Lucca Lucca	K. Naples Naples

36. *The principal Seas, Gulfs, and Bays in Europe, are*

The *Mediterranean Sea*, having Europe on the N. and Africa on the S.

The *Adriatic Sea*, between Italy and Turkey.

The *Euxine, or Black Sea*, in Turkey, between Europe and Asia,

The *White Sea*, in the N. N. W. parts of Muscovy.

The *Baltic Sea*, between Sweden, Denmark, and Poland.

The *German Ocean, or Sea*, between Germany and Britain.

The *English Channel*, between England and France,

St. George's Channel, between Britain and Ireland.

The *Bay of Biscay*, formed between France and Spain.

The *Gulf of Bothnia*, in the N. E. parts of Sweden.

The *Gulf of Finland*, between Sweden and Russia,

The *Gulf of Venice*, the N. W. end of the Adriatic Sea.

37. *The principal Islands in Europe, are*

The *British Isles*; viz. Great Britain, Ireland, Orkneys, and Western Isles.

The *Spanish Isles*; Majorca, Minorca, Ivica, in the Medit. Sea.

Italian Isles; Sicily, Sardinia, Corsica, Lipari, in the Medit. Sea.

Turkish Isles; Candia, Archipelago Isles, in the Medit. Sea.

Swedish Isles; Gothland, Oeland, Alan, Rugen, in the Baltic Sea.

Danish Isles; Zeland in the Baltic Sea; and Iceland, Faro Isles, E. and

W. Greenlands in the Northern ocean.

The *Azores Isles* in the Atlantic ocean, belonging to Portugai,

OF ASIA.

38. The continent of Asia is bounded on the north by the Northern or frozen ocean, on the east by the Pacific ocean, on the south by the Indian ocean, and by Africa and Europe on the west. It lies, including its islands, between the latitudes of 10 degrees south, and 72 degrees north; and is between the longitudes of 25 and 148 degrees east of London; its length, exclusive of the isles, being about 4800 miles, and breadth about 4300 miles.

39. The positions and names of the chief countries, cities, rivers, and mountains, are contained in the following table:

Countries.	Position.	Chief Cities.	Rivers.	Course.	Mountains.
E. China	S. E.	{ Pekin Nankin Canton	Yellow R. Kiam Ta	S. to E. E. S. E.	Ottorocoran Nd. Damafian Wd.
K. Korea	E.	Kingkitau	Yalu	S.	Shanalin
Chinese Tartary	E.	Chynian	Yamour	N.E. to E.	Fongwanshan
Mongalia	Mid.	Kudak	Yellow R.	N. to W.	
K. Thibet	Mid.	Eskerdu	Yaru	E.	Kantes
Bukharia, or Ulbses	Mid.	Samarkand	Amu	N. W.	Belurlag
Karazm	Mid.	Urjenz	Amu	S. W.	Irder
Kalmucks	Mid.		Tekis		Tubratubusluk
E. Siberia	N.	{ Tobolski Astracan	Oby Jeniska	N. N. N. W.	Stolp
E. Turkey	W.	Smyrna	Euphrates	S. E.	Taurus
K. Syria	W.	Aleppo	Euphrates	S.	Lebanon
Arabia	S. W.	Medina	Euphrates	S. E.	Gabel el ared
E. Persia	S.	Isfahan	{ Oxus Araxes	W. S. W.	Caucasus Taurus
India West of the Ganges	{ S.	Agra Delli	Indus Ganges	S. W. S. W.	Caucasus Balagate
India East of the Ganges	{ S.	Ava Pegu Siam Cambodia	Domea Mecon Menan Ava	S. S. S. S.	{ Damascene

Some of these countries contain several others.

Asiatic Turkey contains

Countries	Georgia N.	Turcomania E. or Armenia	Curdistān E. or Assyria	Diarbec E.
Ch. cities	Teffis	Erzerum	Betlis	Moufol
Countries	Eyraca S. E.	Arabia desert S.	Natolia W.	Syria W.
Ch. cities	Bagdat		Smyrna	Aleppo
				India

India west of the Ganges contains

Northern Parts.	Countries	Indoſtan N.	Cambaya S. W.	Bengala S. E.
	Ch. cities	Delli	or Guzarat Surat	Patna
Malabar Coaſt	Countries	Decan	Bisnagar W.	
	Ch. cities	or Viſapour Goa	or Carnate Calcut and Cochin	
Coromandel Coaſt	Countries	Bisnagar	K. Golconda	K. Orixá
	Ch. cities	Carnate, E. ſide Madras	Golconda	Orixá

India eaſt of the Ganges contains

Countries	K. Ava N. W.	K. Pegu W.	K. Siam S.	K. Malacca S.
Ch. cities	Ava	Pegu	Siam	Malacca
Countries	K. Cambodia S.	K. Cochin China E.	K. Laos N.	K. Tonquin N. E.
Ch. cities	Cambodia	Thoanoa	Lanchan	Keccio

40. The principal Seas, Gulfs, and Bays in Aſia, are

Caspian Sea, quite ſurrounded by Siberia on the North, Korazm eaſt, by Perſia on the ſouth, and by Georgia on the weſt.

Korean Sea, between Korea and the iſlands of Japan.

Yellow Sea, between China and the Japan iſles.

Gulf of Cochin China, on the borders of Tonquin and Cochin China.

Bay of Siam, formed by the countries of Siam and Malacca.

Bay of Bengal, between India eaſt, and India weſt of the Ganges.

Gulf of Perſia, having Perſia on the N. E. and Arabia on the S. W.

41. The principal Iſlands belonging to Aſia, are

Ladrone, or Marian Iſles, whoſe chief iſland is Guam.

Japan Iſles	Ch. iſles	Japan	Bongo	Tonfa
	Ch. cities	Jeddo	Bongo	Tonfa
Philippines	Ch. iſles	Luconia	Mindanao	Samar
	Ch. cities	Manilla	Mindanao	
Chineſe Iſles	C. iſles	Formoſa	Ainan	Makao
	Ch. cities	Taywanfu	Tan	Makau
Moluccos	Ch. iſles	Celebes	Gilolo	Ceram
	Ch. cities	Macaffer	Gilolo	Ambay
Sunda Iſles	Ch. iſles	Borneo	Sumatra	Java
	Ch. cities	Banjar	Achin	Batavia

The *Andaman Iſles* to the weſt of Siam.

Nicobar Iſlands weſt of Malacca.

Maldivé Iſlands to the S. W. of Biſnagar.

The *Iſland of Ceylon* S. E. of Biſnagar; the chief city is Candy, or Candy Uta.

OF AFRICA.

42. This large continent is a peninsula, joined to Asia by the Isthmus of Suez. On the N. E. it is separated from Asia by the Red Sea; it has the Indian Ocean on the east, the Southern on the south, the Atlantic on the west, and the Mediterranean Sea on the north, which separates it from Europe. It is situated between the latitudes of 37 degrees N. and 35 degrees S.; and between the longitudes of 18 degrees W. and 50 degrees E. from London; is about 4300 miles long, and 4200 miles broad.

43. The positions and names of the chief countries, cities, rivers, and mountains, are contained in the following table :

Countries.	Position.	Chief Cities.	Rivers.	Course.	Mountains.
E. Morocco	N. W.	Fez	Mulvia	N.	Atlas
K. Algiers	N.	Algiers	Saffran	N.	Atlas
K. Tunis	N.	Tunis	Bagrada	N.	Atlas
K. Tripoli	N.	Tripoli	Salines	N. E.	Atlas
K. Barca	N.	Docra			Meies
K. Egypt	N. E.	Cairo	Nile	N.	Gianadel
E. Abyssinia or Ethiopia }	E.	Ambamarjam	Nile	N.	
Ajan	E.	Adea	Madadoxa	S.	
Zanguebar	E.	Melinda	Cuama	E. S. E.	
Sofala	S. E.	Sofala	Amara	E.	Amara
Terra de Natal	S. E.	Natal	St. Esprit	E.	Amara
Cafraria	S.	Cape Town	St. Christopher	E.	Table
Mataman	S. S. W.		Angri	W.	Sunda
K. Benguela	S. S. W.	Benguela	Negros	W.	Sunda
K. Angola	S. W.	Loando	Coanza	W.	Sunda
K. Congo	S. W.	St. Salvador	Zaara	S. W.	Sunda
K. Loango	S. W.	Loango	Zette	S. W.	St. Esprit
Biafara	S. W.	Biafara	Camerones	S. W.	St. Esprit
K. Benin	S. W.	Benin	Formosa	S. W.	
Guinea	S. W.	Cape Coast	Volta	S.	Sierra Leon
Mandinga	W.	James Fort	Gambia	W.	C. Verd
Sanhaga	W.	Sanhaga	Senegal	N. W.	
Bildulgerid	Mid. N.	Dara	Dara	S.	Atlas
Zara	Mid.	Zuenzega	Nubia	E.	
Nubia	Mid.	Nubia	Nubia	E.	
Negroland	Mid.	Tombute	Niger	W.	
Ethiopia inter.	Mid.	Chaxumo	Niger	W.	Luna
Monomugi	Mid. S.	Merango	Cuama	E. S. E.	Luna
E. Monomotapa	Mid. S.	Morgar	Amara	S. E.	Amara

Many parts of the Coast of Africa are subject to the European nations: Thus the Kingdoms of Algiers, Tunis, Tripoli, Barca, and Egypt, are either subject to the Ottoman, or Turkish empire, or acknowledge themselves under its protection.

Abyssinia

Abyssinia is governed by its own Emperor.

Ajan or Anian is peopled by a few wild Arabs.

In Zanguebar and Sofala, the Portuguese have many black Princes tributary to them.

Cafraria, or the county of the Hottentots, belongs to the Dutch.

The sea coasts of Guinea are usually distinguished by the names of the *Slave Coast*, *Gold Coast*, *Ivory Coast*, *Grain Coast*, and *Sierra Leon*.

The English, Dutch, French, Portuguese, and others, have several settlements along these coasts, and even many miles up the country, particularly the English on the rivers Gambia and Senegal.

In the general table the countries are taken in a very large sense; for many of them contain a great number of states independent one of the other, the particulars of which are not known to Geographers.

44. The principal Seas, Gulfs, and Bays in Africa, are

The *Red Sea*, between Africa and Asia: It washes the coast of Arabia on the Asiatic side, and the coasts of Egypt and Abyssinia on the African side.

Mosambique Sea, between Africa and the island of Madagascar eastward.

Saldanna Bay in Cafraria, on the Ethiopic Ocean.

Bight of Benin on the coast of Guinea, in the Ethiopic Ocean.

45. The principal African Islands, are

	Chief isles.	Chief Towns.	Situation.
<i>Madeira isles</i>	Madeira	Funchal	N. Atlantic Ocean
<i>Canary isles</i>	Canaria	Palma	N. Atlantic Ocean
<i>C. Verd isles</i>	St. Jago	St. Jago	N. Atlantic Ocean
<i>Ethiopian isles</i>	St. Helena		Ethiopic Ocean
<i>Komora isles</i>	Johanna	Demani	Indian Ocean
<i>Sokotora isles</i>	Zocotora	Calansia	Indian Ocean
<i>Almirante isles</i>	But little known.		Indian Ocean

The island of *Madagascar*, one of the largest in the world, lies in the Indian Ocean: It is divided into a multitude of little states; some of them formed by the European privateers, and their successors descended from a mixture with the natives.

The islands of *Bourbon* and *Mauritius* lie in the Indian Ocean, to the east of Madagascar; these belong to the French.

The *Madeiras*, and *Cape de Verd Isles*, belong to the Portuguese.

The *Canary Isles* to Spain; *St Helena* to England.

OF AMERICA.

46. This vast continent, called by some the new world, having been discovered by the Europeans since the year 1492, is usually divided into two parts, one called North, and the other South America, being joined to one another by the Isthmus of Darien.

North America lies between the latitudes of 10 degrees and 80 degrees north; and chiefly between the longitudes of 50 degrees and 130 degrees west of London; is about 4200 miles from north to south, and about 4800 from east to west. It is bounded on the east by the north Atlantic Ocean, by the Gulf of Mexico on the south, on the west by the Pacific Ocean, and by the Northern continent and ocean to the northward.

South America is bounded on the east by the south Atlantic Ocean, by the Southern Ocean to the south, by the Pacific Ocean on the west, and on the north by the Caribbean Sea. It lies between the latitudes of 12 degrees north, and 56 degrees south; and between the longitudes of 45 degrees and 83 degrees west from London; is about 4200 miles long, and about 2200 miles in breadth.

47. The positions and names of the chief countries, cities, rivers, and mountains, in North America, are in the following table:

Countries.	Position.	Chief Cities.	Rivers.	Course.	Mountains.
California	W.	St. Juan			
New Mexico	S.	Sainte Fe	N. River	S. S. E.	
Old Mexico	S. W.	Mexico	Panuco	E.	
Louisiana	S.	New Orleans	Mississippi	S.	
Florida	S.	St. Augustine	St. John	N. to E.	Apalachian
Georgia	S. S. E.	Savannah	Altamaha	E. S. E.	Apalachian
Carolina	S. E.	Charles Town	Ashly	S. E.	Apalachian
Virginia	E.	James Town	Powtomack	S. E.	Apalachian
Maryland	E.	Annapolis	Powtomack	S. E.	Apalachian
Pennsylvania	E.	Philadelphia	Delawar	S.	Apalachian
Jerseys	E.	New York	Albany	S.	
New England	E.	Boston	Connecticut	S.	
Nova Scotia	N. E.	Halifax	St. John	S. S. E.	Ladies
Canada	Mid.	Quebec	St. Lawrence	N. E.	
New Britain	N. N. E.	Fort Rupert	Rupert	W.	
New Wales	N.	York Fort	Nelson	W.	

California, Old Mexico, and New Mexico, Louisiana, and Florida, belong to Spain.

Georgia, Carolina, Virginia, Maryland, Pennsylvania, the Jerseys, and New England, form the United States of America.

Nova Scotia, Canada, New Britain, and New Wales, belong to England.

In South America the position and names of the chief countries, cities, rivers, and mountains, are as follow :

Countries.	Position.	Chief Cities.	Rivers.	Courfe.	Mountains.
Terra Firma.	N.	Panama	Oronoque	N. E.	
Peru	W.	Lima	Chuquimayo	W. N. W.	Andes
Chili	S. W.	St. Jago	Valpariso	W.	Andes
Patagonia	S.		Desaguadero	S.	Andes
La Plata	S. E.	Buenos Ayres	La Plata	S.	Andes
Paraguay	Mid.	Assumption	Paragua	S.	
Brazil	E.	St. Salvador	Rio Real	N. E.	
Amazonia	Mid.		Amazons	E.	
Guiana	N. E.	Surinam	Esquebe	N. N. E.	

Terra Firma, Peru, Chili, La Plata, and Paraguay, are in the possession of the Spaniards.

Brazil belongs to the Portuguese.

Patagonia, Amazonia, and Guiana, are possessed by the native Indians, except some parts of the coasts of Guiana, in the hands of the Dutch and French.

48. *The principal Seas, Gulfs, and Bays in America.*

The *Caribbean Sea*, bounded by Terra Firma on the south, and a range of islands on the north and east.

Gulf of Mexico, formed by Old Mexico, Louisiana, and Florida.

Bay of Campeachy, part of the Gulf of Mexico, on the Mexican coast.

Bay of Honduras, part of the Caribbean Sea, next to Mexico.

Bay of Panama, in the Pacific Ocean, next the Isthmus of Darien.

Bay of California, in the Pacific Ocean, having California on the west.

Bay of Fundy, in Nova Scotia, north Atlantic Ocean.

Gulf of St. Lawrence, in the North Atlantic Ocean, bounded by Nova Scotia, New Britain, and some islands eastward and south-eastward.

Hudson's Bay, between New Britain E. and New Wales W.

49. *The chief American Islands in the Atlantic Ocean.*

Newfoundland, and *Cape Breton*, east of the Gulf of St. Lawrence.

Bermudas, or *Summer Islands*, east of Carolina.

Bahama Isles, south east of Florida.

Great Antilles	{	Cuba	ch. town Havanna	} lying E. of the Mexican Gulf, and N. of the Car. Sea.
		Hispaniola	ch. town St. Domingo	
		Jamaica	ch. town Kingston.	

Caribbee Isles, bounding the Caribbean Sea on the E. and N. E.

Lesser Antilles, on the N. N. E. of Terra Firma, in the Caribbee Sea.

Terra del Fuego, on the south of Patagonia, in the Southern Ocean.

Gallipago Isles, lying N. W. of Peru in the Pacific Ocean.

SECTION

SECTION IV.

Geographical Problems.

50. PROBLEM I. *Given the latitudes of two places :
Required their difference of latitude.*

CASE I. When the latitudes of the given places have the same name :

RULE. Subtract the lesser latitude from the greater, the remainder is the difference of latitude.

EXAM. I. *What is the difference of latitude between London and Rome ?*

London's lat. $51^{\circ} 31' N.$
Rome's lat. $41 \quad 54 N.$

Diff. lat. $\begin{array}{r} 9 \quad 37 \\ 60 \end{array}$

577 miles.

EXAM. II. *What is the difference of latitude between the Lizard and the Island of Madeira ?*

Lizard's lat. $49^{\circ} 57' N.$
Madeira's lat. $32 \quad 38 N.$

Diff. lat. $17 \quad 19 = 1039m.$

EXAM. III. *What is the difference of latitude between the Island of St. Helena and the Cape of Good Hope ?*

C. Good Hope's lat. $34^{\circ} 29' S.$
St. Helena's lat. $15 \quad 55 S.$

Diff. lat. $18 \quad 34 = 1114m.$

EXAM. IV. *A ship from the latitude of $43^{\circ} 18' N.$ is come to the lat. of $34^{\circ} 49' N.$ Required the diff. of latitude.*

Lat. from $43^{\circ} 18' N.$
Lat. in $34 \quad 49 N.$

Diff. lat. $8 \quad 29 = 509m.$

CASE II. When the latitudes of the given places have contrary names :

RULE. Add the latitudes together, and the sum will be the difference of latitude.

EXAM. I. *Required the diff. of lat. between C. Finisterre and C. St. Roque.*

C. Finisterre lat. $42^{\circ} 52' N.$
C. St. Roque lat. $5 \quad 00 S.$

Diff. lat. $\begin{array}{r} 47 \quad 52 \\ 60 \end{array}$

2872 miles.

EXAM. II. *What is the difference of latitude between the Island of Barbadoes and C. Negro ?*

I. of Barbadoes's lat. $13^{\circ} 00' N.$
C. Negro's lat. $16 \quad 30 S.$

Diff. lat. $29 \quad 30 = 1770m.$

EXAM. III. *Required the difference of latitude between Cape Horn and Cape Corientes in Mexico.*

Cape Horn's lat. $55^{\circ} 59' S.$
C. Corientes's lat. $20 \quad 18 N.$

Diff. lat. $76 \quad 17 = 4577m.$

EXAM. IV. *A ship from the lat. of $8^{\circ} 28' S.$ has sailed north to the lat. $6^{\circ} 45' N.$ Required the diff. of latitude.*

Lat. from $8^{\circ} 28' S.$
Lat. in $6 \quad 45 N.$

Diff. lat. $15 \quad 13 = 913m.$

The situation of about 1500 particular places are contained in a Geographical Table, Art. 137, at the end of this book ; where the latitudes and longitudes of places are to be sought, as they follow in alphabetical order.

51. PRO-

51. PROBLEM II. *Given the latitude of one place, and the difference of latitude between it and another : Required the latitude of the latter place.*

CASE I. When the given latitude and difference of latitude have the same name :

RULE. To the given latitude add the degrees and minutes in the diff. of latitude, that sum is the other latitude, of the same name.

EXAM. I. *A ship from the latitude of $38^{\circ} 14'$ N. sails north till her difference of latitude is $12^{\circ} 32'$: What latitude is she come to ?*

Lat. from	$38^{\circ} 14' \text{ N.}$
Diff. lat.	$12 \quad 32 \text{ N.}$
Lat. in	$50 \quad 46 \text{ N.}$

EXAM. II. *A ship from the island of Ascension runs south till her diff. of latitude is $5^{\circ} 37'$: What is the present latitude of the ship ?*

I. Ascension's lat.	$7^{\circ} 56' \text{ S.}$
Diff. lat.	$5 \quad 37 \text{ S.}$
Ship's lat.	$12 \quad 33 \text{ S.}$

EXAM. III. *A ship from the island of Madeira sails N. 675 miles : What lat. is she in ?*

I. Madeira's lat.	$32^{\circ} 38' \text{ N.}$
Diff. lat.	$\frac{675}{60} \text{ (I. 22)} = 11 \quad 15 \text{ N.}$
Ship's lat.	$43 \quad 53 \text{ N.}$

EXAM. IV. *Three days ago we were in the latitude of the Cape of Good Hope, and have run each day 92 miles directly S. What is our present latitude ?*

C. Good Hope's lat.	$34^{\circ} 29' \text{ S.}$
Diff. lat.	$\frac{92 \times 3}{60} \text{ (I. 22)} = 4 \quad 36 \text{ S.}$
Present latitude	$39 \quad 05 \text{ S.}$

CASE II. When the given latitude and difference of latitude have contrary names :

RULE. Take the difference between the given latitude, and the degrees and minutes in the diff. of latitude, the remainder is the other latitude, of the same name with the greater.

EXAM. I. *A ship from the latitude of $38^{\circ} 14'$ N. sails south till her difference of latitude is $12^{\circ} 32'$: What latitude is she come to ?*

Lat. from	$38^{\circ} 14' \text{ N.}$
Diff. lat.	$12 \quad 32 \text{ S.}$
Lat. in	$25 \quad 42 \text{ N.}$

EXAM. II. *A ship from the island of Ascension runs north till her diff. lat. is $5^{\circ} 37'$: What is the present latitude of the ship ?*

I. Ascension's lat.	$7^{\circ} 56' \text{ S.}$
Diff. lat.	$5 \quad 37 \text{ N.}$
Ship's lat.	$2 \quad 19 \text{ S.}$

EXAM. III. *A ship from Sierra Leona sails S. 839 miles : What latitude is she in ?*

Sierra Leon's lat.	$8^{\circ} 30' \text{ N.}$
Diff. lat.	$\frac{839}{60} \text{ (I. 22)} = 13 \quad 59 \text{ S.}$
Ship's lat.	$5 \quad 29 \text{ S.}$

EXAM. IV. *Four days ago we were in the latitude of the island of St. Matthew, and sailed due north 6 miles an hour : What latitude is the ship in ?*

St. Matthew's lat.	$1^{\circ} 23' \text{ S.}$
Diff. lat.	$\frac{6 \times 24 \times 4}{60} \text{ (I. 22)} = 9 \quad 36 \text{ N.}$
Ship's latitude	$- 8 \quad 13 \text{ N.}$

52. PROBLEM. III. *Given the longitudes of two places :
Required their difference of longitude.*

RULE. If the longitudes are of the same name, their difference is the difference of longitude required.

But if the longitudes are of different names, their sum is the difference of longitude.

And if this sum exceed 180 degrees, take it from 360 degrees, and there will remain the difference of longitude.

EXAM. I. *Required the difference of longitude between London and Naples ?*

London's long.	00° 00'
Naples' long.	14 19 E.
Diff. long.	<u>14 19</u> 60 859

EXAM. II. *A ship in longitude 14° 45' W. is bound to a port in longitude 48° 18' west : What diff. of longitude must she make ?*

Ship's long.	14° 45' W.
Long. bound to	48 18 W.
Diff. longitude	<u>33 33</u> 60 2013 miles.

EXAM. III. *What is the difference of longitude between Cape Gardafuir and Cape Comorin ?*

C. Gardafuir's long.	50° 25' E.
C. Comorin's long.	78 17 E.
Diff. longitude	<u>27 52</u> 60 1672 miles.

EXAM. IV. *Required the difference of longitude between St. Christopher's and Cape Negro.*

St. Christopher's long.	62° 54' W.
C. Negro's long.	11 30 E.
Diff. longitude	<u>74 24</u> 60 4464 miles,

EXAM. V. *A ship in longitude 140° 20' W. is bound to a place in longitude 139° 35' E. what diff. of longitude must she make.*

Ship's long.	140° 20' W.
Long. bound to	139 35 E.
Diff. longitude	<u>279 55</u> 360 00 80 05

EXAM. VI. *What is the difference of longitude between Cape Horn and Manila ?*

Cape Horn's long.	67° 21' W.
Manila's long.	120 58 E.
Diff. longitude	<u>188 19</u> 360 00 171 41

Sometimes the diff. lon. between two places is estimated by the diff. of time, allowing an hour to every 15 degrees of longitude, and one min. of time for every 15 min. of a deg. or a deg. for every 4 min. of time.

EXAM. *At 6 h. 48 m. P. M. having observed at sea a certain appearance in the heavens, which I knew was seen the same instant at 3 h. 25 m. P. M. in London : Required the diff. longitude between the places of observation.*

From	6 h. 48 m.	3 h. = 45 deg.
Take	3 35	13 m. = 3 15
Remain	3 13 = diff. time.	Sum 48 15 = diff. long.

And because the hour of appearance at London was least, therefore I know myself to be to the eastward of London.

53. PROBLEM IV. *Given the longitude of one place, and the difference of longitude between that and another : Required the longitude of the second place.*

RULE. If the given longitude and difference of longitude are of contrary names, their difference is the longitude required; and is of the same name with the greater.

But if the given longitude and difference of longitude are of the same name, the sum is the longitude sought, of the same name with the given place.

And if the sum is greater than 180 degrees, take it from 360 degrees, and there will remain the longitude required, of a contrary name to that of the given place.

EXAM. I. *A ship from the longitude of $41^{\circ} 12'$ E. sails westward until her difference of longitude is $15^{\circ} 47'$: What is her present longitude?*

Ship's longitude	$41^{\circ} 12' \text{ E.}$
Diff. long.	$15 \quad 47 \text{ W.}$
Pref. long.	$25 \quad 25 \text{ E.}$

EXAM. II. *A ship from Cape Charles, in Virginia, sails eastward until she has altered her longitude $22^{\circ} 53'$: What longitude is she in?*

C. Charles's long.	$76^{\circ} 07' \text{ W.}$
Diff. long.	$22 \quad 53 \text{ E.}$
Ship's long.	$53 \quad 14 \text{ W.}$

EXAM. III. *Four days ago I departed from C. St. Sebastian, in Madagascar, and I have made each day 75 miles of east longitude : Required the longitude the ship is in?*

75	C. Sebas. long.	$46^{\circ} 30' \text{ E.}$
4	Diff. long.	$5 \quad 00 \text{ E.}$
6,0)30,0	Ship's long.	$51 \quad 30 \text{ E.}$
5 ⁰		

EXAM. IV. *A ship from Cape Finisterre sails westward, and finds she has altered her longitude 587 miles : What longitude is she arrived in?*

C. Finisterre's long.	$9^{\circ} 12' \text{ W.}$
Diff. lon.	$\frac{587}{60} \quad 9 \quad 47 \text{ W.}$
Long. in.	$18 \quad 59 \text{ W.}$

EXAM. V. *A ship from Cape St. Lucar, in California, has made $87^{\circ} 18'$ of west longitude : What longitude is she in?*

C. St. Lucar's long.	$109^{\circ} 40' \text{ W.}$
Diff. long.	$87 \quad 18 \text{ W.}$
	$196 \quad 58 \text{ W.}$
	$360 \quad 00$
Ship's longitude	$163 \quad 02 \text{ E.}$

EXAM. VI. *Seven days ago my longitude was $172^{\circ} 17' \text{ W.}$ and I have made each day 132 miles of west longitude : Required my present longitude?*

132	Departed long.	$172^{\circ} 17' \text{ W.}$
7	Diff. long.	$15 \quad 24 \text{ W.}$
6,0)92,4		$187 \quad 41$
15 24		$360 \quad 00$
	Present long.	$172 \quad 19 \text{ E.}$

S E C T I O N V.

54.

Of the Use of the Globes.

By the globes are here meant two spherical bodies, called the Terrestrial and Celestial Globes, the convex surfaces of which are supposed to give a true representation of the earth and heavens.

The TERRESTRIAL GLOBE has delineated on its convexity the whole surface of the *earth* and *sea* in their relative size, form, and situation.

The CELESTIAL GLOBE has drawn on its surface the images of the several constellations and stars; the relative magnitude and position which the stars are observed to have in the heavens, being preserved on this globe.

The globes are fitted up with certain machinery, by means of which a great variety of useful problems are neatly solved.

The BRAZEN MERIDIAN is that ring, or hoop, in which the globe hangs on its axis; which is represented by two wires passing through its poles. This circle is divided into four quarters, of 90° each; in one semicircle the divisions begin at each pole, and end at 90° , where they meet. In the other semicircle, the divisions begin at the middle, and proceed thence toward each pole, where they end at 90 degrees. The graduated side of this brazen circle serves as a meridian for any point on the surface of the earth, the globe being turned about till that point comes under the circle.

The HOUR CIRCLE is a small circle of brass, which is divided into 24 hours, the quarters, and half quarters. It is fixed on the brazen meridian, equally distant from the north end of the axis, to which an index is fitted, that points out the divisions of the hour circle as the globe is turned about.

The HORIZON is represented by the upper surface of the wooden circular frame encompassing the globe about its middle. On this wooden frame is a kind of perpetual calendar, contained in several concentric circles: The inner one is divided into four quarters, of 90 degrees each: the next circle is divided into the twelve months, with the days in each, according to the new style; the next contains the 12 equal signs of the zodiac, each being divided into 30 degrees: the next is the 12 months and days, according to the old style; and there is another circle, containing the 32 winds, with their halves and quarters. Although these circles are on all horizons, yet their disposition is not always the same.

The QUADRANT of ALTITUDE is a thin straight slip of brass, one edge of which is graduated into 90 degrees and their quarters, equal to those of the meridian. To one end of this is fixed a brass nut and screw, by which it is put on, and fastened to the meridian: and if it is fixed to the zenith or pole of the horizon, then the graduated edge represents a vertical circle passing through any point.

Besides these, there are several circles described on the surfaces of both globes; such as the equinoctial, ecliptic, circles of longitude and right ascension, the tropics, polar circles, parallels of lat. and decl. on the celestial globe; and on the terrestrial, the equator, ecliptic, tropics, polar circles, parallels of latitude, hour circles, or meridians to every 15 degrees, and the spiral rhumbs flowing from several centers, called Flies.

P R O B L E M I.

55.

To find the latitude and longitude of any place on the terrestrial globe.

- 1st. Bring the given place under that side of the graduated brazen meridian where the degrees begin at the equator, by turning the globe about.
- 2d. Then the degree of the meridian over it shews the latitude.
- 3d. And the degree of the equator under the merid. shews the long.

On some globes the longitude is reckoned on the equator from the meridian where it begins, eastward only, until it ends at 360° : On such globes, when the longitude of a place exceeds 180° , take it from 360, and call the remainder the longitude westward.

P R O B L E M II.

56.

To find any place on the globe, the latitude and longitude of which are given.

- 1st. Bring the given longitude, found on the equator, to the meridian.
- 2d. Then under the given latitude, found on the meridian, is the place sought.

P R O B L E M III.

57.

To find the distance and bearing of any two given places on the globe.

- 1st. Lay the graduated edge of the quadrant of altitude over both places, the beginning, or 0 degree, being on one of them, and the degrees between them shew their distance; these degrees, multiplied by 60, give sea miles, and by 70 give the distance in land miles nearly; or multiplied by 20, give leagues.

2d. Observe, while the quadrant lies in this position, what rhumb of the nearest fly, or compass, runs mostly parallel to the edge of the quadrant, and that rhumb shews the bearing sought, nearly.

P R O B L E M IV.

58.

To find the Sun's place and declination on any day.

- 1st. Seek the given day in the circle of months on the horizon, and right against it in the circle of signs is the Sun's place.

Thus it will be found that the Sun enters

The spring signs, Aries, March 20. Taurus, April 20. Gemini, May 21.

The summer signs, Cancer, June 21. Leo, July 23. Virgo, Aug. 23.

Autumnal signs, Libra, Sept. 22. Scorpio, Oct. 23. Sagittar. Nov. 22.

The winter signs, Capric. Dec. 21. Aquarius, Jan. 20. Pisces, Feb. 18.

- 2d. Seek the Sun's place in the ecliptic on the globe, bring that place to the meridian, and the division it stands under is the Sun's declination on the given day.

On the globes, the ecliptic is readily distinguished from the equator, not only by the different colours they are stained with, but also by the ecliptic's approaching toward the poles, after its intersection with the equator. The marks of the signs are also put along the ecliptic, one at the beginning of every successive 30 degrees.

59.

P R O B L E M V.

To rectify the globe for the latitude, zenith, and noon.

1st. Set the globe upon an horizontal plane with its parts answering to those of the world; move the meridian in its notches, by raising or depressing the pole, until the degrees of latitude cut the horizon; then is the globe rectified for the latitude.

2d. Reckon the latitude from the equator towards the elevated pole, there screw the bevil edge of the nut belonging to the quadrant of altitude, and the rectification is made for the zenith.

3d. Bring the Sun's place (found by the last problem) to the meridian, set the index to the XII at noon, or upper XII, and the globe is rectified for the Sun's southing, or noon.

60.

P R O B L E M VI.

To find where the Sun is vertical, at any given time, in a given place.

1st. Bring the Sun's place, found for the given day (58), to the meridian, and note the degree over it.

2d. Bring the place, for which the time is given, to the meridian, and set the index to the given hour.

3d. Turn the globe till the index comes to 12 at noon, then the place under the said noted degree has the Sun in the zenith at that time.

4th. All the places that pass under that degree, while the globe is turned round, will have the Sun vertical to them on that day.

61.

P R O B L E M VII.

To find on what days the Sun will be vertical, at any given place, in the torrid zone.

1st. Note the latitude of the given place on the meridian.

2d. Turn the globe, and note what two points of the ecliptic pass under the latitude noted on the meridian.

3d. Seek those points of the ecliptic in the circle of signs on the horizon, and right against them, in the circle of months, stand the days required.

In this manner it will be found, that the Sun will be vertical to the Island of St. Helena on the 6th of November, and on the 4th of February. And at Barbadoes on the 24th of April, and the 18th of August.

62.

P R O B L E M VIII.

At any given hour in a given place, to find what hour it is in any other place.

1st. Bring the place where the time is given to the meridian, and set the index to the given hour.

2d. Bring the other given place to the meridian, and the index shews the hour corresponding to the given time.

63. PRO

P R O B L E M IX.

63.

At any given time to find all those places of the Earth where the Sun is then rising or setting, and where it is mid-day or midnight.

Find the place where the Sun is vertical at the given time (60), rectify the globe for the latitude of that place, and bring it to the meridian.

Then all those places, that are in the western half of the horizon, have the Sun rising; and those in the eastern half have it setting.

Those under the meridian, above the horizon, have the Sun culminating, or noon; and those under the meridian, below the horizon, have midnight.

Those above the horizon have day; those below it have night.

P R O B L E M X.

64.

To find the angle of position of two places, or the angle made by the meridian of one place, and a great circle passing through both places.

Rectify for the latitude of one of the given places, and bring it to the meridian; there fix the quadrant of altitude, and set its graduated edge to the other place: then will that edge of the quadrant cut the horizon in the degree of position sought.

Thus, the angle of position at the Land's End to Barbadoes is south $71\frac{1}{2}^{\circ}$ westerly: but the angle of position at Barbadoes to the Land's End is north $37\frac{1}{2}^{\circ}$ easterly.

Hence neither of those positions can be the true bearing; for the rhumb passing through both places, will be opposite one way to what it is the other.

P R O B L E M XI.

65.

The latitude of any place not within the polar circle being given, to find the time of Sun-rising and setting, and the length of the day and night.

Rectify for the latitude and the noon; bring the Sun's place to the eastern side of the horizon, and the index shews the time of rising: the Sun's place being brought to the western side of the horizon, the index gives the setting.

Or, the time of rising taken from 12 hours gives the time of setting.

The time of setting being doubled gives the length of the day.

And the time of rising being doubled gives the length of the night.

Thus, at London, on April 15th, the day is $13\frac{1}{2}$ hours; the night $10\frac{1}{2}$ hours.

P R O B L E M XII.

66.

To find the length of the longest and shortest days in any given place.

Rectify for the latitude; bring the solstitial point of that hemisphere to the eastern part of the horizon, set the index to 12 at noon, turn the globe till the solstitial point comes to the western side of the horizon, the hours past over by the index give the length of the longest day or night; and its complement to 24 hours gives the length of the shortest night or day.

67.

P R O B L E M XIII.

A place being given in either frigid or frozen zone, to find the time when the Sun begins to appear at, or depart from, that place: also how many successive days he is present to, or absent from, that place.

Rectify for the latitude, turn the globe, and observe what degrees in the first and second quadrants of the ecliptic are cut by the north point of the horizon, the latitude being supposed to be north.

Find those degrees in the circle of signs on the horizon, and their corresponding days of the month; and all the time between those days the Sun will not set in that place.

Again: Observe what degree in the third and fourth quadrants of the ecliptic will be cut by the south point of the horizon, and the days answering: then the Sun will be quite absent from the given place during the intermediate days; that day in the third quadrant shews when he begins to disappear; and that in the fourth quadrant shews when he begins to shine in the place proposed.

Thus at the North cape, in lat. 71° , the Sun never sets from May 15 to July 28, which is 74 days; and never rises from November 16 to January 24, which is 69 days.

68.

P R O B L E M XIV.

To find the antæci, peræici, and antipodes of any place.

Bring the given place to the meridian, tell as many degrees of latitude on the contrary side of the equator, and it gives the place of the antæci; that is, of those who have opposite seasons of the year, but the same times of the day.

The given place being under the meridian, set the index to 12 at noon, turn the globe until the index points to 12 at night, and the point under the meridian in the given latitude is the place of the peræci; that is, of those who have the same seasons of the year, but opposite times of the day.

The globe remaining in this position, seek on the contrary side of the equator for the degrees of latitude given, and the point under the meridian, thus found, will be the antipodes to the given place; that is, there the seasons of the year and times of the day are directly opposite to those of the given place.

69.

P R O B L E M XV.

To find the beginning and end of the twilight in any place.

Rectify the globe for the latitude, zenith, and noon.

(59)

Seek the point of the ecliptic opposite to the Sun's place, turn the globe and quadrant of altitude, till the said opposite point of the ecliptic stands against 18 degrees on the quadrant of altitude; then will the index shew the beginning or end of the twilight; that is, the beginning in the morning, when those points meet in the western hemisphere; or the end in the evening, when the said points meet in the eastern hemisphere.

70. P R O-

P R O B L E M XVI.

70.

The latitude of a place and day of the month being given, to find the Sun's declination, meridian altitude, right ascension, amplitude, oblique ascension, ascensional difference; and thence the time of rising, setting, length of the day and night.

Rectify for the latitude and noon. Then,

The degree of the meridian over the Sun's place is the declination.

The meridian altitude is shewn by the degrees the Sun is above the horizon; and is equal to the sum or diff. of the co-lat. and decl.

The Sun's right ascen. is that degree of the equator under the meridian.

Bring the Sun's place to the eastern part of the horizon. Then,

The amplitude is that degree of the horizon opposite the Sun.

The oblique ascension is that degree of the equator cut by the horizon.

The ascen. diff. is the diff. between the right and oblique ascensions.

The ascen. diff. converted into time, will give the time the Sun rises before or after the hour of six, according as his amplitude is to the northward or southward of the east point of the horizon.

P R O B L E M XVII.

71.

Given the latitude of the place and day of the month, to find the Sun's altitude and azimuth, either when he is due east or west, at 6 o'clock, or at any other hour while he is above the horizon.

Rectify the globe for the latitude, zenith, and noon.

Set the quadrant of altitude to the east point of the horizon, turn the globe till the Sun's place comes to the quadrant's edge, and it shews the altitude, his azimuth being now 90° , and the index shews the hour.

Turn the globe till the index points at 6, there stay it, and move the quadrant until its edge cuts the Sun's place; then the degrees at the Sun shew its altitude, and the degrees cut by the quadrant in the horizon shew the azimuth, reckoning from the north.

In like manner, the globe being turned till the index is against any other hour, suppose 10 in the forenoon; then,

The graduated edge of the quadrant of altitude being turned to cut the Sun's place, will give both the altitude and azimuth at that time.

P R O B L E M XVIII.

72.

Given the latitude, day of the month, and Sun's altitude, to find the azimuth and hour of the day.

Rectify the globe for the latitude, zenith, and noon.

Turn the globe and quadrant, until the Sun's place coincide with the altitude on the graduated edge of the quadrant.

Then will that edge of the quadrant cut the degrees of azimuth on the horizon, reckoned from the north; and the index will shew the hour of the day.

73.

P R O B L E M X I X.

To represent the appearance of the heavens at any time in a given place.

Rectify the celestial globe for the latitude, zenith, and noon, and turn the globe till the index points at the given hour. Then,

The stars in the eastern half of the horizon are rising; those in the western are setting: and those on the meridian are culminating.

The quadrant being set to any given star will shew its altitude, and at the same time its azimuth, reckoned on the horizon.

Now by turning the globe round it will readily appear, what stars never set in that place, and what never rise: those of perpetual apparition never go below the horizon, those of perpetual absence never come above it.

74.

P R O B L E M X X.

To find the latitude and longitude of any star.

Put the center of the quadrant of altitude on the pole of the ecliptic, and its graduated edge on the given star. Then,

The latitude is shewn by the degrees between the ecliptic and star.

The longitude is the degrees cut on the ecliptic by the quadrant.

75.

P R O B L E M X X I.

To find the declination and right ascension of a star.

Bring the star to the meridian, the degree over it is the declination; and the degree of the equator under the meridian is the right ascension.

76.

P R O B L E M X X I I.

On any day, and in any given place, to find when a proposed star rises, sets, or culminates.

Rectify the globe for the latitude and noon.

Bring the star to the eastern side of the horizon, and the index shews the time of its rising.

Turn the globe till the star comes to the meridian, and the index shews the time of its culminating; and in like manner when it sets, the time will be shewn by the index.

Its meridian altitude, oblique ascension, and ascensional difference, are found in the same manner as for the Sun, at Art. 70.

S E C T I O N VI.

Of Winds.

77. A FLUID is a body, the particles of which readily give way to any impressed force; and by this readiness of yielding, the particles are easily put into motion.

Thus, not only liquids, but streams or vapours, smoke or fumes, and others of the like kind, are reckoned as fluids.

From all parts of the Earth vapours and fumes are constantly arising to some distance from its surface.

This is known by observation; it is caused chiefly by the heat of the Sun, and sometimes by subterraneous fires arising from the accidental mixing of some bodies.

78. AIR is a fine invisible fluid surrounding the globe of the Earth, and extended to some miles above its surface.

The ATMOSPHERE is that collection of air, and of bodies contained in it, which circumscribes the Earth.

79. From a multitude of experiments, air is found to be both heavy and springy.

By its weight it is capable of supporting other bodies, such as vapours and fumes, in the same manner as wood is supported by water.

By its springiness or elasticity a quantity of air is capable of being expanded, or of spreading itself so as to fill a larger space *; and of being compressed or confined in a smaller compass †.

80. Air is compressed or condensed by cold, and expanded or rarefied by heat.

This is evident from a multitude of experiments.

An alteration being made by heat or cold in any part of the atmosphere, its neighbouring parts will be put in motion, by the endeavour which the air always makes to restore itself to its former state.

For experiments shew, that condensed or rarefied air will return to its natural state, when the cause of that condensation or rarefaction is removed.

81. WIND is a stream or current of air which may be felt; it usually blows from one part of the horizon to its opposite part.

82. The *horizon*, beside being divided into 360 degrees, like all other circles, is by mariners supposed to be divided into four *quadrants*, called the north-east, north-west, south-east, and south-west quarters; each of these quarters they divide into eight equal parts, called points, and each point into four equal parts, called quarter-points.

So that the horizon is divided into 32 points, which are called *rhumbs*, or *winds*; to each wind is assigned a name, which shews from what point of the horizon the wind blows.

The points of North, South, East, and West, are called *cardinal points*; and are at the distance of 90 degrees, or 8 points, from one another.

* Near 14000 times. *Wallis's Hydrosf.* p. 13.

† Into the $\frac{1}{8}$ part. *Phil. Trans.* N^o 181.

83. Winds are either constant or variable, general or particular.

Constant winds are such as blow the same way, at least for one or more days; and *variable winds* are such as frequently shift within a day.

A *general wind* is that which blows the same way over a large tract of the Earth, almost the whole year.

A *particular wind* is that which blows in any place, sometimes one way, and sometimes another, indifferently.

If the wind blows gently, it is called a breeze; if it blows harder, it is called a gale, or a *ft. T* gale; and if it blows very hard, it is called a storm*.

84. The following observations on the wind have been made by skillful seamen; and particularly by the great Dr. Halley.

1st. Between the limits of 60 degrees, namely, from 30° of north latitude to 30° of south latitude, there is a constant easterly wind throughout the year, blowing on the Atlantic and Pacific Oceans; and this is called the *trade-wind*.

For as the Sun, in moving from east to west, heats the air more immediately under him, and thereby expands it; the air to the eastward is constantly rushing toward the west to restore the equilibrium, or natural state of the atmosphere; and this occasions a perpetual easterly wind in those latitudes.

2d. The trade-winds near the northern limits blow between the north and east; and near the southern limits they blow between the south and east.

For as the air is expanded by the heat of the Sun near the equator; therefore the air from the northward and southward will both tend toward the equator to restore the equilibrium. Now those motions from the north and south, joined with the foregoing easterly motion, will produce the motions observed near the said limits between the north and east, and between the south and east.

3d. These general motions of the wind are disturbed on the continents, and near their coasts.

For the nature of the soil may cause the air to be either heated or cooled; and hence will arise motions that may be contrary to the foregoing general one.

4th. In some parts of the Indian ocean there are periodical winds, called *MONSOONS*; that is, such as blow one half the year one way, and the other half-year the contrary way.

For air that is cool and dense, will force the warm and rarefied air in a continual stream upwards, where it must spread itself to preserve the equilibrium: so that the upper course or current of the air shall be contrary to the under current; for the upper air must move from those parts where the greatest heat is; and so, by a kind of circulation, the N. E. trade-wind below will be attended with a S. W. above; and a S. E. below with a N. W. above: and this is confirmed by the experience of seamen, who, as soon as they get out of the trade-winds, immediately find a wind blowing from the opposite quarter.

* The swiftness of the wind in a storm is not more than 50 or 60 miles in an hour; and a common brisk gale is about 15 miles an hour.

5th. In the Atlantic ocean near the coasts of Africa, at about 100 leagues from the shore, between the latitudes of 28° and 10° north, seamen constantly meet with a fresh gale of wind blowing from the N. E.

6th. Those bound to the Caribbee Islands, across the Atlantic ocean, find, as they approach the American side, that the said N. E. wind becomes easterly; or seldom blows more than a point from the east, either to the northward or southward.

These trade-winds, on the American side, are extended to 30 , 31 , or even to 32° of N. latitude; which is about 4° farther than what they extend to on the African side. Also to the southward of the equator, the trade-winds extend 3 or 4 degrees farther toward the coast of Brasil on the American side, than they do near the Cape of Good Hope on the African side.

7th. Between the latitudes of 4° North, and 4° South, the wind always blows between the south and east: on the African side the winds are nearest to the south; and on the American side nearest to the east. In these seas Dr. Halley observed, that when the wind was eastward, the weather was gloomy, dark, and rainy, with hard gales of wind; but when the wind veered to the southward, the weather generally became serene, with gentle breezes next to a calm.

These winds are somewhat changed by the seasons of the year; for when the Sun is far northward, the Brasil S. E. wind gets to the south, and the N. E. wind to the east; and when the Sun is far south, the S. E. wind gets to the east, and the N. E. winds on this side of the equator veer more to the north.

8th. Along the coast of Guinea, from Sierra Leon to the Island of St. Thomas (under the equator) which is above 500 leagues, the southerly and south-west winds blow perpetually. For the S. E. trade-wind having passed the equator, and approaching the Guinea coast within 80 or 100 leagues, inclines toward the shore, and becomes south, then S. E. and by degrees, as it comes near the land, it veers about to south, S. S. W. and in with the land it is S. W. and sometimes W. S. W. This tract is troubled with frequent calms, violent sudden gusts of wind, called Tornadoes, blowing from all points of the horizon.

The reason why the wind sets in west on the coast of Guinea, is, in all probability, owing to the nature of the coast, which being greatly heated by the Sun, rarefies the air exceedingly; and consequently the cool air from off the sea will keep rushing in to restore the equilibrium.

9th. Between the 4th and 10th degrees of north latitude, and between the longitudes of Cape Verd, and the easternmost of the Cape Verd isles, there is a tract of sea which seems to be condemned to perpetual calms, attended with terrible thunder and lightnings, and such frequent rains, that this part of the sea is called the *Rains*. Ships in sailing these 6 degrees have been sometimes detained whole months, as it is reported.

The cause of this seems to be, that the westerly winds setting in on this coast, and meeting the general easterly wind in this tract, balance each other, and so cause the calms; and the vapours carried thither by each wind meeting and condensing, occasion the almost constant rains.

The last three observations shew the reason of two things which mariners experience in sailing from Europe to India, and in the Guinea trade.

First. The difficulty which ships in going to the southward, especially in the months of July and August, find in passing between the coast of Guinea and Brasil, notwithstanding the width of this sea is more than 500 leagues. This happens, because the S. E. winds at that time of the year commonly extend some degrees beyond the ordinary limits of 4° N. latitude; and besides come so much southerly, as to be sometimes south, sometimes a point or two to the west; it then only remains to ply to windward. And if, on the one side, they steer W. S. W. they get a wind more and more easterly; but then there is a danger of falling in with the Brasilian coast, or shoals; and if they steer E. S. E. they fall into the neighbourhood of the coast of Guinea, from whence they cannot depart without running easterly as far as the island of St. Thomas; and this is the constant practice of all the Guinea ships.

Secondly. All ships departing from Guinea for Europe, their direct course is northward; but on this course they cannot go, because the coast bending nearly east and west, the land is to the northward: therefore as the winds on this coast are generally between the S. and W. S. W. they are obliged to steer S. S. E. or south, and with these courses they run off the shore; but in so doing they always find the winds more and more contrary; so that when near the shore, they can lie south, at a greater distance they can make no better than S. E. and afterwards E. S. E.; with which courses they commonly fetch the island of St. Thomas and Cape Lopez, where finding the winds to the eastward of the south, they sail westerly with it, until they come to the latitude of 4 degrees south, where they find the S. E. wind blowing perpetually.

On account of these general winds, all those who use the West India trade, even those bound to Virginia, reckon it their best course to get as soon as they can to the southward, that so they may be certain of a fair and fresh gale to run before to the westward. And for the same reason the homeward-bound ships from America endeavour to gain the latitude of 30 degrees, where they first find the winds begin to be variable; though the most ordinary winds in the north Atlantic ocean come from between the south and west.

10th. Between the southern latitudes of 10 and 30 degrees in the Indian ocean, the general trade-wind about the S. E. by S. is found to blow all the year long in the same manner, as in the like latitude in the Ethiopic ocean: and during the six months from May to December, these winds reach to within 2 degrees of the equator; but during the other six months, from November to June, a N. W. wind blows in the tract lying between the 3d and 10th degrees of southern latitude, in the meridian of the north end of Madagascar: and between the 2d and 12th degree of south latitude, near the longitude of Sumatra and Java.

11th. In the tract between Sumatra and the African coast, and from 3 degrees of south latitude quite northward to the Asiatic coasts, including the Arabian sea and the Gulf of Bengal, the Monsoons blow from September to April on the N. E. ; and from March to October on the S. W. In the former half year the wind is more steady and gentle, and the weather clearer than in the latter six months ; and the wind is more strong and steady in the Arabian sea than in the Gulf of Bengal.

12th. Between the Island of Madagascar and the coast of Africa, and thence northward as far as the equator, there is a tract, where from April to October there is a constant fresh S. S. W. wind ; which to the northward changes into the W. S. W. wind, blowing at that time in the Arabian sea.

13th. To the eastward of Sumatra and Malacca, on the north of the equator, and along the coasts of Cambodia and China, quite through the Philippines as far as Japan, the Monsoons blow northerly and southerly, the northern setting in about October or November, and the southern about May ; the winds are not quite so certain as those in the Arabian seas.

14th. Between Sumatra and Java to the west, and New Guinea to the east, the same northerly and southerly winds are observed ; but the first half year Monsoon inclines to the N. W. and the latter to the S. E. These winds begin a month or six weeks after those in the Chinese seas set in, and are quite as variable.

15th. These contrary winds do not shift from one point to its opposite all at once ; in some places the time of the change is attended with calms, in others by variable winds. And it often happens on the shores of Coromandel and China towards the end of the Monsoons, that there are most violent storms, greatly resembling the hurricanes in the West Indies ; when the wind is so vastly strong, that hardly any thing can resist its force.

All navigation in the Indian ocean must necessarily be regulated by these winds ; for if mariners should delay their voyages till the contrary Monsoon begins, they must either sail back, or go into harbour, and wait for the return of the trade-wind.

SECTION VII.

Of the Tides.

85. A **TIDE** is that motion of the waters in the seas and rivers, by which they are found regularly to rise and fall.

The general cause of the tides was discovered by Sir Isaac Newton, and is deduced from the following considerations :

86. Daily experience shews, that all bodies thrown upward from the Earth, fall down to its surface in perpendicular lines ; and as lines perpendicular to the surface of a sphere tend toward the center, therefore the lines, along which all heavy bodies fall, are directed toward the Earth's center.

As these bodies apparently fall by their weight, or gravity ; therefore the law by which they fall, is called the **LAW OF GRAVITATION**.

87. A piece of glass, amber, or sealing-wax, and some other things, being rubbed against the palm of a hand, or against a woollen cloth, until they are warmed, will draw bits of paper or other light substances, towards them, when held sufficiently near those substances.

Also a magnet, or loadstone, being held near the filings of iron or steel, or other small pieces of these metals, will draw them to itself ; and a piece of hammered iron or steel, that has been rubbed by a magnet, will have a like property of drawing iron or steel to itself. And this property is called **ATTRACTION**.

88. Now as bodies by their gravity fall toward the Earth, it is not improper to say the Earth attracts those bodies ; and therefore in respect to the Earth, the words gravitation and attraction may be used one for the other, as by them is meant no more than the power, or law, by which bodies tend toward its center.

And it is likely, that this is the cause why the parts of the Earth adhere and keep close to one another.

89. The incomparable Sir Isaac Newton, by a sagacity peculiar to himself, discovered from many observations, that this law of gravitation or attraction was universally diffused throughout the solar system ; and that the regular motions observed among the heavenly bodies were governed by this same principle ; so that the Earth and Moon attracted each other, and both of them are attracted by the Sun. He discovered also that the force of attraction, exerted by these bodies one on the other, was less and less as the distance increased, in proportion to the squares of those distances ; that is, the power of attraction at double the distance was four times less, at triple the distance nine times less, at quadruple the distance sixteen times less, and so on.

90. Now as the Earth is attracted by the Sun and Moon, therefore all the parts of the Earth will not gravitate toward its center in the same manner, as if those parts were not affected by such attractions. And it is very evident that, were the Earth entirely free from such actions of the Sun and Moon, the ocean being equally attracted toward its center, on all sides, by the force of gravity, would continue in a perfect stagnation without ever ebbing or flowing. But since the case is otherwise, the water in the ocean must needs rise higher in those places where the Sun and Moon

Moon diminish its gravity, or where the Sun and Moon have the greatest attraction.

As the force of gravity must be diminished most in those parts of the Earth to which the Moon is nearest, that is, where she is in the **ZENITH**, or vertical, and, consequently, where her attraction is most powerful; therefore the waters in such places will rise highest, and it will be *full sea* or *flood* in such places.

91. *The parts of the Earth directly under the Moon, and also those in her NADIR, viz. such places as are diametrically opposite to those where the Moon is in the Zenith, will have the flood, or high water, at the same time.*

For either half of the Earth would gravitate equally toward the other half, were they free from all external attraction.

But by the action of the Moon, the gravitation of one half-earth toward its center is diminished, and of the other is increased.

Now in the half-earth next the Moon, the parts in the zenith being most attracted, and thereby their gravitation toward the Earth's center diminished, the waters in these parts must be higher than in any other part of this half-earth.

And in the half-earth farthest from the Moon, the parts in the nadir being less attracted by the Moon than the parts nearer to her, gravitate less toward the Earth's center, and consequently the waters in these parts must be higher than they are in any other part of this half-earth.

92. *Those parts of the Earth, where the Moon appears in the horizon, or is 90 degrees distant from the zenith and nadir, will have the ebbs or lowest waters.*

For as the waters in the zenith and nadir rise at the same time, the waters in their neighbourhood will press toward those places to maintain the equilibrium; and to supply the places of these, others will move the same way, and so on to places of 90° distant from the said zenith and nadir; consequently, in those places where the Moon appears in the horizon, the waters will have more liberty to descend toward the center; and therefore in those places they will be the lowest.

93. Hence it plainly follows, that the ocean, if it covered the surface of the Earth, must put on a spheroidal, or egg-like figure; in which the longest diameter passes through the place where the Moon is vertical; and the shortest diameter will be in the horizon of that place. And as the Moon apparently shifts her position from east to west in going round the Earth every day, the longer diameter of the spheroid following her motion, will occasion the two floods and ebbs observable in about every 25 hours, which is about the length of a lunar day, or the time spent between the Moon's leaving the meridian of any place, and coming to it again.

94. Hence the greater the Moon's meridian altitude is at any place, the greater those tides will be which happen when she is above the horizon; and the greater her meridian depression is, the greater will those tides be which happen while she is below the horizon.

Moreover the summer day, and the winter night-tides have a tendency to be highest, because the Sun's summer altitude and his winter depression are greatest; but this is more especially to be noted when the

Moon

Moon has north declination in summer, and south declination in winter.

95. *The time of high water is not precisely at the time of the Moon's coming to the meridian, but about an hour after.*

For the moon acts with some force after she has past the meridian, and by that means adds to the libratory, or waving motion, which she had put the water into, whilst she was in the meridian; in the same manner as a small force applied upward to a ball, already raised to some height, will raise it still higher.

96. *The tides are greater than ordinary twice every month; that is, about the times of new and full Moon: these are called SPRING-TIDES.*

For at these times the actions of both Sun and Moon concur to draw in the same right line; and therefore the sea must be more elevated. In conjunction, or when the Sun and moon are on the same side of the Earth, they both conspire to raise the water in the zenith, and consequently in the nadir. And when the Sun and Moon are in opposition, that is, when the Earth is between them, whilst one makes high water in the zenith and nadir, the other does the same in the nadir and zenith.

97. *The tides are less than ordinary twice every month; that is, about the times of the first and last quarters of the Moon: and these are called NEAP TIDES.*

Because in the quarters of the Moon the Sun raises the water where the Moon depresses it; and depresses where the Moon raises the water; so that the tides are made only by the difference of their actions.

It must be observed, that the spring tides happen not directly on the new and full moons, but rather a day or two after, when the attractions of the Sun and Moon have conspired together for a considerable time. In like manner the neap-tides happen a day or two after the quarters, when the Moon's attraction has been lessened by the Sun for several days together.

98. *When the Moon is in her PERIGÆUM, or nearest approach to the Earth, the tides increase more than in the same circumstances at other times.*

For according to the laws of gravitation, the Moon must attract most when she is nearest to the Earth.

99. *The spring-tides are greater about the time of the EQUINOXES, that is, about the latter ends of March and September, than at other times of the year; and the neap-tides then are less.*

Because the longer diameter of the spheroid, or the two opposite floods, will at that time be in the Earth's equator; and consequently will describe a great circle of the Earth; by the diurnal rotation of which those floods will move swifter, describing a great circle in the same time they used to describe a lesser circle parallel to the equator, and consequently the waters being thrown more forcibly against the shores, must rise higher.

100. The following observations have been made on the rise of the tides:

1st. The morning tides generally differ in their rise from the evening tides

2d. The new and full Moon spring tides rise to different heights.

3d. In winter the morning tides are highest.

4th. In

4th. In summer the evening-tides are highest.

So that after a period of about six months the order of the tides is inverted; that is, the rise of the morning and evening-tides will change places, the winter morning high-tides becoming the summer evening high-tides.

Some of these effects arise from the different distances of the Moon from the Earth after a period of six months, when she is in the same situation with respect to the Sun; for, if she is in perigee at the time of new Moon, in about six months after she will be in perigee about the time of full Moon.

These particulars being known, a pilot may chuse that time, which is most convenient for conducting a ship in or out of a port, where there is not sufficient depth at low-water.

Small inland seas, such as the Mediterranean and Baltic, are little subject to tides; because the action of the Sun and Moon is always nearly equal at both extremities of such seas. In very high latitudes the tides are also very inconsiderable. For the Sun and Moon acting toward the equator, and always raising the water toward the middle of the torrid zone, the neighbourhood of the poles must consequently be deprived of those waters, and the sea must, within the frigid zones, be low, with relation to other parts.

101. All the things hitherto explained would exactly obtain, were the whole surface of the Earth covered with sea. But since it is not so, and there being a multitude of islands, besides continents, lying in the way of the tide, which interrupt its course; therefore in many places near the shores, there arises a great variety of other appearances beside the foregoing ones which require particular solutions, in which the situations of the shores, straits, shoals, winds, and other things, must necessarily be considered. For instance:

102. As the sea * has no visible passage between Europe and Africa, let them be supposed to be one continent, extending from 78 degrees north to 34 degrees south, the middle between these two would be in latitude 19 degrees north, near Cape Blanco, on the west coast of Africa. But it is impossible that the flood-tide should set to the westward upon the western coast of Africa (for the general tide following the course of the Moon must set from east to west), because the continent, for above 50 degrees, both northward and southward, bounds that sea on the east; therefore if any regular tide, proceeding from the motion of the sea, from east to west, should reach this place, it must come either from the north of Europe southward, or from the south of Africa northward, to the said latitude upon the west coast of Africa.

103. This opinion is further corroborated, or rather fully confirmed by common experience, which shews that the flood-tide sets to the southward along the west coast of *Norway*, from the north cape to the *Naze*, or entrance of the *Baltic Sea*, and so proceeds to the southward along the east coast of *Great Britain*, and in its passage supplies all those ports with the tide one after another, the coast of *Scotland* having the tide first, be-

* *Varenius's Geogr.* p. 813. Edit. 1734.

cause it proceeds from the northward to the southward; and thus on the days of the full, or change, it is high-water at *Aberdeen* at 12h. 45m. but at *Tinmouth Bar*, the same day, not till 3h. From thence rolling to the southward, it makes high-water at the *Spurn* a little after 5h.; but not till 6h. at *Hull*, by reason of the time required for its passage up the river; from thence passing over the *Welbank* into *Yarmouth Road*, it makes high-water there a little after 8h. but in the *Pier* not till 9h., and it requires near an hour more to make high-water at *Yarmouth* town; in the mean time setting away to the southward; it makes high-water at *Harwich* at 10h. 30m., at the *Nore* at 12, at *Gravesend* 1h. 30m., and at *London* at 3h. all on the same day. And although this may seem to contradict that hypothesis of the natural motion of the tide being from east to west, yet as no tide can flow west from the main continent of *Norway* or *Holland*, or out of the *Baltic*, which is surrounded by the main continent, except at its entrance, it is evident that the tide we have been now tracing by its several stages from *Scotland* to *London*, is supplied by the tide, the original motion of which is from east to west; yet as water always inclines to the level, it will in its passage fall toward any other point of the compass, to fill up vacancies where it finds them, and yet not contradict, but rather confirm, the first hypothesis.

104. While the tide, or high-water, is thus gliding to the southward along the east coast of *England*, it also sets to the southward along the west coasts of *Scotland* and *Ireland*, a branch of it falls into *St. George's* channel, the flood running up north-east, as may be naturally inferred from its being high-water at *Waterford* above three hours before it is high-water at *Dublin*, or thereabout, on that coast; and it is three quarters of an hour ebb at *Dublin* before it is high-water at the *Isle of Man*, &c.

But not to proceed farther in particulars than to our own, or the *British* channel, we find the tides set to the southward from the coast of *Ireland*, and in their passage a branch falls into the *British* channel between the *Lizard* and *Ushant*; this progress to the southward may be easily proved, by its being high-water on the day of the full and changes at *Cape Clear* a little after 4h., and at *Ushant* about 6h., and at the *Lizard* after 7h. The *Lizard* and *Ushant* may properly be called the chaps of the *British* channel, between which the flood sets to the eastward along the coasts of *England* and *France* till it comes to the *Goodwin* or *Galloper*, where it meets the tide before mentioned, which sets to the southward along the eastern coast of *England* to the *Downs*, where these two tides meeting, contribute very much towards sending a powerful tide up the river *Thames* to *London*. And when the natural course of these two tides has been interrupted by a sudden shift of the wind, by which means that tide was accelerated which had before been retarded, and that driven back which was before hurried in by the wind, it has been known to occasion twice high-water in 3 or 4 hours, which, by those who did not consider this natural cause, was looked upon as a prodigy.

105. But now it may be objected, that this course of the flood-tide, east, or east-north-east, up the channel, is quite contrary to the hypothesis of the general motion of the tides being from east to west, and consequently of its being high-water where the moon is vertical, or any where else in the meridian.

In answer. This particular direction of any branch of the tide does not at all contradict the general direction of the whole; a river, with a western course, may supply canals which wind north, south, or even east, and yet the river keep its natural course; and if the river ebbs and flows, the canals supplied by it would do the same, although they did not keep exact time with the river, because it would be flood, and the river advanced to some height, before the flood reached the farther part of the canals; and the more remote the canals are, the longer time it would require; and it may be added, that if it was high-water in the river just when the Moon was on the meridian, she would be far past it before it could be high-water in the remotest part of those canals, or ditches, and the flood would set according to the course of those canals that received it, and could not set west up a canal of a different position; and as *St. George's* channel, the *British* channel, &c. are no more in proportion to the vast ocean, than these canals are to a large navigable river; it will evidently follow, that among those obstructions and confinements the flood may set upon any other point of the compass as well as west, and may make high-water at any other time as well as when the Moon is upon the meridian, and yet no way contradict the general theory of the tides before asserted.

106. When the time of high-water at any place is, in general, mentioned, it is to be understood on the days of the syzygies, or days of new and full Moon, when the Sun and Moon pass the meridian of that place at the same time. Among pilots it is customary to reckon the time of flood, or high-water, by the point of the compass the Moon is on at that time, allowing $\frac{1}{4}$ of an hour for each point: Thus on the full and change days, in places where it is flood at noon, the tide is said to flow north and south, or at 12 o'clock; in other places, on the same days, where the Moon bears 1, 2, 3, 4, or more points to the east or west of the meridian, when it is high-water, the tide is said to flow on such point: Thus, if the Moon bears S. E. at flood, it is said to flow S. E. and N. W., or 3 hours before the meridian, that is, at 9 o'clock; if it bears S. W., it flows S. W. and N. E., or at 3 hours after the meridian; and in like manner for other points of the Moon's bearing.

107. In some places it is high water on the shore, or by the ground, while the tide continues to flow in the stream, or offing; and according to the length of time it flows longer in the stream than on the shore, it is said to flow tide, and such part of tide; allowing 6 hours to a tide; Thus 3 hours longer in the offing than on the shore, make tide and half-tide; an hour and half longer make tide and quarter-tide; three quarters of an hour longer make tide and half-quarter tide; &c.

108. Along an extent of coast next to the ocean, such as the western coast of Africa, and the eastern and western coasts of South America, it is generally high-water about the same hour. But ports on the coasts of narrow seas, or within land, have the times of their high-water sooner or later on the same day, according as those ports are farther removed from the tide's way, or have their entrances more or less contracted.

109. The times of high-water, in any place, fall about the same hours after a period of 15 days nearly, which is the time between one spring-tide

tide and another: and during that period, the times of high-water fall each day later by about 48 minutes.

110. From the observations of different persons, the times when it is high-water on the days of the new and full Moon, on most of the sea-coasts of Europe, and many other places, have been collected. These times are usually put in a table against the names of the places, digested in an alphabetical order; the like is followed in this work, only they are not given in a table by themselves, but make a column in the table of the latitudes and longitudes of places; which column is not filled up against many of the names in that table, for want of a sufficient collection of observations. This may be supplied by those who have opportunity and inclination.

The use of such a table is to find the time when it is high-water at any of the places mentioned in it. But as this depends upon knowing the time when the Moon comes to the meridian, and this on the Moon's age, and this on the knowledge how to find some of the common notes in the Calendar; therefore it was thought convenient to introduce in this place a compendium of Chronology, containing the several articles above-mentioned.

SECTION VIII.

Of Chronology.

111. CHRONOLOGY is the art of estimating, and comparing together, the times when remarkable events have happened, such as are related in history.

AN ÆRA, or EPOCH, is a time when some memorable transaction occurred; and from which some nations date and measure their computations of time.

	Years of the Julian Period.	Years before Christ.
Some have dated their events from the creation of the world, and suppose it to have happened }	710	4004
Others, from the deluge or flood — — }	2366	2348
The Greeks, from their Olympiads of 4 years each — — — — }	3938	776
The Romans, from the building of Rome — }	3961	753
The Astronomers, from Nabonassar king of Babylon — — — — }	3967	747
Some Historians, from the death of Alexander the Great. — — — — }	4390	324.
The Christians, from the birth of Christ — }	4713	A. D.
The Mahometans, from the flight of Maho- met, called the Hegira — — — }	5335	622

In

In order to assign the distance between these, and other events, the ancients found it necessary to have a large measure of time, the limits of which were naturally pointed out to them by the return of the seasons; and this interval they called a year.

112. The most natural division of the year appeared to them to be the returns of the new Moon; and as they observed 12 new Moons to happen within the time of the general return of the seasons, they therefore first divided the year into 12 equal parts, which they called months; and as they reckoned about 30 returns of morning and evening between the times of the new moon and new moon, therefore they reckoned their month to consist of 30 days, and their year, or 12 months, to contain 360 days; and this is what is generally understood by the lunar year of the ancients.

113. But in length of time it was found, that this year did not agree with the course of the sun, the seasons gradually falling later in the year than they had been formerly observed; this put them upon correcting the method of estimating their year, which they did from time to time, by taking a day or two from the month, as often as they found it too long for the course of the moon; and by adding a month, called an intercalary month, as often as they found 12 lunar months to be too short for the return of the four seasons and fruits of the Earth. This kind of year, so corrected from time to time by the priests, whose business it was, is what is to be understood by the *luni solar* year; which was anciently used in most nations, and is still among the *Arabs* and *Turks*.

As a great variety of methods was used in different countries to correct the length of the year, some by intercalating days in every year, and others by inserting months and days in certain returns, or periods of years; and these different methods being observed by some eminent men to create a considerable difference in the accounts of time kept by neighbouring nations, introducing a confusion in the chronological order of times, they therefore invented certain periods of years, called *cycles*, with which they compared the most memorable occurrences.

114. At length *Julius Cæsar* observing the confusion which this variety of accounts occasioned, and knowing that his order, as Emperor of the Romans, would be followed by a very considerable part of the world; he therefore, about 40 years before the birth of Christ, decreed that every fourth year should consist of 366 days, and the other three of 365 days each. This he did in consequence of the information given him by *Sosigenes*, an eminent mathematician of Alexandria in Egypt; for at that time the philosophers of the Alexandrian school knew from a length of experience, that the year consisted of about 365 days and a quarter; and this was the reason of ordering every fourth year to consist of 366 days, thereby compensating for the quarter day omitted in each of the preceding three years. This method, called the *Julian Account*, or *Old Stile*, continued to be used in most Christian states until the year 1582.

The Astronomers, since the time of *Julius Cæsar*, have found that the true length of the solar year, or common year, is 365 days, 5 hours, 48 minutes, 55 seconds, nearly; being less than the Julian, of 365 days, 6 hours, by about 11 minutes, 5 seconds, which is about the 130th part

part of 86,400, the seconds contained in a day; so that in 130 Julian years there would be one day gained above 130 solar years; and in 400 Julian years there would be gained 3 days, 1 hour, 53 minutes, 20 seconds; consequently one day omitted in every 130 common years, would bring the current account of time to agree very nearly with the motion of the Sun.

115. In the year of our Lord 325, when the *Council of Nice* settled the day for the celebration of Easter, the *Vernal Equinox* (that is, the day in the spring when the Sun rose at six and set at six) happened on the 21st of March; but about the year 1580 the *Vernal Equinox* fell on the 11th of March, making a difference of about 10 days. Now Gregory the XIIIth, who was Pope at that time, observing that this difference of time in the falling out of the *Equinox* would affect the intention of the *Nicene Council*, concerning the time of the year appointed by them for the celebration of Easter; he therefore, in the year 1581, published a *Bull*, ordering that in the year 1582, the 5th of October should be called the 15th, and so on; thus the 10 days taken off would cause the time of the *Vernal Equinox* to fall on the 21st of March, as at the time of the *Nicene Council*: and because a little more than three days were gained in every 400 years by the *Julian* account; therefore to prevent any future difference, every century, or number of 100 years, not divisible by 4, such as 17 hundred, 18 hundred, 19 hundred, &c. should contain only 365 days, which, by the *Julian* account, should have contained 366; and the centuries divisible by 4, such as 16 hundred, 20 hundred, 24 hundred, &c. should be leap-years of 366 days; and thus the three days would be omitted, which the anticipation of the equinoxes would gain in 400 years; the small excess of 1 hour, 53 minutes, 20 seconds, not amounting to a whole day in less than 5082 years, being rejected as inconsiderable; the intermediate years to be reckoned as they used to be in the *Julian* or *Old Stile*. This Pope's alteration, called the *Gregorian* or *New Stile*, was received in most of the Christian states: But some at that time chose to continue the *Julian*; among whom were the English; and they, in the year 1752, reformed their account, and introduced among themselves a new one, that nearly corresponds with the *Gregorian*.

A certain length of the year being once settled, and a regular account of time in consequence of it being so fitted, as to conform invariably to the seasons; it was natural for the States who received such account, to fit to it a register of the days in each month, and to note the days when any remarkable occurrence was to be commemorated; and such register has obtained the name of *Calendar*.

116. The *Calendar* now in use among most of the Christian states consists of 12 months, called *January, February, March, April, May, June, July, August, September, October, November, December*; these months are called *civil*, the number of days in each may be readily remembered by the following rule.

117. Thirty days has *September, April, June, and November*;

February has twenty-eight alone: All the rest have thirty-one;

When the year consists of 365 days: But in every fourth, which consists of 366 days, *February* has 29. This additional day was intercalated

lated after the 24th of *February*, which in the Old Roman Calendars was called the *sixth of the calends of March*, and being this year reckoned twice over, the year was called *Bissextilis*, or LEAP-YEAR.

Beside the months, time is also divided into weeks, days, hours, minutes, &c. a year containing 52 weeks, a week 7 days, a day 24 hours, an hour 60 minutes, &c.

In the Calendars it has been usual to mark the seven days of the week with the seven first letters of the alphabet, always calling the first of January A, the 2d B, the 3d C, the 4th D, the 5th E, the 6th F, and the 7th G. and so on, throughout the year: and that letter answering to all the Sundays for a year, is called the DOMINICAL LETTER.

According to this disposition, the letters answering to the first day of every month in the year, will be known by the following rule:

118. At Dover Dwells George Brown, Esquire,
Good Caleb Finch, and David Frier;

Where the first letter of each word answers to the letter belonging to the first day of the months in the order from January to December.

119. A year of 365 days contains 52 weeks and 1 day; and a leap-year has 52 weeks and 2 days; therefore the first and last days of a common year fall on the same week-day, suppose it *Monday*; then the next year begins on a *Tuesday*, the next year on *Wednesday*, and so on to the eighth year, which would be on *Monday* again, did every year contain 365 days; also the Dominical letter would run backward through all the seven letters. But this round of seven years is interrupted by the leap-years: for then *February* having a 29th day annexed, the first Dominical letter in *March* must fall a day sooner than in the common year; so that leap-year has two Dominical letters, the one (supposing G) serving for *January* and *February*, and the other, which is the preceding letter (F) serving for the rest of the Sundays in that year.

120. The SOLAR CYCLE, or cycle of the Sun, is a period of 28 years, in which all the varieties of the Dominical letters will have happened, and they will return in the same order as they did 28 years before. At the birth of Christ 9 years had past in this cycle.

For the changes, were all the years common ones, would be 7;

But the interruptions by leap-year being every fourth year;

Therefore the changes will be 4 times 7, or 28 years.

This return of the Dominical letter is constant in the *Julian* account. But in the *Gregorian*, where among the complete centuries, or hundredth years, only every fourth is leap-year; the other three hundred years, which according to the *Julian*, would be leap-years, are by the *Gregorian* only common years of 365 days: in these all the letters must be removed one place forward in a direct order; and either year, instead of having two Dominical letters (as suppose D, C), will have only one (as D), the Dominical letters moving retrograde.

121. The LUNAR CYCLE, or cycle of the Moon (and sometimes called the *Metonic Cycle*, from *Meton*, an Athenian, who invented it about 432 years before the time of *Christ*), is a period of nineteen years, containing all the variations of the days on which the new and full Moons happen; after which they fall on the same days they did 19 years before.

The PRIME, or GOLDEN NUMBER, is the number of years elapsed in this cycle.

At the birth of *Christ* the golden number was 2.

For many years after the *Nicene Council*, it was thought that 19 solar years, or 228 solar months, were exactly equal to 235 synodical, or lunar, months: and that the same yearly *golden number* set in their calendars against the days when the new Moons happened throughout one lunar cycle, would invariably serve for the new Moons of corresponding years throughout every successive lunar cycle. But later observations shew, that this cycle is less than 19 years, by a little more than one hour, twenty-eight minutes; therefore, the new Moons will, in a little less than 311 years, happen a day earlier than by the Metonic account; and consequently all the festivals depending on the new Moons, will in time be removed into other seasons of the year than those which they fell in at their first institution: thus the new Moons in the year 1750 happened above $4\frac{1}{2}$ days earlier than the times shewn by the calendar. But were the golden numbers, when once prefixed to the proper new Moon days in a Metonic period, to be set a day earlier at the end of every 310,7 years, a pretty regular correspondence might be preserved between the solar and lunar years.

122. The EPACT of any year is the Moon's age the beginning of that year; that is, the days past since the last new Moon.

The time between new Moon and new Moon is in the nearest round numbers $29\frac{1}{2}$ days; therefore the lunar year consisting of 12 lunations must be equal to 354 days, which is 11 days less than the solar year of 365 days. Now supposing the solar and lunar years to begin together, the epact is 0; the beginning of the next solar year, the epact is 11; the 3d year the epact is 22; the fourth 33, &c. But when the epact exceeds 30, an intercalary month of 30 days is added to the lunar year, making it consist of 13 months; so that the epact at the beginning of the 4th year is only 3, the 5th 14, the 6th 25, the 7th 36, or only 6, on account of the intercalary month; and so on to the end of the cycle of 19 years; at the expiration of which the same epacts would run over again, were the cycle perfect; and the epact would always be 11 times the prime.

123. By the *Nicene Council* it was enacted,

1st. That Easter-day should be celebrated after the vernal equinox, which at that time happened on the 21st of March.

2d. That it should be kept after the full, or 14th day of that Moon which happened first after the 21st of March in common years, and first after the 20th of March in leap-years.

3d. That the Sunday next following the 14th, or day of full Moon, should be Easter-Sunday: which must always fall between the 20th or 21st of March, and 25th of April.

124. The MOON'S SOUTHING at any place is the time when she comes to the meridian of that place, which is every day later by about $\frac{4}{5}$ of an hour; because 24, the hours in a day, being divided by 30, the number of times which she passes the meridian between new Moon and new Moon, will give $\frac{4}{5}=48'$ for the retardation of her passage over the meridian in one day.

The

The Sun and Moon come to the meridian at the same time on the day of the change, or at new Moon; also the Moon comes to the opposite part of the same meridian, when she is in opposition, or at full Moon. Hence between new and full she comes to the meridian in the afternoon; at full she comes to the meridian at mid-night; and when past the full, after mid-night, or in the morning.

125. The ROMAN INDICTION is a cycle of 15 years, used by the ancient Romans for the times of taxing the provinces. Three years of this cycle were elapsed at the birth of Christ.

The DIONYSIAN PERIOD is a cycle of 532 years, arising by multiplying together 28 and 19, the solar and lunar cycles; it was contrived by *Dionysius Exiguus*, a Roman abbot, about the year of Christ 527, as a period for comparing chronological events.

The JULIAN PERIOD contains 7980 years; it arises by multiplying together 28, 19, 15, the cycles of the Sun, Moon, and Indiction. This was also contrived as a period for chronological matters; and its beginning falls 710 years before the usual date of the creation.

On the principles laid down in the preceding articles depend the solution of the following problems.

126. PROBLEM I. *To find whether any given year is leap-year.*

RULE. Divide the given year by 4; if 0 remains, it is leap-year; if 1, 2, or 3 remain, it is so many years after.

Observing that the years 1800, 1900, 2100, &c. are common years.

EXAM. I. *Is 1796 leap-year?*

$$\begin{array}{r} 4 \overline{) 1796} \quad (449 \\ \underline{} \\ 0 \end{array}$$

Remains 0, so it is leap-year.

EXAM. II. *Is 1797 leap-year?*

$$\begin{array}{r} 4 \overline{) 1797} \quad (449 \\ \underline{} \\ 1 \end{array}$$

Remains 1 year past leap-year.

127. PROBLEM II. *To find the years of the solar, lunar, and indiction cycles.*

RULE. To the given year add 9 for the solar, 1 for the lunar, 3 for the indiction: Divide the sums in order by 28, 19, 15; the remainder in each shews the year of its respective cycle.

EXAM. *Required the years of the solar, lunar, and indiction cycles for the year 1797?*

$$\begin{array}{r} 1797 \\ 9 \\ \hline 28 \overline{) 1806} \quad (64 \end{array}$$

$$\begin{array}{r} 1797 \\ 1 \\ \hline 19 \overline{) 1798} \quad (94 \end{array}$$

$$\begin{array}{r} 1797 \\ 3 \\ \hline 15 \overline{) 1800} \quad (120 \end{array}$$

Remains 14=solar cycle.

12=lunar cyc. or golden No 0=indict. cycle.

Whereby it appears
that the year 1797
is the

14th year of the 65th solar cycle
12th year of the 95th lunar cycle
1st year of the 120th indiction cycle

since the
birth of
Christ.

128. PROBLEM III. *To find the Dominical letter till the year 1800.*

RULE. To the given year add its fourth part, divide the sum by 7; the remainder taken from 7 leaves the index of the letter in common years, reckoning A 1, B 2, C 3, &c.

But in leap-year, this letter and its preceding one (in the retrograde order which these letters take), are the Dominical letters.

EXAM. I. *For the year 1797.*

$$\begin{array}{r} 4) 1797 \\ \underline{449} \\ 7) 2246(320 \\ \underline{6} \end{array}$$

Remains 6. Then $7 - 6 = 1 = A$.
So A is the Dominical letter.

After 1799 the index of A will be 0, of B 1, C 2, D 3, &c.

EXAM. II. *For the year 1804.*

$$\begin{array}{r} 4) 1804 \\ \underline{451} \\ 7) 2255(322 \\ \underline{1} \end{array}$$

Remains 1. Then $7 - 1 = 6 = G$.
So A and C are the Dominical letters.

And in this manner were the following numbers computed.

For the Dominical letters during the 18th century.

Solar cycles	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Dom. letters	DC	B	A	G	FE	D	C	B	AG	F	E	D	CB	A
Solar cycles	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Dom. letters	G	F	ED	C	B	A	GF	E	D	C	BA	G	F	E

The year 1800 being a common year, stops the above order, and the following are the Dominical letters for the 19th century.

Solar cycles	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Dom. letters	ED	C	B	A	GF	E	D	C	BA	G	F	E	DC	B
Solar cycles	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Dom. letters	A	G	FE	D	C	B	AG	F	E	D	CB	A	G	F

129. PROBLEM IV. *To find the Epact till the year 1900.*

RULE. Multiply the golden number for the given year by 11, and divide the product by 30; from the remainder take 11, and it will leave the epact.

If the remainder is less than 11, add 19 to it, and it gives the epact.

Ex. I. *To find the epact for 1797.*

The Golden number is 12 (127)
Multiply by 11

$$\begin{array}{r} 30) 132(4 \\ \underline{120} \end{array}$$

Remains 12
Add 11

Consequently the Epact is 1

Ex. II. *To find the epact for 1800.*

The Golden number is 15 (127)
Multiply by 11

$$\begin{array}{r} 30) 165 \\ \underline{150} \end{array}$$

Subtract 15
11

Remains the Epact 4

And thus might the following numbers be found.

Gold. N^o 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
Epacts 29 11 22 3 14 25 6 17 28 9 20 1 12 23 4 15 26 7 18.
The epacts here proceed by the difference 11, rejecting thirties.

130. PROBLEM V. *To find the Moon's age.*

RULE. To the epact add the number and day of the month; their sum, if under 30, is the Moon's age; but if it be above 30, take 30 from it, and the remainder will be the Moon's age, or days since the last conjunction.

The numbers of the months, or monthly epacts, are the Moon's age at the beginning of each month, when the solar and lunar years begin together;

And are { 0 2 1 2 3 4 5 6 7 8 9 10.
Jan. Feb. Mar. Apr. May, June, July, Aug. Sept. Oct. Nov. Dec.

EXAM. I. *What is the Moon's age on the 14th of October, 1797?*

The epact is 1 (129)
The N^o of month 8
The day of the month 14
—
The sum is 23 the Moon's age.

EXAM. II. *What is the Moon's age on the 29th of March, 1800?*

The epact is 4 (129)
Then $4 + 1 + 29 = 34$ is the sum of the epact, number and day of the month.
And $34 - 30 = 4$ is the Moon's age.

131. The day of next new Moon is readily found by taking her age from 30.

The day of new Moon in any month is equal to the difference between the sum of the year's and month's epacts, and 30. Thus;

On March 29, the Moon is 4 days old.

So that new Moon is on the 25th.

Now $4 + 1 = 5$, is the sum of the epacts.

Then $30 - 5 = 25$, the day of new Moon, as it should be.

132. PROBLEM VI. *The day of the month in any year being given, to know on what week-day it will fall.*

RULE. Find the Dominical letter (128): also the week day on which the first of the proposed month falls (118); and hence the name of the proposed day of the month will be known; observing that the 1st, 8th, 15th, 22d, and 29th days of any month fall on the same week-days.

Ex. I. *On what day of the week does the 14th of Oct fall, in 1797?*

The Dominical letter is A. (128)
The 1st of October is A, 118)
Therefore October 1st is Sunday.
Consequently the 14th is on Saturday.

Ex. II. *In 1800 on what week-day does the 20th of March fall?*

The Dominical letter is E. (128)
The 1st of March is D, (118)
Then March 2d is Sunday.
And to March 20th is Thursday.

133. PROBLEM VII. *To find when Easter-day will fall in any year between 1700 and 1899.*

RULE. Find what day that new Moon falls on which is nearest to the 21st of March in common years, or to the 20th in leap-years; then the Sunday next after the full, or 15th day of that new Moon, will be Easter-day.

If the 15th day fall on a Sunday, the next Sunday is Easter-day.

Ex. I. *When does Easter-day fall in the year 1797?*

The Dominical letter is A. (128)
 March 21, Moon's age is 23. (130)
 New Moon on March 28th.
 The 15th day is April 11th.
 April the 1st is G, on Saturday.
 Then Easter-Sunday is April 16th.

Ex. II. *Required the time of Easter-day in the year 1800?*

The Dominical letter is E. (128)
 March 21st, Moon's age 26. (130)
 New Moon on March 25th.
 Full Moon on April 8th.
 April the 1st is c, on Tuesday.
 Then Easter-Sunday is April 13th.

134. Easter-day is always found by the Paschal full Moons, and these are readily found in the following curious table, which was communicated to the Royal Society in the year 1750, by the Earl of Macclesfield, and published in the Philosophical Transactions for the same year; and its use shewn in the following precepts.

“ To find the day, on which the Paschal limit, or full Moon, falls in any given year; look, in the column of golden numbers belonging to that period of time wherein the given year is contained, for the golden number of that year; over-against which, in the same line continued to the column intitled Paschal full Moons, you will find the day of the month, on which the Paschal limit, or full Moon, happens in that year. And the Sunday next after that day is Easter-day in that year, according to the Gregorian account.”

His Lordship also gave with the following table an account of the principles upon which he constructed it; and which the more inquisitive readers may consult, if they please.

A TABLE, shewing, by means of the Golden Numbers, the several days on which the Paschal limits, or full Moons, according to the Gregorian account, have already happened, or will hereafter happen; from the Reformation of the Calendar in the year 1582, to the year 4199, inclusive.

Golden Numbers from the year 1583 to 1699, and so on to 4199, all inclusive.																Paschal full Moons	
1583 to 1699	1700 to 1899	1900 to 2199	2200 to 2399	2400 to 2499	2500 to 2599	2600 to 2899	2900 to 3099	3100 to 3399	3400 to 3499	3500 to 3599	3600 to 3699	3700 to 3799	3800 to 4099	4100 to 4199	Days of the month and Sun. letters.		
3	14	—	6	17	6	17	—	9	—	1	12	1	12	—	4	March 21 C	
—	3	14	—	6	—	6	17	—	9	—	1	—	1	12	—	22 D	
11	—	3	14	—	14	—	6	17	—	9	—	9	—	1	12	23 E	
—	11	—	3	14	3	14	—	6	17	—	9	—	9	—	1	24 F	
19	—	11	—	3	—	3	14	—	6	17	—	17	—	9	—	25 G	
8	19	—	11	—	11	—	3	14	—	6	17	6	17	—	9	26 A	
—	8	19	—	11	—	11	—	3	14	—	6	—	6	17	—	27 B	
16	—	8	19	—	19	—	11	—	3	14	—	14	—	6	17	28 C	
5	16	—	8	19	8	19	—	11	—	3	14	3	14	—	6	29 D	
—	5	16	—	8	—	8	19	—	11	—	3	—	3	14	—	30 E	
13	—	5	16	—	16	—	8	19	—	11	—	11	—	3	14	April 31 F	
2	13	—	5	16	5	16	—	8	19	—	11	—	11	—	3	1 G	
—	2	13	—	5	—	5	16	—	8	19	—	19	—	11	—	2 A	
10	—	2	13	—	13	—	5	16	—	8	19	8	19	—	11	3 B	
—	10	—	2	13	2	13	—	5	16	—	8	—	8	19	—	4 C	
18	—	10	—	2	—	2	13	—	5	16	—	16	—	8	19	5 D	
7	18	—	10	—	10	—	2	13	—	5	16	5	16	—	8	6 E	
—	7	18	—	10	—	10	—	2	13	—	5	—	5	16	—	7 F	
15	—	7	18	—	18	—	10	—	2	13	—	13	—	5	16	8 G	
4	15	—	7	18	7	18	—	10	—	2	13	2	13	—	5	9 A	
—	4	15	—	7	—	7	18	—	10	—	2	—	2	13	—	10 B	
12	—	4	15	—	15	—	7	18	—	10	—	10	—	2	13	11 C	
1	12	—	4	15	4	15	—	7	18	—	10	—	10	—	2	12 D	
—	1	12	—	4	—	4	15	—	7	18	—	18	—	10	—	13 E	
9	—	1	12	—	12	—	4	15	—	7	18	7	18	—	10	14 F	
—	9	—	1	12	1	12	—	4	15	—	7	—	7	18	—	15 G	
17	—	9	—	1	—	1	12	—	4	15	—	15	—	7	18	16 A	
6	17	—	9	—	9	—	1	12	12	4	15	4	15	15	7	17 B	
14	6	6	17	9	17	9	9	1	1	12	4	12	4	4	15	18 C	

135. PROBLEM VIII. To find the time of the Moon's southing on a given day.

RULE. The Moon's age in days, multiplied by 0,8, gives the time of her southing, nearly, in hours and tenth parts.

That time, if less than 12 hours, is the time after mid-day.

But if greater, the excess is the time after last midnight.

Ex. I. At what time does the Moon come to the meridian of London, on the 14th of October, 1797?

The Moon's age is 23 days.

Which multiplied by 0,8

Moon So. $18^h 24^m = 18,4$

Ex. II. Required the time of the Moon's southing on the 21st of March, 1800?

The Moon's age 26 days. (130)

Which multiplied by 0,8

Moon So. $20^h 48^m = 20,8$

Each tenth part of an hour being 6 minutes, any number of such tenth parts multiplied by 6, produces minutes.

136. PRO-

136. PROBLEM IX. *To find the time of high-water at any place.*

RULE. To the time of the Moon's southing add the time the Moon has passed the meridian on the full and change days to make high-water at that place; the sum shews the time of high-water on the given day.

The time of high-water, on the full and change days, is found in the right-hand column of the geographical table, art. 137, against the name of the place.

Ex. I. On the 14th of October 1797, at what time will it be high-water at London?

Moon souths at 18h. 24m. (135)

H. W. at Lond. 3 00 on syzygies

Sum 21 24
Subtract 12

H. W. at 9h 24m. A.M. on the day proposed.

Ex. II. Required the time when it will be high-water at Ushant on March 21st, 1800.

Moon souths at 20h. 48m. (135)
High-water at Ushant 4 30 P.M.

High-water at 1 18 A.M. on the day proposed.

The v. viii. ix. problems preceding have solutions, such as are common in books of pilotage, and which in some cases will produce conclusions considerably wide of the truth; it has therefore been judged necessary to consider these articles in a more accurate manner in Book IX. of Days works.

SECTION IX.

137. *A Geographical Table;*

Containing the Latitudes and Longitudes of the chief towns, islands, bays, capes, and other parts of the sea-coasts in the known world, collected from the most authentic observations and charts extant; with the times of high-water on the days of the new and full Moon.

The longitudes are reckoned from the meridian of London. By the latitude and longitude of an island, or harbour, is meant the middle of that place.

Note. B. stands for bay; C. for cape; R. for river; P. for port; Pt. for point; I. for Isle; St. for saint; G. for gulf; M. for mount; Eu. for Europe; Am. for America; Atl. for the Atlantic; Ind. for Indian; Med. Sea for Mediterranean Sea; Wh. Sea for White Sea; Archip. for Archipelago; Nov. Sco. for Nova Scotia; Phil. I. for Philippine Isles; Adriat. for Adriatic; Eng. for England; D. Neth. for the Dutch Netherlands: Besides other contractions which will be easily understood.

Names

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
A.						
I. Abacco, } N. point	Am.	Bahama I.	Atl. Ocean	27 12 N.	77 05 W.	
or Lucayos } S. point	Eu.	France	Inland	26 15 N.	77 01 W.	
Abbeville	Eu.	France	Eng. Channel	50 7 N.	1 55 E.	
Abbreviak	Eu.	Scotland	Germ. Ocean	48 32 N.	4 15 W.	4h. 30m.
St. Abb'shead	Eu.	Anian	Iudian Sea	55 55 N.	5 56 W.	
I. Abdeleur	Eu.	Scotland	Germ. Ocean	11 55 N.	51 45 E.	
Aberdeen	Eu.	Finland	Baltic Sea	57 06 N.	01 44 W.	0 45
Abo	Am.	Brasil	Atl. Ocean	60 27 N.	22 15 E.	
Abrolhos Bank	Am.	Bahama	Atl. Ocean	18 22 S.	38 45 W.	
Abrollo Bank, N. part	Asia	I. Sumatra	Indian Ocean	21 33 N.	69 50 W.	
Achen	Asia	Arabia	Indian Sea	5 22 N.	95 40 E.	
Aden	Eu.	Nova Zem.	North Ocean	12 55 N.	45 35 E.	
I. Admiralties	Asia	Soc. Isles	Pacif. Ocean	75 05 N.	52 50 E.	
Adventure Island	Africa	Madagascar	Indian Ocean	17 6 S.	144 18 W.	
I. Agalega, or Gallega	Asia	India		10 15 S.	54 46 E.	
Agimere	Am.	Patagonia	S. Atl. Ocean	26 35 N.	75 25 E.	
C. St. Agnes	Asia	India	Mogul's	53 55 S.	66 29 W.	
Agra	Eu.	Dalmatia	Adriatic Sea	26 43 N.	76 49 E.	
I. St. Agusta	Am.	Peru	Pacif. Ocean	42 40 N.	18 57 E.	
C. Ajuga	Eu.	Turkey	Danube	6 38 S.	80 50 W.	
Akerman	Africa	Caffers	Indian Ocean	46 12 N.	30 48 E.	
B. Alagoa	Eu.	Sweden	Baltic Sea	25 30 S.	33 33 E.	
If. Aland	Am.	New S. Wales	Hud. Bay	60 20 N.	21 30 E.	
R. Albany	Africa	Algiers	Medit. Sea	52 15 N.	81 48 W.	
I. Alboran	Eu.	England	Germ. Ocean	36 00 N.	2 27 W.	
Aldborough	Eu.	England	Eng. Channel	52 20 N.	1 25 E.	9 45
I. Alderney	Asia	Syria	Medit. Sea	49 48 N.	2 11 W.	12 00
Aleppo	Asia	Syria	Medit. Sea	35 45 N.	37 25 E.	
Alexandretta	Africa	Egypt	Medit. Sea	36 35 N.	36 20 E.	
Alexandria	Africa	Canaries	Atl. Ocean	31 11 N.	30 17 E.	
I. Algeranca	Africa	Algiers	Medit. Sea	29 23 N.	15 53 W.	
Algiers	Eu.	Spain	Medit. Sea	36 49 N.	2 18 E.	
Alicant	Eu.	Italy	Medit. Sea	38 34 N.	0 07 W.	
I. Alicur, Lipari If.	Africa	Egypt	Red Sea	38 31 N.	14 37 E.	
Alkofir	Am.	Brasil	Atl. Ocean	26 20 N.	34 41 E.	
B. All Saints, or } Todos Sanctos	Eu.	Spain	Medit. Sea	13 05 S.	38 45 W.	
Almeria	Africa	Zanguebar	Indian Sea	36 51 N.	2 15 W.	
If. Almirante, } limits	Am.	T. del Fuego	Pacif. Ocean	5 45 S.	52 30 E.	
St. Alphonso's If.	Asia	Arabia	Red Sea	4 30 S.	55 40 E.	
Altur	Am.	Terra Firma	Atl. Ocean	55 51 S.	69 28 W.	
R. Amazons, } mouths	Am.	Molucca I.	Indian Ocean	28 10 N.	34 19 E.	6 00
I. Amboyne	Asia	N. Hebrides	Pacif. Ocean	0 30 S.	47 35 W.	
Ambrym	Am.	Chili	Pacif. Ocean	4 25 N.	127 25 E.	
If. Ambrofa	Eu.	D. Neth.	Germ. Ocean	16 10 S.	168 12 E.	
I. Ameyland	Eu.	France	Inland	26 40 S.	82 30 W.	
Amiens	Asia	China	Pacif. Ocean	53 30 N.	6 20 E.	7 30
I. Amoy	Eu.	D. Neth.	Germ. Ocean	49 54 N.	2 23 E.	
Amsterdam	Asia	Madagascar	Indian Ocean	24 30 N.	118 45 E.	
I. Amsterdam	Am.	Curacao	Carib. Sea	52 23 N.	04 52 E.	3 00
Amsterdam Har.	Asia	Friendly If.	Pacif. Ocean	37 55 S.	75 15 E.	
I. Amsterdam, or } Tonga-Tabu	Africa	Eth. Coast	Atl. Ocean	12 08 N.	68 16 W.	
I. Anabona	Eu.	Italy	Mediterran.	21 09 S.	174 41 W.	8 30
Anadirskoi Nofs	Asia	India	B. Bengal	2 36 S.	5 35 E.	
Ancona	Am.	Mexico	Atl. Ocean	64 14 N.	173 26 W.	
If. Andaman, Great } Little	Eu.	Scotland	Germ. Ocean	43 38 N.	13 31 E.	
I. Andaro	Asia	India	Indian Ocean	13 17 N.	93 02 E.	
I. St. Andro, Sotovento	Am.	Madagascar	Indian Ocean	10 40 N.	92 30 E.	
C. St. Andrea	Am.	Madagascar	Indian Ocean	10 00 N.	73 40 E.	
St. Andrew's	Eu.	Scotland	Germ. Ocean	12 30 N.	81 35 W.	
				15 46 S.	45 22 E.	
				56 18 N.	2 37 W.	2 15

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
If. Androsa { N. point { S. point	Am.	Bahama I.	Bahama I.	{ 25 00 N. 23 30 N.	77 58 W 77 00 W.	
If. Angafay	Africa	Madagascar	Indian Ocean	17 00 S.	58 40 E.	
C. St. Angelo	Eu.	Turkey	Archipelago	36 27 N.	33 38 E.	
Mount St. Angelo	Eu.	Italy	Medit. Sea	41 42 N.	16 16 E.	
Angra	Eu.	Tercera	Atlantic	38 39 N.	27 7 W.	
R. d'Angra	Africa	Ethiopia	N. Atl. Ocean	01 00 N.	9 35 E.	
C. d'Anguilhas	Africa	Caffers	Indian Ocean	34 50 S.	20 06 E.	
I. Anguilla	Am.	Antilles If.	Atl. Ocean	18 15 N.	62 57 W.	
C. Anguille	Am.	Newfoundl.	Atl. Ocean	47 55 N.	59 11 W.	
I. Anholt (Lights)	Eu.	Denmark	Sound	56 44 N.	12 44 E.	oh. com.
C. Anne	Am.	New Eng.	West. Ocean	42 50 N.	70 27 W.	
C. Queen Anne	Am.	Greenland	North Ocean	64 15 N.	50 30 W.	
Q. Anne's Foreland	Am.	N. Main	Hudson's Str.	64 08 N.	74 36 W.	
Annamocka, or Rotterdam }	Asia	Friendly If.	Pacif. Ocean	20 16 S.	174 30 W	
Annapolis Royal	Am.	Nova Scotia	B. Fundy	44 52 N.	64 00 W.	
I. Antego	Am.	Caribbee If.	Atl. Ocean	16 57 N.	62 4 W.	
Antibes	Eu.	France	Medit. Sea	43 35 N.	7 09 E.	
I. Ante- { W. point { E. point	Am.	Canada	{ B. St. Lau- { rence	49 52 N. 49 10 N.	64 04 W. 61 42 W.	
Antiochetta	Asia	Syria	Medit. Sea	36 08 N.	36 17 E.	
C. d'Antifer	Eu.	France	Eng. Channel	49 47 N.	00 34 E.	
C. Antonio	Am.	Isle Cuba	Atl. Ocean	21 45 N.	84 05 W.	
I. St. Antonio	Africa	Cape Verd	Atl. Ocean	17 00 N.	25 02 W.	
C. St. Antony	Am.	Magellan	Atl. Ocean	54 46 S.	63 42 W.	
Antwerp	Eu.	Flanders	R. Scheld	51 13 N.	4 24 E.	6 00
B. Apalaxy	Am.	Florida	G. Mexico	30 00 N.	83 53 W.	
I. Apaloria	Asia	India	Indian Ocean	9 08 S.	79 40 E.	
Aquapulco	Am.	Mexico	Pacif. Ocean	17 10 N.	101 40 W.	
Aquatulco	Am.	Mexico	Pacif. Ocean	15 27 N.	96 03 W.	
Archangel	Eu.	Russia	White Sea	64 34 N.	38 59 E.	6 00
Arcot	Asia	India	Inland	12 51 N.	79 33 E.	
I. d'Areas	Am.	Mexico	G. Mexico	20 45 N.	92 35 W.	
Arensburg	Eu.		Baltic	58 15 N.	22 11 E.	
Arica	Am.	Peru	Pacif. Ocean	18 27 S.	71 05 W.	
I. Arran	Eu.	Ireland	St. Geo. Ch.	54 48 N.	8 59 W.	11 00
Aruba	Am.	West Indies	Atlantic	12 35 N.	69 25 W.	
I. Ascension	Am.	Brasil	Atl. Ocean	7 56 S.	14 16 W.	
I. Affinaria, Sardinia }	Eu.	Italy	Medit. Sea	41 06 N.	8 36 E.	
R. Ashley	Am.	Carolina	Atl. Ocean	33 22 N.	79 50 W.	0 45
R. Assene	Africa	Guinea	Atl. Ocean	5 30 N.	2 20 W.	
I. Astores	Africa	Madagascar	Indian Ocean	10 22 S.	53 25 E.	
Astracan	Asia	Siberia	Caspian Sea	46 21 N.	48 8 E.	
Athens	Eu.	Turkey	Archipelago	38 5 N.	23 52 E.	
Atkin's Key	Am.	Bahama Isles	Atl. Ocean	22 07 N.	74 26 W.	
Atwood's Keys	Am.	Bahama Isles	Atl. Ocean	21 22 N.	72 04 W.	
C. Ava	Asia	Japan	Pacif. Ocean	34 45 N.	141 00 E.	
If. Aves, Sotovento	Am.	Terra Firma	Atl. Ocean	15 26 N.	66 15 W.	
C. St. Augustine	Am.	Brasil	Atl. Ocean	8 48 S.	35 08 W.	
C. St. Augustine	Asia	Mindanao	Pacif. Ocean	6 40 N.	126 25 E.	
St. Augustine	Am.	Florida	Atl. Ocean	30 10 N.	81 29 W.	7 30
St. Augustine	Africa	Madagascar	Indian Ocean	23 35 S.	43 13 E.	
Aurora	Asia	N. Hebrides	Pacif. Ocean	15 8 S.	168 17 E.	
Aydhah	Africa	Egypt	Red Sea	21 53 N.	36 26 E.	
Aylah	Asia	Arabia	Red Sea	29 08 N.	35 41 E.	
B						
Babelmondel Straits	Africa	Abyssinia	Red Sea	12 50 N.	43 50 E.	
C. Baba	Asia	Natolia	Archipelago	39 33 N.	26 22 E.	
I. Bachian	Asia	Molucca Isles	Pacif. Ocean	00 40 N.	123 00 E.	
Bagdad	Asia	Mesopotamia	Inland	33 20 N.	44 26 E.	
I. Bahama	Am.	Bahama Isles	Atl. Ocean	26 45 N.	78 35 W.	
Bahama Bank, N. pt.	Am.	Bahama Isles	Atl. Ocean	27 50 N.	78 43 W.	
C. Bajader	Africa	Negroland	Atl. Ocean	26 29 N.	14 36 W.	0 00

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Baker's Doken	Am.	Labrador	Hudson's Bay	57 0 N.	0	
Balafor	Asia	India	B. Bengal	21 20 N.	86 00 E.	
Baldivia	Am.	Chili	Pacif. Ocean	39 38 S.	73 20 W.	
I. Bali	Asia	Sunda Isles	Indian Ocean	8 05 S.	114 30 E.	
Baltimore	Eu.	Ireland	West. Ocean	51 16 N.	9 26 W.	4b 30m.
I. Banca { S. end	Asia	Sunda Isles	Indian Ocean	3 15 S.	107 10 E.	
{ N. W. end				1 50 S.	105 30 E.	
I. Banda	Asia	Molucca Isles	Indian Ocean	4 30 N.	127 25 E.	
Bangalore	Asia	Myfore	Inland	13 0 N.	77 42 E.	
Banjar	Asia	I. Borneo	Indian Ocean	2 27 S.	113 50 E.	
Banks's Isle	Asia	N. Zealand.	Pacif. Ocean	43 45 S.	172 40 E.	
Bantam	Asia	I. Java	Indian Ocean	6 15 S.	106 25 E.	
B. Bantry	Eu.	Ireland	Atl. Ocean	51 45 N.	10 46 W.	
I. Barbadoes	Am.	Caribbee Isles	Atl. Ocean	13 05 N.	59 36 W.	
Bridge-town						
C. Barbas	Africa	Sanaga	N. Atl. Ocean	21 50 N.	16 26 W.	
I. Barbuda	Am.	Caribbee Isles	Atl. Ocean	17 46 N.	61 47 W.	
C. Barcam	Eu.	Greenland	North Ocean	78 18 N.	20 06 E.	
Barcelona	Eu.	Spain	Medit. Sea	41 26 N.	2 18 E.	
C. Barfleur	Eu.	France	Eng. Channel	49 38 N.	1 16 W.	7 30
Bargazar Point	Eu.	Iceland	North Ocean	66 30 N.	17 12 W.	
I. Bardsey	Eu.	Wales	St. Geo. Cha.	52 44 N.	5 00 W.	
Barren Isle	Asia	Andamans	Bengal Bay	12 14 N.	93 47 E.	
C. Barfo	Eu.	Russia	White Sea	66 30 N.	38 00 E.	
I. Barnevelt	Am.	T. del Fuego	South. Ocean	55 49 N.	66 53 W.	
I. Bartholomew	Am.	Caribbee Isles	Atl. Ocean	17 56 N.	63 11 W.	
I. Bartholomew	Am.	N. Hebrides	Pacif. Ocean	15 42 N.	167 22 E.	
I. de Bas	Eu.	France	Eng. Channel	48 50 N.	4 00 W.	3 45
Bassora	Asia	Arabia	Persian Gulf	29 45 N.	47 40 E.	
C. Bassos, or Baxos	Africa	Anian	Indian Sea	4 12 N.	47 07 E.	
Bassos de Banhos	Africa	Zanguebar	Indian Ocean	5 00 S.	48 08 E.	
Bassos de Chagos	Asia	India	Indian Ocean	6 42 S.	68 20 E.	
I. Bassus des Indes	Africa	Zanguebar	Indian Ocean	21 19 S.	41 43 E.	
Batavia	Asia	I. Java	Indian Ocean	6 12 S.	106 45 E.	
Bayonne	Eu.	France	B. Biscay	43 30 N.	1 30 W.	3 30
Bayona Isles	Eu.	Spain	Atl. Ocean	41 45 N.	9 01 W.	
Beachy Head	Eu.	England	Eng. Channel	50 44 N.	0 25 E.	0 00
Beary-bay	Eu.	Greenland	North Ocean	79 10 N.	24 15 E.	
N. Bear	Am.	Labradore	Hudson's Bay	54 40 N.	80 0 W.	12 00
S. Bear				54 25 N.		
I. Beerenberg	Eu.		North Ocean	71 45 N.	4 30 E.	
Beicher's Isles	Am.	Labradore	Hudson's Bay	56 0 N.	83 4 W.	
Belfast	Eu.	Ireland	Irish Sea	54 43 N.	5 52 W.	10 0
Bellisle	Eu.	France	B. Biscay	47 21 N.	3 01 W.	3 30
Bellisle	Am.	Newfound.	Atl. Ocean	51 55 N.	55 25 W.	
Bembridge Point	Eu.	Isle Wight	Eng. Channel	50 40 N.	1 0 W.	
Straits of Bellisle	Am.	Newfoundl.	Atl. Ocean	51 48 N.	56 00 W.	
Bell Sound	Eu.	Greenland	North Ocean	77 15 N.	12 40 E.	
Bencolin	Asia	I. Sumatra	Indian Ocean	3 49 S.	102 5 E.	
Bender	Eu.	Turkey	Niefter	46 50 N.	29 41 E.	
Bengal	Asia	India	B. Bengal	22 00 N.	92 45 E.	
Bergen	Eu.	Norway	Western Oc.	60 10 N.	6 14 E.	
Berlin	Eu.	Germany	R. Elbe	52 33 N.	13 26 E.	
I. Bermudas	Am.	Bahama Isles	Atl. Ocean	32 35 N.	63 23 W.	7 00
I. Bermaja	Am.	Mexico	G. Mexico	21 40 N.	92 53 W.	
Berwick	Eu.	England	Germ. Ocean	55 45 N.	1 50 W.	2 30
Berry Point	Eu.	England	Eng. Channel	50 37 N.	3 49 W.	
Birds Island	Am.	Acadia	G. St. Lawr.	47 44 N.	60 24 W.	
Bilboa	Eu.	Spain	B. Biscay	43 26 N.	3 18 W.	
I. du Bic	Am.	Acadia	R. St. Lawr.	48 30 N.	68 36 W.	2 0
Blackney	Eu.	England	Germ. Ocean	53 20 N.	0 55 E.	6 00
Black Point	Eu.	Greenland	North Ocean	78 00 N.	10 50 E.	
Black Isle	Eu.	Nova Zem.	North Ocean	72 52 N.	52 35 E.	
C. Blanco	Africa	Negroland	Atl. Ocean	20 55 N.	17 5 W.	9 45
C. Blanco	Am.	Patagonia	Atl. Ocean	47 20 S.	64 37 W.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
C. Blanco	Eu.	Greenland	North Ocean	77 58 N.	20 04 E.	
C. Blanco	Am.	Mexico	Pacif. Ocean	43 32 N.	124 02 W.	
I. Blanco, Sotovento	Am.	Terra Firma	Atl. Ocean	11 42 N.	64 20 W.	
Blanchart Race	Eu.	France	Eng. Channel	49 42 N.	2 08 W.	oh. com.
Blasques	Eu.	Ireland	Atl. Ocean	52 00 N.	10 56 W.	
Blavet, or Port Louis	Eu.	France	B. Biscay	47 45 N.	3 13 W.	3 0
Bocachica	Am.	Terra Firma	Carib. Sea	10 20 N.	75 30 W.	
C. Bojador	Africa	Negroland	Atl. Ocean	26 12 N.	14 22 W.	
Bolabola	Asia	Society Isl.	Pacif. Ocean	16 33 S.	151 52 W.	
R. B. Ikhaya	Asia	Siberia	Pacif. Ocean	52 48 N.	157 00 E.	
Bolt Head	Eu.	England	Eng. Channel	50 17 N.	3 48 W.	
I. Bombay	Asia	India	Indian Ocean	18 57 N.	72 43 E.	
Bona	Africa	Tunis	Mediterran.	37 08 N.	7 10 E.	
C. Bona	Africa	Tunis	Mediterran.	37 10 N.	10 00 E.	
C. Bona vista	Am.	Newfoundl.	Atl. Ocean	48 54 N.	52 33 W.	
I. Bona vista	Africa	C. Verd Isles	Atl. Ocean	16 05 N.	22 42 W.	
C. Bona fortuna	Eu.	Russia	White Sea	65 35 N.	38 25 E.	
I. Bonayre, Sotovento	Am.	Terra Firma	Atl. Ocean	11 52 N.	67 20 W.	
B. Bonaventura	Am.	Terra Firma	Pacif. Ocean	3 18 N.	76 50 W.	
C. Bon Esperance	Africa	Caffers	Indian Ocean	34 29 S.	18 23 E.	3 00
Borneo { East point	Asia		Indian Ocean	1 12 N.	117 10 E.	
West point				3 15 N.	108 57 E.	
North point				7 05 N.	113 40 E.	
I. South point				3 32 S.	112 05 E.	
I. Bor- } Borneo	Asia		Indian Ocean	5 00 N.	112 15 E.	
neo } Succadano				0 50 S.	108 35 E.	
I. Bornholm	Eu.	Sweden	Baltic Sea	55 12 N.	15 50 E.	
Boston	Eu.	England	Germ. Ocean	53 10 N.	0 25 E.	
Boston	Am.	New Eng.	Atl. Ocean	42 25 N.	70 32 W.	
Botany Isle	Asia	N. Caledonia	Pacif. Ocean	22 27 S.	167 12 E.	
Botany Bay	Asia	N. Holland	Pacif. Ocean	34 06 S.	151 20 E.	8 0
Boulogne	Eu.	France	Eng. Channel	50 44 N.	1 40 E.	10 30
I. Bourbon, St. Den.	Africa	Madagascar	Indian Ocean	20 52 S.	55 35 E.	
Bourdeaux	Eu.	France	B. Biscay	44 50 N.	0 30 W.	3 00
I. St. Brandon	Africa	Madagascar	Indian Ocean	16 45 S.	64 48 E.	
B. Brandwyns	Eu.	Greenland	North Ocean	79 50 N.	26 20 E.	
I. Bravas	Africa	C. Verd	Atl. Ocean	14 54 N.	24 45 W.	
Bremen	Eu.	Germany	R. Weser	53 30 N.	9 00 E.	6 00
Breeseound, a sand	Eu.	D. Neth.	Germ. Ocean	53 12 N.	5 15 E.	4 30
Breslau	Eu.	Silesia	R. Oder	51 03 N.	17 13 E.	
Brest	Eu.	France	B. Biscay	48 23 N.	4 26 W.	3 45
B. Brest	Am.	New Britain	West. Ocean	52 10 N.	52 30 W.	
Cape Bret	Asia	N. Zealand	Pacif. Ocean	35 07 S.	173 52 E.	
Bridge Town	Am.	I. Barbadoes	Atl. Ocean	13 05 N.	59 36 W.	
Bridlington Bay	Eu.	England	Germ. Ocean	54 07 N.	00 04 E.	3 45
Brill	Eu.	D. Neth.	Germ. Ocean	51 56 N.	4 10 E.	1 30
Brion Isle	Am.	Acadia	G. St. Lawr.	47 50 N.	60 47 W.	
Bristol	Eu.	England	St. Geo. Ch.	51 28 N.	2 30 W.	6 45
C. Bristol	Am.	Sandwich L.	Atl. Ocean	59 2 S.	26 46 W.	
Cape { Louisbourg	Am.	Acadia	Atl. Ocean	45 54 N.	59 55 W.	
Britain { I. Scateri				46 01 N.	61 57 W.	
North Cape				47 05 N.	60 8 W.	
I. Mathias	Asia	New Guinea	Pacif. Ocean	2 00 S.	147 50 E.	
North point				2 30 S.	148 40 E.	
S. W. point				6 00 S.	146 37 E.	
Strait Dampier				6 15 S.	146 15 E.	
C. St. George				5 30 S.	150 55 E.	
I. St. John.				4 20 S.	152 40 E.	
Brussells	Eu.	Flanders	Inland	50 51 N.	4 27 E.	
Buchanefs	Eu.	Scotland	Germ. Ocean	57 29 N.	1 23 W.	3 00
Buenos Ayres	Am.	Brasil	Atl. Ocean	34 35 S.	58 26 W.	
Bukarost	Eu.	Walachia	Inland	44 27 N.	26 13 E.	
C. Buller	Am.	S. Georgia	Atl. Ocean	53 58 S.	37 40 W.	
Burgaford point	Eu.	Iceland	North Ocean	66 03 N.	16 34 W.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Burgeo Isles	Am.	Newfoundl.	Atl. Ocean	47 36 N.	57 31 W.	
Burlings, rocks	Eu.	Portugal	Atl. Ocean	39 20 N.	9 32 W.	
Burlington	Eu.	England	Germ. Ocean	54 00 N.	0 08 E.	
Butron's Isles	Am.	New Britain	Hudf. Straits	60 35 N.	65 20 W.	6h. 50m.
Cape Byron	Asia	N. Zealand	Pacif. Ocean	28 39 S.	153 31 E.	
Byron's Isle	Asia		Pacif. Ocean	1 18 S.	170 6 W.	
C						
P. Cabello	Am.	Terra Firma	Carib. Sea	10 31 N.	67 27 W.	
I. Cabrera	Eu.	Italy	Mediterran.	43 10 N.	9 11 E.	
Cadiz	Eu.	Spain	Atl. Ocean	36 31 N.	6 07 W.	4 30
Caen	Eu.	France	Eng. Channel	49 11 N.	0 17 W.	9 00
Cagliari, I. Sardinia	Eu.	Italy	Medit. Sea	39 25 N.	9 38 E.	
Cajanebourg	Eu.	Finland	Baltic Sea	64 13 N.	27 51 E.	
Isles Caricos, or }	Am.	Bahama Isles	Atl. Ocean	21 27 N.	71 24 W.	
Cankrofs, from }				22 5 N.	72 15 W.	
Calabar { Old }	Africa	Guinea	Eth. Ocean	4 30 N.	8 10 E.	
{ New }				5 00 N.	7 00 E.	
C. Caiaberno	Asia	Natolia	Archipelago	38 42 N.	26 44 E.	
Calais	Eu.	France	Eng. Channel	50 58 N.	01 51 E.	11 30
C. Calamadon	Asia	India	B. Bengal	10 22 N.	80 40 E.	
Calcutta	Asia	India	B. Bengal	22 35 N.	88 34 E.	
Caldera	Asia	I. Mindano	Pacif. Ocean	7 00 N.	121 25 E.	
I. Caldy	Eu.	England	St. Geo. Ch.	51 33 N.	5 14 W.	5 15
Calcut	Asia	India	Indian Ocean	11 15 N.	75 39 E.	
Cairo	Africa	Egypt	R. Nile	30 02 N.	31 26 E.	
Callao	Am.	Peru	Pacif. Ocean	12 2 S.	76 53 W.	
P. Calymere	Asia	India	B. Bengal	10 20 N.	80 00 E.	
I. Great Camanis	Am.	West Indies	Atl. Ocean	19 18 N.	80 29 W.	
I. Little Camanis	Am.	West Indies	Atl. Ocean	19 42 N.	79 20 W.	
Camboida	Asia	India	Indian Ocean	10 35 N.	104 45 E.	
Cambridge	Eu.	England	—	52 13 N.	0 9 E.	
Cambridge	Am.	N. England	—	42 25 N.	71 5 W.	
C. Cambron, or }	Africa	Algiers	Medit. Sea	37 18 N.	4 58 E.	
Carbon }						
C. Cameron	Am.	New Spain	Atl. Ocean	15 35 N.	83 29 W.	
R. Camerones	Africa	Guinea	Atl. Ocean	3 30 N.	9 10 E.	
B. Camerones	Am.	Magellan	Atl. Ocean	44 50 S.	67 10 W.	
Camfer, a sand	Eu.	D. Neth.	Germ. Ocean	53 33 N.	5 30 E.	1 30
Camin	Eu.	Germany	Baltic	54 04 N.	15 40 E.	
J. Campbell	Asia	N. Zealand	Pacif. Ocean	41 51 S.	174 41 E.	
Compeachy	Am.	Yucatan	Atl. Ocean	19 36 N.	90 53 W.	
I. Canaria	Africa	Canaries	Atl. Ocean	28 01 N.	15 0 W.	3 0
C. Candemose	Eu.	Russia	North Ocean	69 25 N.	45 30 E.	
Candia { C. St. John, W. Pt. }				35 12 N.	23 54 E.	
{ Candia }	Eu.	Turkey	Medit. Sea	35 19 N.	25 23 E.	
{ C. Solomon, E. Pt }				34 57 N.	27 06 E.	
Candia	Asia	I. Ceylon	Indian Ocean	7 54 N.	81 53 E.	
Candlemas Isles	Am.	Sandwich L.	Atl. Ocean	57 10 S.	27 13 W.	
I. Candu	Asia	India	Indian Ocean	7 30 S.	77 55 E.	
C. Canfo	Am.	Nova Scotia	Atl. Ocean	45 18 N.	60 48 W.	
Canfo Passage	Am.	Nova Scotia	Atl. Ocean	45 30 N.	61 00 W.	
C. Cantin	Africa	Barbary	Atl. Ocean	32 41 N.	9 01 W.	0 00
Canire, Mul	Eu.	Scotland	West Ocean	55 22 N.	5 45 W.	
Canton	Asia	China	Pacif. Ocean	23 08 N.	113 07 E.	
Cape Town	Africa	Cassers	Atl. Ocean	33 55 S.	18 23 E.	2 30
I. Capri	Eu.	Italy	Medit. Sea	40 34 N.	14 11 E.	
I. Capraya	Eu.	Italy	Medit. Sea	43 03 N.	10 15 E.	
C. Carapar	Asia	India	B. Bengal	19 22 N.	86 05 E.	
Caraccas	Am.	Terra Firma	Atl. Ocean	10 06 N.	66 45 W.	
Caries, or Hacluits Isles	Am.	Greenland	Baffin's Bay	77 15 N.	62 00 W.	
Carinda Point	Am.	California	Pacif. Ocean	38 24 N.	124 25 W.	
Carlescroon	Eu.	Sweden	Baltic	56 20 N.	15 31 E.	
Carlisle	Eu.	England	Irish Sea	54 47 N.	2 35 W.	
C. Carmel	Asia	Syria	Levant	33 08 N.	35 35 E.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Waters.
Caroline Isles, limits	Asia		Pacif. Ocean	7 10 N. 12 00 N.	137 25 E. 127 25 E.	
C. Carthage	Africa	Barbary	Medit. Sea	36 52 N.	10 31 E.	
Carthage	Am.	Terra Firma	Caribbean Sea	10 27 N.	75 22 W.	
Carthage	Eu.	Spain	Medit. Sea	37 37 N.	1 03 W.	
Carteret's Isle	Asia	New Britain	Pacif. Ocean	8 26 S.	159 14 E.	
Cary's Swan's Nest	Am.		Hudson's Bay	62 20 N.	83 30 W.	
Calan	Asia	Siberia	Inland	55 47 N.	49 20 E.	
Caskets	Eu.	I. Guernsey	Eng. Channel	49 50 N.	2 26 W.	8h. 15m.
C. Cassander	Eu.	Turkey	Archipelago	40 02 N.	23 41 E.	
I. St. Catherine's	Am.	Brazil	Atl. Ocean	27 35 N.	49 12 W.	
Cat Isle { N. point S. point	Am.	Bahama	Atl. Ocean	24 50 N. 23 48 N.	75 38 W. 75 35 W.	
Cathness Point Dinnnet Head }	Eu.	Scotland	West. Ocean	53 42 N.	3 17 W.	9 00
Catene	Eu.	I. Sicily	Medit. Sea	42 40 N.	20 30 E.	
Cavan	Eu.	Ireland	Inland	54 52 N.	7 18 W.	
C. Catocha	Am.	New Spain	Caribbean Sea	20 48 N.	86 35 W.	
I. Cayenne	Am.	Terra Firma	Atl. Ocean	4 56 N.	52 10 W.	
I. Cephalonia	Eu.	Turkey	Medit. Sea	38 20 N.	20 11 E.	
Ceuta	Africa	Barbary	Medit. Sea	35 49 N.	5 25 W.	
Chain Island	Asia	Society Isles	Pacif. Ocean	17 25 S.	145 30 W.	
Chandernagar	Asia	Bengal	River Ganges	22 51 N.	88 34 E.	
Charles Town	Am.	Carolina	Ashley River	33 22 N.	79 50 W.	3 0
C. Charles	Am.	Virginia	Atl. Ocean	37 11 N.	76 07 W.	
I. of } East end Charles } West end	Am.	Labradore	Hudson's Str.	62 46 N. 62 48 N.	74 15 W. 75 30 W.	10 15
C. Charles	Am.	New Britain	West. Ocean	51 50 N.	51 10 W.	
Charlotte's Isles	Asia	Guadalcanal	Pacif. Ocean	11 0 S.	164 0 E.	
C. Charlotte	Am.	S. Georgia	Atl. Ocean	54 32 S.	36 12 W.	
Q. Charlotte's Sound	Asia	N. Zealand	Pacif. Ocean	41 6 S.	174 19 E.	9 00
Q. Charlotte's Foreld.	Asia	N. Caledonia	Pacif. Ocean	22 15 S.	167 18 E.	
I. Charlton	Am.	New Wales	Hudson's Bay	52 03 N.	79 00 W.	
Chateaux Bay	Am.	Labradore	Atl. Ocean	52 1 N.	55 50 W.	
B. Chebucto	Am.	Nova Scotia	Atl. Ocean	44 45 N.	63 18 W.	
Cheignecto	Am.	Nova Scotia	B. Fundy	46 15 N.	63 11 W.	0 45
Cherbourg	Eu.	France	Eng. Channel	49 38 N.	01 33 W.	7 30
Cherry Isle	Eu.	Greenland	North Ocean	74 35 N.	18 05 E.	
Chester	Eu.	England	Irish Sea	53 10 N.	2 25 W.	
Chiddock	Eu.	England	Eng. Channel	50 47 N.	3 00 W.	
C. Chidley	Am.	New Britain	Hudf. Straits	60 22 N.	65 00 W.	
I. Chiloe { N. point S. point	Am.	Patagonia	Pacif. Ocean	41 45 S. 43 50 S.	73 05 W.	
C. Chiokotskago	Asia	Siberia	North Ocean	64 00 N.	174 45 W.	
Christiana	Eu.	Norway	Sound	59 25 N.	10 30 E.	
Christianople	Eu.	Sweden	Baltic Sea	55 55 N.	15 10 E.	
Christianstadt	Eu.	Sweden	G. Bothnia	62 47 N.	22 50 E.	
Christmas Sound	Am.	T. del Fuego	Pacif. Ocean	55 22 S.	70 01 W.	2 30
Christmas Harbour	Africa	Kergulons L.	Ind. Ocean	48 41 S.	69 7 E.	10 00
Christmas Isle	Am.		Pacif. Ocean	1 58 N.	157 30 W.	
I. St. Christopher's	Am.	Carib. Isles	Atl. Ocean	17 15 N.	62 38 W.	
R. St. Christopher's	Africa	Cassers	Indian Ocean	32 47 S.	30 00 E.	
C. Chukchenge	Asia	Siberia	North Ocean	66 30 N.	171 10 W.	
C. Churchill }	Am.	New Wales	Hudson's Bay	58 48 N. 58 47 N.	93 10 W. 94 03 W.	7 20
R. Churchill }	Am.	New Wales	Hudson's Bay	58 48 N. 58 47 N.	93 10 W. 94 03 W.	7 20
I. Chusan	Asia	China	Chinese Sea	30 00 N.	121 50 E.	
Civita Vecchia	Eu.	Italy	Medit. Sea	42 5 N.	11 51 E.	
C. Clear	Eu.	Ireland	West. Ocean	51 18 N.	9 50 W.	4 30
Clerk's Isles	Am.	S. Georgia	Atl. Ocean	55 6 S.	34 37 W.	
I. Cloate	Asia	India	Indian Ocean	22 00 S.	95 40 E.	
Cochin	Asia	India	Indian Ocean	9 50 N.	76 05 E.	
I. Cocos	Asia	India	Indian Ocean	12 20 S.	98 10 E.	
I. Cocos	Am.	Mexico	Pacif. Ocean	5 00 N.	88 45 W.	
I. Cocos	Asia	India	B. Bengal	14 0 N.	93 16 E.	
C. Cod	Am.	New Eng.	Atl. Ocean	42 15 N.	69 27 W.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Colchester	Eu.	England	Germ. Ocean	52 00 N.	0 58 E.	
C. Cold	Eu.	Greenland	North Ocean	79 00 N.	10 00 E.	
C. Colnet	Asia	N. Caledonia	Pacif. Ocean	20 30 S.	164 56 E.	
R. Colerado	Am.	New Spain	G. California	31 40 N.	115 25 W.	
I. Colgau	Eu.	Russia	North Ocean	69 20 N.	45 00 E.	
Collioure	Eu.	Spain	Medit. Sea	42 31 N.	3 10 E.	
C. Colone	Asia	Turkey	Archipelago	37 43 N.	24 41 E.	
C. Colone	Asia	Natolia	Archipelago	39 10 N.	27 04 E.	
C. Colonne	Eu.	Italy	Medit. Sea	38 56 N.	18 05 E.	
C. Colville	Asia	N. Zealand	Pacif. Ocean	36 27 S.	174 48 E.	
Comana	Am.	Terra Firma	Atl. Ocean	10 00 N.	65 07 W.	
C. Comarin	Asia	India	Indian Ocean	7 55 N.	78 7 E.	
C. Comfort	Am.	New Wales	Hudson's Bay	64 45 N.	82 30 W.	
Concarneau	Eu.	France	B. Biscay	47 54 N.	3 50 W.	3h. oom.
C. Conception	Am.	California	Pacif. Ocean	35 40 N.	126 01 W.	
B. Conception Entra	Am.	Newfoundl.	Atl. Ocean	48 25 N.	50 07 W.	
Conception	Am.	Chili	Pacif. Ocean	36 43 S.	73 13 W.	
R. Congo	Africa	Congo	Eth. Ocean	5 45 S.	11 53 E.	
I. Coningen	Asia	N. Zealand	Pacif. Ocean	34 30 S.	164 25 E.	
Coningsburgh	Eu.	Poland	Baltic Sea	54 44 N.	21 53 E.	
Conquet	Eu.	France	Eng. Channel	48 30 N.	4 35 W.	2 15
C. Conquibaco	Am.	Terra Firma	Atl. Ocean	12 15 N.	69 57 W.	
Constantinople	Eu.	Turkey	Archipelago	41 00 N.	28 53 E.	
Cook's Straits	Asia	N. Zealand	Pacif. Ocean	41 6 S.	174 30 E.	
Cooper's Isle	Am.	S. Georgia	Atl. Ocean	54 57 S.	36 0 W.	
Cook's Town	Eu.	Ireland	Atl. Ocean	54 38 N.	6 35 W.	
Copenhagen	Eu.	Denmark	Baltic Sea	55 41 N.	12 40 E.	
Coperwic	Eu.	Norway	Sound	59 20 N.	10 10 E.	
I. Copland	Eu.	Ireland	Irish Sea	54 40 N.	6 40 W.	
I. Coquet	Eu.	England	Germ. Ocean	55 20 N.	1 25 W.	3 00
R. Coquimbo	Am.	Chili	Pacif. Ocean	29 54 S.	71 10 W.	
C. Corbau	Asia	Natolia	Archipelago	38 03 N.	26 58 E.	
Cordoue	Eu.	France	B. Biscay	45 30 N.	01 10 W.	
Corea, South limit	Asia	China	Pacif. Ocean	34 50 N.	124 25 E.	
I. Corfu	Eu.	Turkey	Mediterran.	39 50 N.	19 48 E.	
C. Corientes	Africa	Caffres	Indian Ocean	24 08 S.	36 49 E.	
C. Corientes	Am.	Mexico	Pacif. Ocean	20 18 N.	108 00 W.	
Corinth	Eu.	Turkey	Archipelago	37 30 N.	23 00 E.	
Corke	Eu.	Ireland	St. Geo. Ch.	51 54 N.	8 30 W.	6 30
P. Cornwallis	Asia	India	B. Bengal	13 17 N.	93 2 E.	10 00
C. Coronation	Asia	N. Caledonia	Pacif. Ocean	22 5 S.	167 8 E.	
C. Corfe	Africa	Guinea	Eth. Sea	5 12 N.	0 23 W.	3 30
C. Corfo, N. pt.	Eu.	Italy	Mediterran.	42 53 N.	9 40 E.	
C. Bonfacio, S. pt.	Eu.	Italy	Mediterran.	42 22 N.	9 26 E.	
I. Corvo	Eu.	Azores	Atl. Ocean	39 42 N.	31 02 W.	
I. Cosmoledo	Africa	Madagascar	Indian Ocean	10 28 S.	51 40 E.	
I. Coudre	Am.	Canada	R. St. Lawr.	47 31 N.	69 2 W.	
Cowes	Eu.	I. of Wight	B. Channel	50 46 N.	1 15 W.	
Cow and Calf	Eu.	Ireland	West Ocean	51 22 N.	10 30 W.	
I. Cozumel	Am.	Yucatan	Atl. Ocean	19 36 N.	86 35 W.	
Cremmunster	Eu.	Germany	Inland	48 3 N.	14 8 E.	
R. Croci	Asia	China	B. Nankin	34 06 N.	120 10 E.	
Cromer	Eu.	England	Germ. Ocean	53 05 N.	0 56 E.	7 00
Crooked I. N. point	Am.	Bahama	Atl. Ocean	22 47 N.	73 50 W.	
Crofs Isle	Eu.	Russia	White Sea	66 31 N.	36 33 E.	
Crofs Point	Eu.	Nova Zem.	North Ocean	72 00 N.	53 12 E.	
Crofs Cape	Am.		Pacif. Ocean	57 58 N.	136 39 W.	
St. Cruz	Africa	Barbary	Atl. Ocean	30 36 N.	9 35 W.	
I. St. Cruz	Am.	Antilles I.	Atl. Ocean	17 53 N.	64 55 W.	
C. Antonio, W. pt.				21 45 N.	84 05 W.	
P. de Mais, E. pt.				20 03 N.	74 52 W.	
Hollow Cape	Am.	Antilles I.	Atl. Ocean	19 42 N.	77 25 W.	
St. Jago				20 03 N.	75 51 W.	
St. Mary				21 26 N.	78 10 W.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Cuba { Le St. Esprit Havannah B. Hondy	Am.	Antilles I.	Atl. Ocean	{ 21 56 N. 23 12 N. 22 54 N.	{ 79 50 W. 81 45 W. 82 40 W.	
Cubbs Isles	Am.	New Wales	Hudson's Bay	54 15 N.	82 34 W.	
Cubello	Asia	Ind. Malab.	Indian Ocean	7 50 N.	71 55 E.	
Cuddalore	Asia	India	B. Bengal	11 41 N.	79 51 E.	
C. Cumberland	Asia	N. Hebrides	Pacif. Ocean	14 40 S.	166 47 E.	
Cumberland Isles	Asia	Society Isles	Pacif. Ocean	19 18 S.	140 36 E.	
B. Cumberland	Am.	North Main	Davis's Str.	66 40 N.	65 20 W.	
Cumberland House	Am.	New Wales	Inland	53 57 N.	102 4 W.	
Curassoa	Am.	Terra Firma	Atl. Ocean	11 56 N.	68 20 W.	
I. Cuzzola	Eu.	Turkey	Medit. Sea	42 50 N.	16 55 E.	
Cusco	Am.	Peru	Inland	12 25 S.	73 35 W.	
Cypre's { C. Bassa, W. end C. St. Andr. E. pt. C. de Gasse, S. pt. D.	Asia	Syria	Medit. Sea	{ 35 04 N. 35 40 N. 34 35 N.	{ 33 04 E. 35 08 E. 33 41 E.	
Dabul	Asia	India	Arabian Sea	18 24 N.	73 33 E.	
Dahlak	Asia	Arabia	Red Sea	15 50 N.	41 44 E.	
Isles of Danger	Asia	Society Isles	Pacif. Ocean	10 15 S.	165 50 W.	
I. Dageroort	Eu.	Livonia	Baltic Sea	58 55 N.	22 32 E.	
Light-house						
Dantzic	Eu.	Poland	Baltic Sea	54 22 N.	18 39 E.	
Str. Dardanel	Eu.	Turkey	Archipelago	40 10 N.	26 26 E.	
Gulph Darien	Am.	Terra Firma	Caribbean Sea	8 45 N.	76 35 W.	
C. Darby	Am.		Beering's St.	64 21 N.	162 55 W.	
Dartmouth	Eu.	England	Eng. Channel	50 27 N.	3 36 W.	6h. 30m.
I. Daffan	Africa	Caffres	Atl. Ocean	33 25 N.	18 7 E.	
I. Dauphin	Am.	Louifiana	G. Mexico	29 40 N.	87 53 W.	
St. David's Head	Eu.	Wales	St. Geo. Ch.	51 55 N.	5 22 W.	6 co
Fort St. David's	Asia	India	Corom. Coast	12 05 N.	80 55 E.	
I. Defeada	Am.	Carib. Isles	Atl. Ocean	16 36 N.	61 10 W.	
C. Defeada	Am.	T. del Fueg	Pacif. Ocean	53 4 S.	74 13 W.	
C. Defire	Eu.	Nova Zem.	North Ocean	77 45 N.	79 20 E.	
C. Desolation	Am.	Greenland	North Ocean	61 45 N.	47 00 W.	
Devicotta	Asia	India	B. Bengal	11 21 N.	80 00 E.	
Devil's Isles	Eu.	Greenland	North Ocean	80 00 N.	11 43 E.	
Dewpoint	Asia	India	B. Bengal	16 07 N.	81 47 E.	
I. Diamond	Asia	India	B. Bengal	15 50 N.	94 18 E.	
C. Diego	Am.	T. del Fuego	St. Le Maire	54 33 S.	65 9 W.	
I. Diego Rayes	Asia	Ind. Malab	Indian Ocean	{ 0 45 S. 0 30 N.	{ 70 25 E.	
I. Diego Garcia	Africa	India	Indian Ocean	7 23 S.	72 25 E.	
Str. Diemen	Asia	Japan Isles	Pacif. Ocean	31 12 N.	130 55 E.	
I. Dieu	Eu.	France	Bay of Biscay	46 26 N.	2 20 W.	
Dieppe	Eu.	France	Eng. Channel	49 55 N.	1 12 E.	10 30
C. Dudley Diggs	Am.	Greenland	Baffin's Bay	76 48 N.	59 07 W.	
C. Diggs	Am.	Labradore	Hudson's Bay	62 45 N.	78 30 W.	
Po. Dia	Asia	India	Indian Ocean	21 37 N.	70 28 E.	
C. Dobbs	Am.	North Wales	Hudson's Bay	65 10 N.	86 25 W.	
Dofare	Asia	Arabia	Indian Ocean	16 24 N.	53 42 E.	
St. Domingo	Am.	Antilles	Atl. Ocean	18 25 N.	69 30 W.	
Hispaniola						
I. Dominico	Am.	Caribbee	Atl. Ocean	15 18 N.	61 28 W.	
Dordrecht	Eu.	D. Neth.	R. Maes	52 00 N.	4 26 E.	
C. Dorfui	Africa	Ajan	Indian Ocean	10 15 N.	50 44 E.	
C. Doro	Eu.	Turkey	Archipelago	38 02 N.	25 12 E.	
Dort	Eu.	D. Neth.	Germ. Ocean	51 47 N.	4 40 E.	3 co
I. Dofel	Eu.	Livonia	Baltic Sea	58 20 N.	23 00 E.	
Is. Dos Banhos	Africa	Zanguebar	Indian Ocean	5 15 N.	49 24 E.	
Dover	Eu.	England	Eng. Channel	51 07 N.	1 24 E.	11 30
Downs	Eu.	England	Germ. Ocean	51 25 N.	1 32 E.	1 15
Po. Dradate	Africa	Egypt	Red Sea	19 56 N.	37 40 E.	
Po. Drake, fir Francis	Am.	California	Pacif. Ocean	38 45 N.	128 35 W.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.	
Drake's Isle	Eu.	England	Ply. Sound	51 21 N.	4 9 W.	5h. 45m.	
Drontheim	Eu.	Norway	North Ocean	63 26 N.	11 08 E.		
Dublin	Eu.	Ireland	Irish Sea	53 21 N.	6 10 W.	9 15	
Dunbar	Eu.	Scotland	Germ. Ocean	55 58 N.	2 22 W.	2 30	
Dundalk	Eu.	Ireland	Irish Sea	53 57 N.	6 28 W.		
Dundee	Eu.	Scotland	Germ. Ocean	56 26 N.	2 48 W.	2 15	
Dungarven	Eu.	Ireland	Atl. Ocean	51 57 N.	7 55 W.	4 30	
Dungeness	Eu.	England	Eng. Channel	50 55 N.	1 03 E.	9 45	
Duncanby Head	Eu.	Scotland	Germ. Ocean	58 40 N.	2 57 W.		
Dunkirk	Eu.	France	Germ. Ocean	51 02 N.	2 27 E.	0 00	
Dunnotie	Eu.	I. White	Eng. Channel	50 34 N.	1 15 W.	9 45	
Durazzo	Eu.	Turkey	Medit. Sea	41 58 N.	25 00 E.		
Dusky Bay	Asia	N. Zealand	Pacif. Ocean	45 47 S.	166 23 E.	10 57	
E.							
Eaooe	Asia	Friend. Isles	Pacif. Ocean	21 24 S.	174 25 W.		
C. East	Asia	Ischutski	Beering's St.	66 5 N.	169 39 W.		
C. East	Am.	Statenland	Stra. le Maire	54 54 S.	64 47 W.		
Easter Isl.	Am.	Chili	Pacif. Ocean	27 7 S.	109 42 W.	2 00	
C. Edgcombe	Am.		Pacif. Ocean	57 4 N.	135 50 W.		
Edinburgh	Eu.	Scotland	Germ. Ocean	55 53 N.	3 7 W.	4 30	
Edystone	Eu.	England	Eng. Channel	50 8 N.	4 20 W.	5 30	
Egmont Isle	Asia	Society Isles	Pacif. Ocean	19 20 S.	138 30 W.		
C. Egmont	Asia	N. Zealand	Pacif. Ocean	39 20 S.	173 45 E.		
Eimeo	Asia	Society Isles	Pacif. Ocean	17 30 S.	149 49 W.		
I. Elba	Eu.	Italy	Mediterran.	42 52 N.	10 38 E.		
R. Elbe mouth	Eu.	Germany	Germ. Ocean	54 18 N.	7 10 E.	0 00	
Elbing	Eu.	Poland	Baltic Sea	54 12 N.	20 35 E.		
Elfsingburgh	Eu.	Sweden	Baltic Sea	56 00 N.	13 35 E.		
Elfsinore	Eu.	Denmark	Baltic Sea	56 00 N.	13 23 E.		
I. Elutheria	{ N. point S. point	Am.	Bahama	Atl. Ocean	25 45 N.	76 42 W.	
					24 57 N.	75 53 W.	
Emden	Eu.	Germany	Germ. Ocean	53 05 N.	7 26 E.	0 00	
R. Emes mouth	Eu.	Germany	Germ. Ocean	53 10 N.	7 20 E.	7 30	
Enchuyfen	Eu.	D. Neth.	Zuyder Sea	52 43 N.	5 06 E.	0 00	
Endeavour R.	Asia	N. Holland	Pacif. Ocean	15 26 S.	145 12 E.		
I. Engano	Am.	Sumatra	Indian Ocean	6 00 S.	102 35 E.		
B. Enhorn	Eu.	Greenland	North Sea	78 45 N.	26 05 E.		
Ephesus	Asia	Natolia	Archipelago	38 00 N.	27 53 E.		
Erramanga	Asia	N. Hebrides	Pacif. Ocean	18 44 S.	169 20 E.		
Erzerum	Asia	Arminea	Black Sea	39 57 N.	48 41 E.		
Eftaples	Eu.	France	Eng. Channel	50 34 N.	1 42 E.	11 00	
Eustatia	Am.	Caribbee	Atl. Ocean	27 50 N.	63 04 W.		
I. Exuma	Am.	Bahama	Atl. Ocean	23 25 N.	75 35 W.		
F.							
Fairhead	Eu.	Ireland	West. Ocean	55 19 N.	6 20 W.		
Fairlight	Eu.	England	Eng. Channel	50 53 N.	0 43 E.		
C. Falcon	Africa	Barbary	Medit. Sea	36 03 N.	0 14 W.		
I. Falkland	{ E. end Isl. A- nifant	Am.	Patagonia	Atl. Ocean	51 05 S.	56 40 W.	
					52 27 S.	61 53 W.	
Falmouth	Eu.	England	Eng. Channel	50 8 N.	4 58 W.	5 30	
C. Falo	Eu.	Turkey	Archipelago	40 12 N.	24 27 E.		
C. Falfo	Africa	Caffers	Indian Ocean	34 16 S.	18 44 E.		
C. Falfo	Africa	Zanguebar	Indian Ocean	8 52 S.	59 55 E.		
Falsterborn	Eu.	Sweden	Baltic Sea	55 20 N.	13 36 E.		
I. Fana	Eu.	Turkey	Medit. Sea	40 14 N.	19 32 E.		
R. Farate	Africa	Egypt	Red Sea	21 40 N.	36 29 E.		
Faro Head	Eu.	Scotland	West. Ocean	58 40 N.	4 50 W.		
C. Farewell	Asia	N. Zealand	Pacif. Ocean	40 35 S.	172 47 E.		
C. Farewell	Am.	Greenland	North Ocean	59 37 N.	42 37 W.		
C. Fartack	Asia	Arabia	Indian Ocean	15 41 N.	51 31 E.		
C. Fehr	Am.	Carolina	Atl. Ocean	34 04 N.	78 09 W.		
I. Fernando Noronha	Am.	Brasil	Atl. Ocean	3 56 S.	32 23 W.		
I. Felicur, Lipari Isles	Eu.	Italy	Medit. Sea	38 33 N.	14 51 E.		
I. Fermina	Eu.	Turkey	Archipelago	37 24 N.	25 05 E.		

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
I. Fernandepo	Africa	Guinea	Atl. Ocean	3 02 N.	3 33 E.	
I. Ferro	Africa	Canaries	Atl. Ocean	27 48 N.	17 40 W.	
C. Finisterre	Eu.	Spain	Atl. Ocean	42 52 N.	9 12 W.	
I. Fironda	Asia	Corea	Pacif. Ocean	33 30 N.	127 25 E.	
Fladstrand	Eu.	Denmark	Cattagat	57 27 N.	10 37 E.	
Flamborough Head	Eu.	England	Germ. Ocean	54 08 N.	0 11 E.	4h. com.
Florence	Eu.	Italy	Medit. Sea	43 46 N.	11 7 E.	
I. Flores	Eu.	Azores	Atl. Ocean	39 34 N.	30 55 W.	
C. Florida	Am.	Florida	G. Mexico	25 50 N.	80 20 W.	
Flushing	Eu.	D. Neth.	Germ. Ocean	51 33 N.	3 20 W.	7 30
I. Fly	Eu.	D. Neth.	Germ. Ocean	53 16 N.	5 35 E.	0 45
Forbisher's Straits	Am.	Greenland	Atl. Ocean	62 05 N.	47 18 W.	7 30
North Foreland	Eu.	England	Germ. Ocean	51 28 N.	1 25 E.	
South Foreland	Eu.	England	Eng. Channel	51 12 N.	1 24 E.	9 45
Foreland Fair	Eu.	Ireland	North Ocean	55 05 N.	6 30 W.	9 45
Foreland Fair	Eu.	Greenland	North Ocean	79 18 N.	10 50 E.	
Foreland Merchants	Am.	Greenland	North Ocean	63 20 N.	17 05 W.	
I. Formetaria	Eu.	Spain	Medit. Sea	38 33 N.	1 15 E.	
I. Formigas	Eu.	Azores	Atl. Ocean	37 17 N.	24 43 W.	
C. Formosa	Africa	Guinea	Eth. Sea	4 22 N.	5 43 E.	
R. Formosa	Africa	Guinea	Eth. Sea	6 10 N.	4 49 E.	
I. Formosa { N. point	Asia	China	Indian Ocean	21 25 N.	121 25 E.	
I. Forteventura, S. } W. end	Africa	Canaries	Atl. Ocean	22 00 N.	120 40 E.	
Foulness	Eu.	England	Germ. Ocean	28 35 N.	14 04 W.	
Foulfound	Eu.	Greenland	North Ocean	52 57 N.	0 58 E.	6 45
Fowey	Eu.	England	Eng. Channel	77 30 N.	12 50 E.	
I. France, P. Louis	Eu.	England	Eng. Channel	50 25 N.	4 30 W.	5 15
C. St. Francis	Africa	Madagascar	Indian Ocean	20 10 S.	57 33 E.	
I. St. Francisco	Am.	Peru	Pacif. Ocean	0 30 N.	80 35 W.	
R. St. Francisco	Africa	Zanguebar	Indian Ocean	0 30 N.	80 35 W.	
C. Francois	Am.	Brasil	Atl. Ocean	6 23 S.	53 22 E.	
Frederickstadt	Am.	Domingo	Atl. Ocean	10 55 S.	36 30 W.	
C. Frehel	Eu.	Norway	Sound	19 47 N.	72 15 W.	
French Keys	Eu.	France	Eng. Channel	59 00 N.	11 10 E.	
Fretum Borough	Eu.	Bahama	Atl. Ocean	48 41 N.	2 15 W.	
C. Frio	Am.	Russia	North Ocean	21 30 N.	72 10 W.	
R. Fugor	Eu.	Brasil	Atl. Ocean	70 00 N.	61 20 E.	
I. Fuego	Am.	Zanguebar	Indian Ocean	23 00 S.	40 11 W.	
Furneaux Island	Africa	De Verd	Atl. Ocean	00 10 N.	42 05 E.	
B. Fushan	Asia	Soc. Isles	Pacif. Ocean	14 55 N.	24 28 W.	
I. Fyal	Asia	China	Pacif. Ocean	17 11 S.	143 07 W.	
G.	Eu.	Azores	Atl. Ocean	23 00 N.	112 35 E.	
I. Galla	Am.	Terra Firma	Pacif. Ocean	38 32 N.	28 36 W.	2 20
R. Gallega	Am.	Patagonia	Atl. Ocean	2 40 N.	79 35 W.	
I. Gallego	Am.	Terra Firma	Pacif. Ocean	51 37 S.	65 35 W.	
Gallipoli	Eu.	Italy	Pacif. Ocean	1 40 N.	104 35 W.	
Gallipoly	Eu.	Turkey	Medit. Sea	40 19 N.	18 08 E.	
I. Gallita	Eu.	Turkey	Archipelago	40 36 N.	27 02 E.	
C. de Galle	Africa	Barbary	Medit. Sea	37 42 N.	9 03 E.	
Is. Gallepago	Asia	I. Ceylon	Indian Ocean	6 03 N.	80 27 E.	
Gally Head	Am.	Peru	Pacif. Ocean	2 00 N.	89 00 W.	
Galway	Eu.	Ireland	West. Ocean	2 00 S.	89 00 W.	
R. Gambia	Eu.	Ireland	West. Ocean	52 40 N.	9 30 W.	
I. Gamo	Africa	Negroland	Atl. Ocean	53 10 N.	10 03 W.	
Gangam	Asia	India	Indian Ocean	13 00 N.	14 58 W.	
C. Gardafui	Asia	India	Indian Ocean	3 05 S.	77 25 E.	
R. Garonne	Asia	India	B. Bengal	19 22 N.	85 25 E.	
Gaspay Bay	Africa	Anian	Indian Ocean	11 48 N.	50 25 E.	
C. de Gatt	Eu.	France	B. Biscay	45 30 N.	1 05 W.	3 00
C. Gear	Am.	Acadia	G. St. Lawr.	48 48 N.	64 36 W.	1 30
Geneva	Eu.	Spain	Medit. Sea	36 32 N.	2 05 W.	
Genoa	Africa	Barbary	Atl. Ocean	30 35 N.	10 01 W.	
	Eu.	Savoy	Inland	46 12 N.	6 5 E.	
	Eu.	Italy	Medit. Sea	44 25 N.	8 41 E.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
C. St. George	Am.	Newfoundl.	Atl. Ocean	48 28 N.	57 43 W.	
C. George	Am.	S. Georgia	Atl. Ocean	54 17 S.	36 33 W.	
C. George	Africa	KerguelensL.	Indian Ocean	49 54 S.	70 17 E.	
B. St. George	Am.	Newfoundl.	Atl. Ocean	48 19 N.	57 30 W.	
I. St. George	Asia	Natolia	Archipelago	38 47 N.	25 07 E.	
I. St. George	Eu.	Azores	Atl. Ocean	38 39 N.	28 00 W.	
St. George's Fort	Asia	India	B. Bengal	13 05 N.	80 34 E.	
Gibraltar	Eu.	Spain	Medit. Sea	36 05 N.	5 17 W.	oh oom.
Glibert's Island	Am.	T. del Fuego	Pacif. Ocean	55 13 S.	71 4 W.	
I. Gilolo { N. point	Asia	Spice Islands	Indian Ocean	2 30 N.	128 00 E.	
I. Gilolo { S. point				1 30 S.	129 25 E.	
Glasgow	Eu.	Scotland	R. Clyde	55 52 N.	4 10 W.	
Gloucester Isles	Asia	Society Isles	Pacif. Ocean	19 11 S.	140 4 W.	
Gloucester Isles	Asia	Society Isles	Indian Ocean	20 36 S.	146 7 W.	
Gluckstad	Eu.	Holstein	Elbe	53 48 N.	9 31 E.	
Goa	Asia	India	Malabar	15 31 N.	73 50 E.	
Goes	Eu.	D. Neth.	Germ. Ocean	51 39 N.	4 05 E.	
Golfe triste	Am.	Terra Firma	Carrib. Sea	10 20 N.	67 40 W.	
Gombroon	Asia	Perfia	Perfian Gulf	27 40 N.	55 20 E.	
I. Gomerio	Africa	Canaries	Atl. Ocean	28 06 N.	17 03 W.	
C. Gondewar	Asia	India	B. Bengal	16 55 N.	82 55 E.	
C. Good Hope	Africa	Caffies	Indian Ocean	34 29 S.	18 28 E.	3 00
I. Gorea	Africa	Negroland	Atl. Ocean	14 40 N.	17 20 W.	
I. Gorgona	Eu.	Italy	Medit. Sea	43 21 N.	9 11 E.	
I. Goth-land { N. end				58 60 N.	20 15 E.	
I. Goth-land { S. end	Eu.	Sweden	Baltic Sea	56 58 N.	19 37 E.	
I. Goth-land { Wisby				57 40 N.	19 50 E.	
I. Goto	Asia	Corea	Pacif. Ocean	34 25 N.	125 50 E.	
Gottenberg	Eu.	Sweden	Sound	57 42 N.	11 44 E.	
Gottingen	Eu.	Germany	Inland	51 32 N.	9 58 E.	
Gower's Isle	Asia	N. Britain	Pacif. Ocean	7 56 S.	158 56 E.	
R. Grand	Am.	Paraguay	Atl. Ocean	31 53 S.	50 35 W.	
Granville	Eu.	France	Eng. Channel	48 50 N.	1 32 W.	7 00
C. De Grat	Am.	Newfoundl.	Atl. Ocean	51 36 N.	55 33 W.	
I. Gratiofa	Africa	Canaries	Atl. Ocean	29 15 N.	13 07 W.	
I. Gratiofa	Eu.	Azores	Atl. Ocean	39 02 N.	27 53 W.	
C. Gratiofa a Dios	Am.	New Spain	Caribbean Sea	14 48 N.	82 15 W.	
Graveline	Eu.	France	Eng. Channel	50 59 N.	2 13 E.	0 00
Gravesend	Eu.	England	R. Thames	51 35 N.	0 20 E.	1 30
I. Grenada	Am.	Caribbee	Atl. Ocean	11 52 N.	61 39 W.	
Greenwich	Eu.	England	R. Thames	51 29 N.	0 05 E.	
C. Gremia	Eu.	Turkey	Archipelago	40 33 N.	26 20 E.	
Gripfswald	Eu.	Germany	Baltic Sea	54 04 N.	13 43 E.	
Griuesby	Eu.	England	Germ. Ocean	53 30 N.	0 56 E.	
Groin, or C. Corunna	Eu.	Spain	B. Biscay	43 28 N.	9 20 W.	3 00
I. Groy	Am.	Newfoundl.	Atl. Ocean	50 56 N.	55 35 W.	
I. Guadaloupe	Am.	Caribbee	Atl. Ocean	16 00 N.	61 55 W.	
Guayaquil	Am.	Peru	Pacif. Ocean	2 10 S.	81 05 W.	
I. Gurnsey	Eu.	England	Eng. Channel	49 30 N.	2 47 W.	1 30
Gulf	Eu.	England	St. Geo. Cha.	50 06 N.	6 00 W.	
Gurief	Asia	Astracan	Caspian Sea	47 7 N.	52 02 E.	
H						
Haeluit's Headland	Eu.	Greenland.	North Ocean	79 55 N.	12 00 E.	
Hague	Eu.	Holland	Ger. Ocean	52 4 N.	4 22 E.	
I. Hai-nam { N. E. point	Asia	China	Indian Ocean	19 45 N.	110 13 E.	
I. Hai-nam { S. W. point				18 22 N.	108 13 E.	
Halifax	Am.	Nova Scotia	Western Oc.	44 46 N.	63 20 W.	7 30
I. Hall	Am.	Greenland	Atl. Ocean	63 56 N.	44 26 W.	
Halliford	Eu.	Iceland	North Ocean	64 30 N.	27 15 W.	
Hamburgh	Eu.	Germany	R. Elbe	53 34 N.	9 55 E.	6 00
C. Hang-lip	Africa	Caffres	Indian Ocean	34 16 S.	18 49 E.	
Hare Isle	Am.	Canada	R. St. Lawr.	48 00 N.	63 26 W.	3 30
Harlem	Eu.	D. Neth.	Germ. Ocean	52 24 N.	4 10 E.	9 00
Hartland Point	Eu.	England	Bristol Chan.	51 06 N.	4 35 W.	
Hartlepool	Eu.	England	Germ. Ocean	54 40 N.	0 56 W.	3 00

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Harwich	Eu.	England	Germ. Ocean	52 11 N.	1 18 E.	11h. 15m.
C. Harteras	Am.	Carolina	Atl. Ocean	35 24 N.	76 20 W.	
Hastings	Eu.	England	B. Channel	50 52 N.	0 46 E.	
Havannah	Am.	I. Cuba	Atl. Ocean	23 12 N.	82 13 W.	
Havre de Grace	Eu.	France	Eng. Channel	49 30 N.	0 11 E.	9 00
Hawke's Bay	Asia.	N. Zealand	Pacif. Ocean	39 30 S.	177 6 E.	
I. St. Helena	Africa	Caffres	Atl. Ocean	15 55 S.	5 44 W.	
Helie's Sound	Eu.	Greenland	North Ocean	79 15 N.	12 50 E.	
Is. Heligh's Land	Eu.	Norway	North Ocean	65 15 N.	9 30 E.	
C. Henlopen	Am.	Maryland	Atl. Ocean	38 48 N.	75 08 W.	
Henly House	Am.	New Wales	Inland	51 14 N.	84 41 W.	
C. Henrietta Maria	Am.	New Wales	Hudson's Bay	55 10 N.	84 00 W.	12 00
C. Henry	Am.	Virginia	Atl. Ocean	37 00 N.	76 23 W.	11 15
Hernofand	Eu.	Sweden	G. Bothnia	62 38 N.	17 58 E.	
Hervey's Isle	Asia.	Soc. Isles	Pacif. Ocean	19 17 S.	158 43 W.	
I. Hestelöe	Eu.	Denmark	Cattagat	56 12 N.	11 48 E.	
I. Heys	Eu.	France	B. Biscay	46 24 N.	2 14 W.	
High Mount	Eu.	Greenland	North Ocean	83 23 N.	26 40 E.	
Hinchbrook I.	Asia	N. Hebrides	Pacif. Ocean	17 25 S.	168 38 E.	
I. Hispaniola	Am.	Antilles	Atl. Ocean	C. Tiberoon	18 17 N.	74 24 W.
				S. Louis	18 19 N.	73 11 W.
				C. St. Nicholas	19 50 N.	73 18 W.
				Po. Grave	18 28 N.	72 42 W.
				St. Domingo	18 25 N.	69 30 W.
I. St. Raphael	Am.	Antilles	Atl. Ocean	C. Raphael	19 05 N.	68 30 W.
				Hoai-Ngham	33 35 N.	118 55 E.
				Hogsties	21 41 N.	73 25 W.
				C. La Hogue	49 45 N.	1 51 W.
New Holland	Asia		Indian Ocean	W. limit	25 30 S.	111 10 E.
				N. Ditto	12 35 S.	141 31 E.
				S. Ditto	43 38 S.	146 00 E.
				E. Ditto	27 10 S.	153 39 E.
Holme Point	Eu.	England	Germ. Ocean	53 0 N.	0 36 E.	
Holy Cape	Asia	Siberia	North Ocean	72 32 N.	179 45 E.	
Holy Head	Eu.	Wales	Irish Sea	53 23 N.	4 40 W.	1 30
C. Honduras	Am.	New Spain	Caribbean Sea	16 18 N.	85 23 W.	
B. Hondy, I. Cuba	Am.	Antilles	Atl. Ocean	22 54 N.	82 40 W.	
Honfleur	Eu.	France	R. Seine	49 24 N.	0 20 E.	9 00
Hood's Isle	Asia	Marquesas	Pacif. Ocean	9 26 S.	138 47 W.	
Hoogstraeten	Eu.	Netherlands	Inland	51 25 N.	4 52 E.	
Hope Isle	Eu.	Greenland	North Ocean	76 22 N.	23 40 E.	
C. Horn	Am.	T. del Fuego	Pacif. Ocean	55 59 S.	67 21 W.	
Hornfound	Eu.	Greenland	North Ocean	76 41 N.	13 36 E.	
Hout Bay	Africa	Caffres	Indian Ocean	34 03 N.	18 24 E.	
Howe's Isle	Asia	Society's Is.	Pacif. Ocean	16 46 S.	154 2 W.	
C. How	Asia	N. Holland	Pacif. Ocean	37 24 S.	150 00 E.	
Huahine	Asia	Society Isles	Pacif. Ocean	16 44 S.	151 1 W.	
Hunaston Light	Eu.	England	Germ. Ocean	52 59 N.	0 33 E.	
R. Hughly	Asia	India	B. Bengal	21 45 N.	89 15 E.	
Hull	Eu.	England	R. Humber	53 50 N.	0 28 W.	6 00
R. Humber, Ent.	Eu.	England	Germ. Ocean	53 55 N.	0 24 E.	5 13
Hydrabad	Asia	India	Inland	17 12 N.	78 56 E.	
I. Hyneago	Am.	Bahama	Atl. Ocean	21 27 N.	73 29 W.	
J.						
Jado	Asia	Japan	Pacif. Ocean	36 00 N.	139 40 E.	
C. Jaffanapatan	Asia	I. Ceylon	Indian Ocean	9 47 N.	80 55 E.	
I. Jago	Africa	C. Verd	Atl. Ocean	15 07 N.	23 30 W.	
Jakutskoi	Asia	Siberia	Pacif. Ocean	62 2 N.	129 48 E.	
J. Jam.	Am.	West Indies	Atl. Ocean	West end	18 45 N.	78 00 W.
				Port Royal	18 00 N.	76 40 W.
				East End	17 58 N.	76 05 W.
James Town	Am.	Virginia	B. Chesapeake	37 30 N.	76 00 W.	
R. Janeiro	Am.	Brasil	Atl. Ocean	22 54 S.	43 10 W.	
Japan Isles	Asia		Pacif. Ocean	40 40 N.	141 25 E.	
				31 45 N.	126 10 E.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Jaffy	Eu.	Moldavia	Inland	47 9 N.	27 35 E.	
Java Head	Asia	Java	Indian Ocean	6 49 S.	105 06 E.	
Ice Cove	Am.	N. Main	Hudf. Straits	62 20 N.	69 00 W.	10h. oom.
Ice Point	Eu.	Nova Zem.	North Ocean	77 40 N.	69 10 E.	
Ice Sound	Eu.	Greenland	North Ocean	78 13 N.	12 00 E.	
Ice Cape	Am.		North Ocean	70 29 N.	161 37 W.	
I. Jerfey	Eu.	England	Eng. Channel	49 07 N.	2 26 W.	
Jerusalem	Asia	Palestine	Inland	31 55 N.	35 25 E.	
I. Hay, S. pt.	Eu.	Scotland	West Ocean	55 39 N.	6 20 W.	
R. Indus	Asia	India	Indian Ocean	25 50 N.	66 33 E.	
Ingolstadt	Eu.	Germany	Danube	48 46 N.	11 28 E.	
Inverness	Eu.	Scotland	Germ. Ocean	57 33 N.	4 02 W.	
I. Joanna	Africa	Zanguebar	Indian Ocean	12 05 S.	45 45 E.	
Juddah	Asia	Arabia	Red Sea	22 00 N.	39 27 E.	
C. St. John	Am.	Newfoundl.	Atl. Ocean	50 08 N.	55 32 W.	
C. St. John	Africa	Maiumbo	Eth. Ocean	1 17 N.	9 34 E.	6 0
St. John's	Am.	Newfoundl.	Atl. Ocean	47 34 N.	52 18 W.	
St. John's	Am.	Antigua	Atlantic	17 4 N.	62 4 W.	
I. St. John { E. pt.	Am.	Canada {	Bay St. Lau-	46 30 N.	62 03 W.	
I. St. John de Nova { N. pt.	Am.	Canada {	rence	47 07 N.	64 05 W.	
St. John de Luz	Africa	Madagascar	Indian Ocean	17 00 S.	44 02 E.	
Cape Jones	Eu.	France	B. Biscay	43 10 N.	1 38 W.	3 30
Joppa	Am.	New Britain	Hudson's Bay	58 50 N.	79 00 W.	
Jones Sound	Asia	Syria	Levant	32 45 N.	36 00 E.	
St. Josegh	Am.	Greenland	Baffin's Bay	71 07 N.	91 30 W.	
Ipswich	Am.	California	Pacif. Ocean	23 4 S.	109 35 W.	
Irraname	Eu.	England	Germ. Ocean	52 14 N.	1 00 E.	
Islamabad	Asia	N. Hebrides	Pacif. Ocean	19 31 S.	170 26 E.	
Ismail	Asia	Chittigong	B. Bengal	22 20 N.	92 0 E.	
Ispahan	Eu.	Turkey	R. Zenduro	45 21 N.	28 55 E.	
C. St. Juan	Asia	Perfia	Atl. Ocean	32 25 N.	52 55 E.	
I. Juan Fernandez	Am.	Statenland	Atl. Ocean	54 47 S.	63 42 W.	
Judda	Am.	Chili	Pacif. Ocean	33 45 S.	78 37 W.	
Port. St. Julian	Asia	Arabia	Red Sea	21 29 N.	39 27 E.	
I. Ivica	Am.	Patagonia	S. Atl. Ocean	49 10 S.	66 10 W.	4 45
	Eu.	Spain	Medit. Sea	38 54 N.	1 15 E.	
K						
Kalmer	Eu.	Sweden	Baltic Sea	56 40 N.	17 25 E.	
Kambaya	Asia	India	Indian Ocean	23 36 N.	72 50 E.	
Kamtchatka { Lower	Asia	Siberia	Pacif. Ocean	56 11 N.	159 25 E.	
I. Karaghinskoy { Upper	Asia	Siberia	Pacif. Ocean	54 48 N.	157 25 E.	
I. St. Katharine's	Am.	Brasil	Atl. Ocean	58 00 N.	162 10 E.	
Kaye's Island	Am.		Pacif. Ocean	27 35 S.	49 12 W.	
Keco	Am.		Pacif. Ocean	59 52 N.	144 55 W.	
Kegor	Asia	Tonquin	Indian Ocean	21 55 N.	106 10 E.	
R. Kennebeck	Eu.	Muscovy	North Ocean	70 18 N.	34 00 E.	
Kentish Knock, a {	Am.	N. England	Atl. Ocean	44 00 N.	69 45 W.	
land	Eu.	England	Germ. Ocean	51 42 N.	1 45 E.	0 00
I. St. Kilda	Eu.	Scotland	West. Ocean	57 44 N.	8 18 W.	
I. Kilduin	Eu.	Lapland	North Ocean	69 30 N.	31 20 E.	7 30
Kinsale	Eu.	Ireland	Atl. Ocean	51 41 N.	8 23 W.	5 15
Kiow	Eu.	Ukraine	Neiper	50 30 N.	13 13 E.	
Klip	Eu.	Greenland	North Ocean	80 10 N.	12 22 E.	
Kola	Eu.	Lapland	North Ocean	68 53 N.	33 13 E.	
C. Kol	Eu.	Sweden	Sound	56 50 N.	13 13 E.	
Port Komol	Africa	Abyssinia	Red Sea	22 30 N.	36 17 E.	
Komero Isles	Africa	Zanguebar	Indian Ocean	10 48 S.	44 46 E.	
Korfor Light	Eu.	Denmark	G. Belt	13 10 S.	46 22 E.	
R. Kowimia	Eu.	Siberia	North Ocean	55 20 N.	11 12 E.	
Kullen Light	Eu.	Sweden	Cattagat	70 40 N.	159 00 E.	
L						
Ladrone, or Marian {	Asia		Pacif. Ocean	21 00 N.	144 00 E.	
Illes	Asia		Pacif. Ocean	13 15 N.	142 55 E.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Ladrone (Grand)	Asia	China	Chinese Sea	22 2 N.	114 1 E.	
Laguna	Africa	Canaries	Atlantic	28 29 N.	16 13 W.	
C. L' Aigulle	Africa	Caffraria	Indian Ocean	34 50 S.	20 06 E.	
Lancaster	Eu.	England	St. Geo. Ch.	54 42 N.	4 36 W.	
I. Lancerota	Africa	Canaries	Atl. Ocean	29 10 N.	13 20 W.	
Land's End	Eu.	England	St. Geo. Ch.	50 06 N.	5 50 W.	7h. 30m.
Landscroon	Eu.	Sweden	Sound	55 52 N.	12 55 E.	
Langeness	Eu.	Nova Zem.	North Ocean	74 40 N.	53 36 E.	
I. Lambay	Eu.	Ireland	Irish Sea	53 24 N.	7 30 W.	8 15
I. Lampadosa	Africa	Tunis	Medit. Sea	35 32 N.	12 45 E.	
B. St. Lazarus	Am.	Patagonia	Pacif. Ocean	48 42 S.	73 35 W.	
C. Ledo	Africa	Angola	Atl. Ocean	9 24 S.	12 55 E.	
Leeds	Eu.	England	Aire	53 48 N.	1 33 W.	
Leith	Eu.	Scotland	Germ. Ocean	55 58 N.	2 59 W.	4 30
Leghorn	Eu.	Italy	Medit. Sea	43 33 N.	10 25 E.	
Leipfic	Eu.	Saxony	Inland	51 19 N.	12 25 E.	
I. Lemnos	Asia	Natolia	Archipelago	40 02 N.	25 36 E.	
C. Lengua	Eu.	Turkey	Medit. Sea	40 44 N.	19 36 E.	
Leostoff	Eu.	England	Germ. Ocean	52 38 N.	1 54 E.	9 45
Lepanto	Eu.	Turkey	Medit. Sea	38 20 N.	22 03 E.	
Leper's Isle	Asia	N. Hebrides	Pacif. Ocean	15 23 S.	167 57 E.	
I. Lefow	Eu.	Denmark	Sound	57 05 N.	11 06 E.	
Liverpool	Eu.	England	Irish Sea	53 22 N.	3 10 W.	11 15
I. Lewes, N. point	Eu.	Scotland	West. Ocean	58 35 N.	6 37 W.	6 30
Leyden	Eu.	Holland		52 10 N.	4 33 E.	
Liampo, or Ningpo	Asia	China	Pacif. Ocean	29 58 N.	120 23 E.	
Lima	Am.	Peru	Pacif. Ocean	12 01 S.	76 44 W.	
Lime	Eu.	England	Eng. Channel	50 45 N.	3 15 W.	7 00
Limerick	Eu.	Ireland	R. Shannon	52 22 N.	10 00 W.	
I. Limofa	Eu.	Italy	Medit. Sea	36 08 N.	13 01 E.	
I. Lipari	Eu.	Italy	Medit. Sea	38 35 N.	15 31 E.	
I. Liqueo	Asia	Japan	Pacif. Ocean	28 00 N.	127 30 E.	
Lisbon	Eu.	Portugal	R. Tagus	38 42 N.	9 4 W.	2 15
Lisbon Rock	Eu.	Portugal	West. Ocean	38 42 N.	9 25 W.	
C. Lisburne	Asia	N. Hebrides	Pacif. Ocean	15 41 S.	166 57 W.	
I. Liffa	Eu.	Dalmatia	Adriatic Sea	42 56 N.	18 32 E.	
Lizard	Eu.	England	Eng. Channel	49 57 N.	5 10 W.	7 30
Isles { S. W. end	Eu.	Norway	North Ocean	68 15 N.	10 20 E.	
Loffout { N. E. end	Eu.	France	B. Biscay	69 00 N.	12 00 E.	
R. Loire, Ent.	Eu.	France	R. Thames	47 07 N.	42 05 W.	3 00
London	Eu.	England	R. Thames	51 31 N.	0 00	3 00
New London	Am.	N. England	West. Ocean	41 50 N.	72 14 W.	1 30
Londonderry	Eu.	Ireland	West. Ocean	55 01 N.	7 31 W.	
Long Isle	Am.	N. England	West. Ocean	41 00 N.	71 59 W.	3 00
I. Longo	Eu.	Dalmatia	Ardriat. Sea	43 45 N.	17 58 E.	
Longsand Head	Eu.	England	Germ. Ocean	51 47 N.	1 41 E.	10 30
Lookout Point	Eu.	Greenland	North Ocean	76 40 N.	16 25 E.	
C. Lopas	Africa	Loango	Atl. Ocean	0 47 S.	8 30 E.	
C. Lorenza	Am.	Peru	Pacif. Ocean	1 2 S.	80 12 W.	
B. St. Louis	Am.	Louisiana	G. Mexico	28 50 N.	97 08 W.	
P. Louis	Am.	Hispaniola	Atl. Ocean	18 19 N.	73 11 W.	
P. Louis	Africa	Mauritius	Indian Ocean	20 10 S.	57 33 E.	
Louisburg	Am.	C. Breton	B. St. Law.	45 54 N.	59 50 W.	
Lubec	Eu.	Germany	Baltic Sea	54 00 N.	11 40 E.	
C. St. Lucar	Am.	California	Pacif. Ocean	23 15 N.	109 40 W.	
R. Lucia	Africa	Caffres	Indian Ocean	27 52 S.	33 28 E.	
I. St. Lucia	Africa	C. de Verd	Atl. Ocean	16 43 N.	24 33 W.	
I. St. Lucia	Am.	Caribbee	Atl. Ocean	13 25 N.	60 46 W.	
Luconia { N. E. point				19 25 N.	121 45 E.	
Luconia { C. Bajador				18 50 N.	120 25 E.	
Luconia { Manila	Asia	Phil. Isles	Pacif. Ocean	14 36 N.	120 58 E.	
Luconia { S. W. point				13 30 N.	119 35 E.	
Luconia { E. point				14 00 N.	124 00 E.	
Lunden	Eu.	Sweden	Baltic Sea	55 42 N.	13 26 E.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
I. Lundy	Eu.	England	St. Geo. Ch.	51 20 N.	0 04 W.	5h. 15m.
Lupis's Head	Eu.	Ireland	West. Ocean	52 24 N.	10 15 W.	
Luxemburg	Eu.	Netherlands		49 37 N.	6 17 E.	
Lynn	Eu.	England	Germ. Ocean	52 46 N.	0 30 E.	6 45
M.						
C. Mabo	Afia	New Guinea	Pacif. Ocean	0 40 S.	130 05 E.	
Macao, or Makau	Afia	China	Pacif. Ocean	22 12 N.	113 46 E.	
Macassar	Afia	I. Celebes	Pacif. Ocean	5 09 S.	119 50 E.	
C. Machian	Eu.	Spain	B. Biscay	43 44 N.	3 05 W.	
C. St. Mary				25 24 S.	45 53 E.	
B. St. Augustine				23 35 S.	43 13 E.	
Terra de Gada				19 36 S.	44 46 E.	
C. St. Andrew				15 46 S.	45 22 E.	
C. St. Sebastian	Africa		Indian Ocean	12 30 S.	46 30 E.	
C. de Ambre				12 15 S.	50 15 E.	
B. d'Antongil				16 00 S.	49 40 E.	
Antavare				20 53 S.	47 48 E.	
Po. Dauphin				24 48 S.	48 06 E.	
I. Ma- { Funchal	Africa	Canaries	Atl. Ocean	32 38 N.	17 06 W.	12 4
deira { W. end				32 25 N.	17 21 W.	12 0
Madrafs	Afia	India	Indian Ocean	13 5 N.	80 34 E.	
Madrid	Eu.	Spain	R. Manzana	40 25 N.	03 21 W.	
P. Madre de Dios	Afia	Marquesas	Pacif. Ocean	9 55 S.	139 4 W.	
Madura	Afia	India	Indian Ocean	9 54 N.	78 18 E.	
R. Maes, Mouth	Eu.	D. Neth.	Germ. Ocean	52 06 N.	3 50 E.	1 30
Str. Le Maire	Am.	Patagonia	Atl. Ocean	54 51 S.	65 00 W.	
Magadoxa	Africa	Zanguebar	Indian Ocean	2 53 N.	45 25 E.	
I. Magdalena	Afia	Marquesas	Pacif. Ocean	10 25 S.	138 44 W.	
Str. Ma- { E. ent.				52 30 S.	67 50 W.	
gellan { W. ent.	Am.	Patagonia	Pacif. Ocean	52 55 S.	74 18 W.	
Magisland	Afia	India	Malabar Coa.	12 10 N.	74 14 E.	
I. Maguana	Am.	Bahama I.	Atl. Ocean	22 36 N.	72 25 W.	
P. Mahon, Isle						
Minorca	Eu.	Spain	Medit. Sea	39 51 N.	3 53 E.	
Majorca, Isle Ma- } jorca	Eu.	Spain	Medit. Sea	39 35 N.	2 35 E.	
C. Mala	Eu.	Turkey	Archipelago	37 20 N.	24 07 E.	
Malacca	Afia	India	Str. Malacca	2 12 N.	102 10 E.	
Malaga	Eu.	Spain	Medit. Sea	36 43 N.	4 02 W.	
Isles Mal- { N. end				7 20 N.	73 03 E.	
dive { S. end	Afia	India	Indian Ocean	0 20 S.	76 10 E.	
Malestrom Whirl- } pool	Eu.	Norway	West Ocean	68 08 N.	10 40 E.	
I. Malique	Afia	Maldiva I.	Indian Ocean	7 45 N.	72 40 E.	
St. Maloes	Eu.	France	Eng. Channel	48 39 N.	1 57 W.	6 00
I. Malta	Eu.	Italy	Medit. Sea	35 54 N.	14 34 E.	
I. Man, W. end	Eu.	England	Irish Sea	53 45 N.	5 00 W.	9 00
Mangalore	Afia	India	Indian Ocean	13 02 N.	75 10 E.	
Manilla	Afia	I. Luconia	Pacif. Ocean	14 36 N.	120 58 E.	
I. Mansfield, N. pt.	Am.	New Britain	Hudson's Bay	62 38 N.	80 33 W.	
I. Manfia	Africa	Zanguebar	Indian Ocean	8 36 S.	40 40 E.	
I. Mardou	Eu.	Norway	Sound	58 14 N.	8 55 E.	
I. Margarita	Am.	Terra Firma	Atl. Ocean	11 15 N.	63 35 W.	
R. Maragnon	Am.	Brasil	Atl. Ocean	1 48 S.	44 17 W.	
Margate	Eu.	England	Eng. Channel	51 29 N.	1 10 E.	11 15
C. St. Maria	Eu.	Portugal	Atl. Ocean	36 45 N.	7 45 W.	
C. St. Maria, or Lucia	Eu.	Italy	Medit. Sea	40 04 N.	18 31 E.	
Marian or { N. lim.				21 00 N.	144 00 E.	
Ladrone { S. lim.	Afia		Pacif. Ocean	13 15 N.	142 55 E.	
I. St. Maries	Eu.	Azores	Atl. Ocean	37 00 N.	25 00 W.	
St. Maries	Eu.	I. Scilly	Eng. Channel	49 57 N.	6 38 W.	
I. Marigallante	Am.	West Indies	Atl. Ocean	16 00 N.	61 10 W.	
I. Maritimo, Sicily	Eu.	India	Medit. Sea	38 04 N.	12 33 E.	
Marquesa Is.	Afia		Pacif. Ocean	9 56 N.	139 00 W.	2 30

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
C. Martelo	Eu.	Turkey	Medit. Sea	38 00 N.	26 00 E.	
St. Martha	Am.	Terra Firma	Atl. Ocean	11 26 N.	74 00 W.	
I. St. Martin	Am.	West Indies	Atl. Ocean	18 06 N.	63 06 W.	
C. St. Martin	Africa	Caffres	Atl. Ocean	32 08 S.	18 58 E.	
C. St. Martin	Eu.	Spain	Medit. Sea	38 44 N.	0 25 E.	
I. Martinique, Port } Royal	Am.	West Indies	Atl. Ocean	14 36 N.	61 04 W.	
Marfeilles	Eu.	France	Medit. Sea	43 18 N.	5 27 E.	
C. St. Mary	Am.	Newfoundl.	Atl. Ocean	46 52 N.	54 01 W.	
C. St. Mary	Am.	Brazil	Atl. Ocean	34 52 S.	52 55 W.	
C. St. Mary	Asia	Natolia	Archipelago	37 46 N.	27 21 E.	
C. St. Mary	Eu.	Spain	N. Atl. Ocean	36 46 N.	7 49 W.	
C. Virgin Mary	Am.	Patagonia	S. Atl. Ocean	52 23 S.	68 10 W.	
Mafafuero	Am.	Chili	Pacif. Ocean	33 45 S.	80 34 W.	
I. Mascarenhas	Africa	Zanguebar.	Indian Ocean	20 52 S.	55 35 E.	
I. Mascarin	Am.	Peru	Pacif. Ocean	1 20 S.	88 50 W.	
Mascot	Asia	Arabia	Indian Ocean	23 10 N.	57 40 E.	
Maskelyne's Isles	Asia	N. Hebrides	Pacif. Ocean	16 32 S.	167 59 E.	
Masterland	Eu.	Sweden	Sound	57 58 N.	12 00 E.	
Mafulipatam	Asia	India	B. Bengal	16 28 N.	81 40 E.	
C. Matapan	Eu.	Turkey	Archipelago	36 25 N.	22 40 E.	
I. Matbare	Asia	Japan	Pacif. Ocean	26 30 N.	137 00 E.	
I. St. Matthew's	Africa	Guinea	Eth. Ocean	1 23 S.	6 11 W.	
St. Matthew's Lights	Eu.	France	B. Biscay	48 20 N.	4 42 W.	
I. Mauritius	Africa	Madagascar	Indian Ocean	20 10 S.	57 33 E.	
Maurua	Asia	Society Isles	Pacif. Ocean	16 26 S.	152 33 W.	
I. May	Africa	C. Verd	Atl. Ocean	15 10 N.	23 00 W.	
C. May	Am.	Pennsylvania	Atl. Ocean	39 15 N.	74 43 W.	
I. Mayette	Africa	Madagascar	Indian Ocean	12 53 S.	46 10 E.	
I. John Mayne	Eu.	Greenland	North Ocean	71 10 N.	9 44 W.	
Mecca	Asia	Arabia	Red Sea	21 40 N.	41 00 E.	
Mechlin	Eu.	Netherlands		51 2 N.	4 34 E.	
Medina	Asia	Arabia	Red Sea	24 58 N.	39 53 E.	
I. Melada	Eu.	Dalmatia	Adriat. Sea	42 40 N.	19 34 E.	
Melinde	Africa	Zanguebar	Indian Ocean	3 07 S.	39 40 E.	
I. Melo	Eu.	Turkey	Archipelago	36 41 N.	25 05 E.	
Memel	Eu.	Courland	Baltic Sea	55 48 N.	22 23 E.	
Memiffan	Eu.	France	B. Biscay	44 20 N.	1 23 W.	3h. 30m.
I. Menado	Asia	I. Celebes	Pacif. Ocean	1 36 N.	122 25 E.	
C. Mendozin	Am.	California	Pacif. Ocean	41 20 N.	130 15 W.	
Mercury Bay	Asia	N. Zealand	Pacif. Ocean	36 50 S.	175 12 E.	
Mergui	Asia	Siam	B. Bengal	12 9 N.	98 25 E.	
R. Metaparovous	Am.	Bahama	Atl. Ocean	21 58 N.	74 13 W.	
Messina	Eu.	I. Sicily	Medit. Sea	38 21 N.	16 21 E.	
C. Mesurato	Africa	Tripoli	Medit. Sea	32 18 N.	16 36 E.	
I. Mety- } lene } C. Sigré Po. Oliviea	Asia	Natolia	Archipelago	39 21 N.	26 08 E.	
I. Meun	Eu.	Denmark	Baltic Sea	39 11 N.	26 47 E.	
Mexico	Am.	Mexico	Inland	39 00 N.	26 50 E.	
Miatea	Asia	Society Isles	Pacif. Ocean	55 00 N.	13 15 E.	
I. St. Michael	Eu.	Azores	Atl. Ocean	19 54 N.	100 01 W.	
Middleburgh	Eu.	D. Neth.	Germ. Ocean	17 52 S.	148 01 W.	
Middleburgh, or } Eaoowe }	Asia	Friendly Is.	Pacif. Ocean	37 47 N.	25 37 W.	
Milan	Eu.	Italy	Medit. Sea	51 37 N.	3 58 E.	
Milford	Eu.	Wales	St. Geo. Ch.	21 21 S.	174 34 W.	
Milo, I. Milo	Asia	Turkey	Archipelago	45 28 N.	9 15 E.	
Mill Isles	Am.	North Main	Hudson's Bay	51 45 N.	5 15 W.	5 15
I. Mindanao { N. point S. E. pt. C. St. Augustine S. W. pt. Cal- dera S. point	Asia	Spice Islands	Pacif. Ocean	36 41 N.	25 05 E.	
				64 36 N.	80 30 W.	
				9 40 N.	124 25 E.	
				6 40 N.	126 25 E.	
				7 00 N.	121 25 E.	
				3 50 N.	124 43 E.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
I. Mindora	Asia	Philip. Isles	Pacif. Ocean	13 00 N.	119 37 E.	
I. Mi- } N. W. pt.	Eu.	Spain	Mediterran.	39 58 N.	3 54 E.	
norca } S. E. pt.				40 24 N.	4 18 E.	
G. Miquelon	Am.	Newfoundl.	Atl. Ocean	47 3 N.	56 13 W.	
L. Miquelon	Am.	Newfoundl.	Atl. Ocean	46 50 N.	56 13 W.	
I. Misco	Am.	Nova Scotia	G. St. Lawr.	48 04 N.	64 19 W.	
C. Miserata	Africa	Guinea	Atl. Ocean	6 25 N.	9 35 W.	
R. Mississippi, mouth	Am.	Louisiana	G. Mexico	29 00 N.	89 17 W.	
Mizen Head	Eu.	Ireland	Atl. Ocean	51 16 N.	10 20 W.	
Mocha	Asia	Arabia	Red Sea	13 45 N.	44 04 E.	
Modena	Eu.	Italy	Inland	44 34 N.	11 18 E.	
Modon	Eu.	Turkey	Medit. Sea	36 55 N.	21 03 E.	
I. Mohilla	Africa	Zanguebar	Indian Ocean	11 55 S.	45 00 E.	
I. Montferat	Am.	West Indies	Atl. Ocean	16 48 N.	62 12 W.	
Montagu Isle	Asia	N. Hebrides	Pacif. Ocean	17 26 S.	168 36 E.	
Montreal	Am.	Canada	R. St. Lawr.	45 52 N.	73 11 W.	
I. Monte Christo	Eu.	Italy	Medit. Sea	42 17 N.	10 28 E.	
C. Monte Sancto	Eu.	Turkey	Archipelago	40 27 N.	24 39 E.	
Monument	Asia	N. Hebrides	Pacif. Ocean	17 14 S.	168 38 E.	
Moose Fort	Am.	New Wales	Hudson's Bay	51 16 N.	80 50 W.	
I. Morgo	Asia	Natolia	Archipelago	36 55 N.	26 30 E.	
Morlaix	Eu.	France	Eng. Channel	48 30 N.	3 50 W.	
Mort Point	Eu.	England	St. Geo. Ch.	51 12 N.	4 40 W.	
Mofambique	Africa	Zanguebar	Indian Ocean	15 00 S.	41 40 E.	
Moscow	Eu.	Russia	R. Moscow	55 45 N.	37 51 E.	
Mokkettos Bank	Am.	Mexico	Atl. Ocean	14 45 N.	80 05 W.	
Mofchetto Cove	Am.	Greenland	North Ocean	64 55 N.	52 51 W.	
C. Mount	Africa	Guinea	Atl. Ocean	7 12 N.	10 44 W.	
Mount's Bay	Eu.	England	Eng. Channel	50 05 N.	3 45 W.	4h. 30m.
Moufe River	Am.	New Wales	Hudson's Bay	51 25 N.	83 15 W.	
C. Mufaldon	Asia	Arabia	Persian Gulf	26 04 N.	55 22 E.	
N.						
C. Nabo	Asia	Japan	Pacif. Ocean	40 35 N.	141 25 E.	
Nagpour	Asia	Berar	—	21 8 N.	79 51 E.	
Namur	Eu.	Netherlands	—	50 29 N.	4 50 E.	
Nancovey Harbour	Asia	Nicobar I.	B. Bengal	7 58 N.	93 31 E.	
Nangafaki	Asia	Japan	Pacif. Ocean	32 32 N.	128 50 E.	
Nankin	Asia	China	Pacif. Ocean	32 07 N.	118 35 E.	
Nantes	Eu.	France	B. Biscay	47 13 N.	1 29 W.	3 00
Nantucket Isle	Am.	New Eng.	West Ocean	41 34 N.	69 40 W.	
Naples	Eu.	Italy	Medit. Sea	40 51 N.	14 19 E.	
Narbonne	Eu.	France	Medit. Sea	43 11 N.	3 05 E.	
Narlinga	Asia	India	B. Bengal	18 05 N.	85 20 E.	
Narva	Eu.	Livonia	G. Finland	59 23 N.	29 27 E.	
I. Nassau	Asia	Sumatra	Indian Ocean	3 00 S.	100 25 E.	
C. Nassau	Am.	Terra Firma	Atl. Ocean	7 53 N.	58 07 W.	
Nassau Str.	Eu.	Russia	North Ocean	69 55 N.	57 30 E.	
B. Natal	Africa	Caffres	Indian Ocean	29 25 S.	33 10 E.	
I. Navassa	Am.	W. Indies	Carib. Sea	18 23 N.	74 56 W.	
I. Naxos	Eu.	Turkey	Archipelago	37 06 N.	25 58 E.	
Naze	Eu.	Norway	West. Ocean	57 50 N.	7 32 E.	11 15
Needles	Eu.	England	Eng. Channel	50 41 N.	1 28 W.	10 15
Negapatam	Asia	India	B. Bengal	10 46 N.	80 02 E.	
C. Negrailles	Asia	Pegu	B. Bengal	16 20 N.	94 15 E.	
C. Negro	Africa	Caffres	Atl. Ocean	16 30 S.	11 30 W.	
C. Negro	Africa	Barbary	Medit. Sea	37 17 N.	9 09 E.	
Negropont	Eu.	Turkey	Archipelago	38 30 N.	24 05 E.	
Port Nelson	Am.	New Wales	Hudson's Bay	57 07 N.	92 37 W.	
Port Nelson's Shoals	Am.	New Wales	Hudson's Bay	57 35 N.	92 07 W.	8 20
I. Nevis	Am.	Caribbee Isles	Atl. Ocean	17 11 N.	62 52 W.	
Newcastle	Eu.	England	Germ. Ocean	55 03 N.	1 28 W.	3 15
R. Nicaragua	Am.	New Spain	Atl. Ocean	11 40 N.	82 47 W.	
Nice	Eu.	Italy	Medit. Sea	43 42 N.	7 22 E.	
If. Nicobar	Asia	Siam	B. Bengal	7 4 N.	93 44 E.	
I. St. Nicholas	Africa	C. Verd I.	Atl. Ocean	16 35 N.	24 06 W.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Nicotera	Eu.	Italy	Medit. Sea	38 33 N.	16 30 E.	
Nieuport	Eu.	Flanders	Germ. Ocean	51 08 N.	2 50 E.	12h. 00m.
Ninhay	Asia	China	Pacif. Ocean	37 10 N.	122 25 E.	
Ningpo, or Liampo	Asia	China	Pacif. Ocean	29 58 N.	120 23 E.	
I. Nio	Eu.	Turkey	Archipelago	36 48 N.	26 02 E.	
I. Noel	Asia	India	Indian Ocean	10 30 S.	105 25 E.	
C. Noir	Am.	T. del Fuego	Pacif. Ocean	54 32 S.	73 3 W.	
Norfolk Isle	Asia	N. Holland	Pacif. Ocean	29 2 S.	168 15 E.	
C. de Non	Africa	Barbary	Atl. Ocean	28 04 N.	10 32 W.	
Nombre de Dios	Am.	Terra Firma	Carribbe. Sea	9 43 N.	78 35 W.	
Nootka Sound	Am.	—	Pacif. Ocean	49 36 N.	126 36 W.	
Nore	Eu.	England	R. Thames	51 28 N.	0 48 E.	0 00
Noriton	Am.	Pensilvania	Inland	40 10 N.	75 17 W.	
C. North	Am.	Terra Firma	Atl. Ocean	1 43 N.	49 00 W.	
C. North	Am.	C. Breton	Atl. Ocean	47 5 N.	60 8 W.	
C. North	Am.	S. Georgia	Atl. Ocean	54 5 S.	38 10 W.	
N. Cape, I. Maggoroe	Eu.	Lapland	North Ocean	71 10 N.	26 02 E.	3 00
North Point	Eu.	Norway	North Ocean	62 15 N.	6 15 E.	
North Bluff	Am.	North Main	Hudson's Str.	62 30 N.	70 59 W.	
I. Nottingham, E. pt.	Am.	New Britain	Hudson's Str.	63 35 N.	77 48 W.	10 00
Nuremberg	Eu.	Germany	Inland	49 27 N.	11 12 E.	
O.						
Oaite-Peha Bay	Asia	Otaheite	Pacif. Ocean	17 46 S.	149 9 W.	
Ochozk	Asia	Tartary	Sea of Ochot.	59 20 N.	143 18 W.	
Oczakow	Eu.	Turkey	Black Sea	45 12 N.	34 40 E.	
I. Oeland { S. end	Eu.	Sweden	Baltic Sea	56 15 N.	18 35 E.	
I. Oeland { N. end				57 23 N.	17 36 E.	
Ohamaneno Bay	Asia	Society Isles	Uliatea	16 46 S.	151 33 W.	11 20
Ohevahoa I.	Asia	Marquesas	Pacif. Ocean	9 41 S.	138 56 W.	
Ohiteroa I.	Asia	Society Isles	Pacif. Ocean	22 27 S.	150 43 W.	
Old Head of Kinfale	Eu.	Ireland	Atl. Ocean	51 30 N.	8 54 W.	
I. Oleron	Eu.	France	B. Biscay	46 03 N.	1 20 W.	
Olinde	Am.	Brasil	S. Atl. Ocean	8 13 S.	35 00 W.	
Oliva	Eu.	Germany	Baltic Sea	54 20 N.	18 30 E.	
Ollone	Eu.	France	B. Biscay	46 32 N.	1 36 W.	3 45
Onateayo I.	Asia	Marquesas	Pacif. Ocean	9 58 S.	138 46 W.	
Oneglia	Eu.	Italy	Medit. Sea	43 57 N.	7 52 E.	
C. Onemak	Am.	Fox Isles	Pacif. Ocean	54 31 N.	167 26 W.	
Oporto	Eu.	Portugal	Atl. Ocean	41 10 N.	8 22 W.	
Oran	Africa	Barbary	Medit. Sea	35 45 N.	0 00	
C. Orange	Am.	Terra Firma	Atl. Ocean	4 27 N.	50 50 W.	
Orbitello	Eu.	Italy	Medit. Sea	42 30 N.	12 00 E.	
I. Orchillo	Am.	Terra Firma	Caribbean Sea	11 32 N.	65 25 W.	
Orenburg	Asia	Astracan	Inland	51 46 N.	55 14 E.	
Orfordness	Eu.	England	Germ. Ocean	52 17 N.	1 11 E.	9 45
Orkney Isles, limits	Eu.	Scotland	West. Ocean	59 24 N.	3 23 W.	
New Orleans	Am.	Louisiana	R. Mississippi	30 00 N.	89 54 W.	
I. Ormus	Asia	Perfia	G. Perfia	27 30 N.	55 17 E.	
C. del Oro, or Olerada	Africa	Negroland	Atl. Ocean	23 30 N.	14 31 W.	
R. Oronoque	Am.	Terra Firma	Atl. Ocean	8 08 N.	59 50 W.	
C. Oropeto	Eu.	Spain	Medit. Sea	40 20 N.	0 49 E.	
Orotava	Africa	Canaries	Atl. Ocean	28 23 N.	16 19 W.	
Orsk	Asia	Tartary	Inland	51 12 N.	58 37 E.	
C. Ortegal	Eu.	Spain	B. Biscay	43 47 N.	7 34 W.	
Ortona	Eu.	Italy	Medit. Sea	42 19 N.	14 37 E.	
I. Oruba	Am.	Terra Firma	Caribbean Sea	12 03 N.	69 03 W.	
Osnaburg Isle	Asia	Society Isles	Pacif. Ocean	22 00 S.	141 34 W.	
Ostend	Eu.	Flanders	Germ. Ocean	51 14 N.	3 00 E.	12 00
C. Otranto	Eu.	Italy	Medit. Sea	40 23 N.	17 41 E.	
Owharre Bay	Asia	Huachine	Pacif. Ocean	16 44 S.	151 3 W.	
Ozaca	Asia	Japan	Pacif. Ocean	35 10 N.	134 05 E.	
P.						
C. Padron	Africa	Congo	Atl. Ocean	6 00 S.	11 40 E.	
Padua	Eu.	Italy	Adriatic	45 22 N.	11 59 E.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Paita	Am.	Peru	Pacif. Ocean	5 20 S.	80 35 W.	
C. Paillouri	Eu.	Turkey	Archipelago	39 59 N.	24 03 E.	
Palermo, I. Sicily	Eu.	Italy	Medit. Sea	38 10 N.	13 43 E.	
Paliakate	Asia	India	B. Bengal	13 40 N.	80 50 E.	
Palliser's Isles	Asia	Society Isles	Pacif. Ocean	15 38 S.	146 25 W.	
C. Palliser	Asia	N. Zealand	Pacif. Ocean	41 40 S.	175 28 E.	
I. Palma	Africa	Canaries	Atl. Ocean	28 36 N.	17 45 W.	
I. Palmaria	Eu.	Italy	Medit. Sea	41 00 N.	13 03 E.	
Palmerston's Isle	Asia	Society Isles	Pacif. Ocean	18 00 S.	162 52 W.	
C. Palmiras	Asia	India	B. Bengal	20 44 N.	87 07 E.	
C. Palmas	Africa	Guinea	Atl. Ocean	4 26 N.	5 56 W.	
Panama	Am.	Mexico	Pacif. Ocean	8 45 N.	80 16 W.	
I. Panaria	Eu.	Italy	Medit. Sea	38 40 N.	15 41 E.	
Panorma	Eu.	Turkey	Medit. Sea	40 05 N.	21 40 E.	
I. Pantalaria	Eu.	Italy	Medit. Sea	36 55 N.	12 31 E.	
R. Panuco	Am.	Mexico	G. Mexico	24 02 N.	100 13 W.	
R. Paraila	Am.	Brazil	Atl. Ocean	21 26 S.	39 50 W.	
Paris	Eu.	France	R. Seine	48 50 N.	2 25 E.	
C. Passero	Eu.	I Sicily	Medit. Sea	36 35 N.	15 22 E.	
C. Patam	Asia	Malacca	Indian Ocean	7 27 N.	101 20 E.	
I. Patmos	Asia	Natolia	Archipelago	37 22 N.	26 48 E.	
Patixfiord	Eu.	Iceland	North Ocean	65 36 N.	24 5 W.	
R. Pattahan	Asia	I. Sumatra	Str. Malacca	0 28 N.	102 25 E.	
C. Paul	Eu.	Spain	Medit. Sea	37 50 N.	0 15 W.	
I. St. Paul	Am.	Newfoundl.	B. St. Lawr.	47 12 N.	59 59 W.	
I. St. Paul	Asia	Madagascar	Indian Ocean	37 51 S.	77 53 E.	
St. Paul de Leon	Eu.	France	Eng. Channel	48 41 N.	3 55 W.	
I. Paxeros	Am.	California	Pacif. Ocean	30 18 N.	120 45 W.	
I. Pearl, or Serana	Am.	West Indies	Atl. Ocean	14 55 N.	79 00 W.	
Pegu	Asia	India	B. Bengal	17 00 N.	96 58 E.	
Pekin	Asia	China	Inland	39 55 N.	116 29 E.	
I. Pelugofa	Eu.	Italy	Adriatic Sea	42 20 N.	18 32 E.	
I. Pemba	Africa	Zanguebar	Indian Ocean	5 38 S.	40 09 E.	
C. Pembroke	Am.	New Wales	Hudson's Bay	63 12 N.	82 54 W.	
I. Penguin	Am.	Newfoundl.	Atl. Ocean	47 23 N.	56 56 W.	
Penmark	Eu.	France	B. Biscay	47 50 N.	4 20 W.	
R. Penobscot	Am.	New Eng.	Atl. Ocean	44 40 N.	68 52 W.	
Pernambuco	Am.	Brazil	S. Atl. Ocean	8 30 S.	35 07 W.	
Petapoli	Asia	India	B. Bengal	16 14 N.	81 10 E.	
I. St. Peter	Am.	Newfoundl.	Atl. Ocean	46 46 N.	56 5 W.	
E. St. Peter	Am.	Martinico	Atl. Ocean	14 44 N.	61 16 W.	
St. Peter and Paul	Asia	Kamtchat.	Beerings St.	53 1 N.	158 46 E.	
Peteriburg	Eu.	Russia	Baltic Sea	59 56 N.	30 24 E.	
C. Petra	Asia	Natolia	Archipelago	37 02 N.	27 38 E.	
Peverel Point	Eu.	England	Eng. Channel	50 34 N.	1 22 W.	
Philadelphia	Am.	Pennsylvania	R. Delaware	39 57 N.	75 8 W.	
St. Philip	Africa	Benguela	Atl. Ocean	12 22 S.	13 20 E.	
I. Pianofa	Eu.	Italy	Medit. Sea	42 45 N.	10 34 E.	
Isle of Pines	Asia	N. Caledonia	Pacif. Ocean	22 38 S.	167 43 E.	
I. Pico (Pike)	Eu.	Azores	Atl. Ocean	38 29 N.	28 19 W.	
C. Pinas	Eu.	Spain	B. Biscay	43 51 N.	6 14 W.	
Pickersgill's I.	Am.	S. Georgia	Atl. Ocean	54 42 S.	36 53 W.	
Pickersgill's Harb.	Asia	N. Zealand	Indian Ocean	45 47 N.	166 23 E.	
Mo. Pintados, or	Am.	California	Pacif. Ocean	27 30 N.	117 15 W.	
St. Martin	Eu.	Italy	Medit. Sea	43 43 N.	10 17 E.	
Pifa	Asia	China	Pacif. Ocean	23 30 N.	119 25 E.	
Piscadore Isles	Am.	Chili	Pacif. Ocean	25 2 S.	133 21 W.	
Pitcairn's Isles	Am.	Newfoundl.	Atl. Ocean	47 15 N.	53 43 W.	
Placentia	Am.	La Plata	Atl. Ocean	36 00 S.	57 40 W.	
R. Plata	Am.	Bahama Isles	Atl. Ocean	20 04 N.	63 37 W.	
R. Platewrack	Am.	England	Eng. Channel	50 22 N.	4 10 W.	
Plymouth	Eu.	Italy	Medit. Sea	40 18 N.	15 45 E.	
Policaastro	Eu.	Italy	Indian Ocean	9 45 N.	109 55 E.	
Is. Polfapate	Asia	Cambaya	Adriatic Sea	42 57 N.	18 14 E.	
I. Poma	Eu.	Dalmatia				

4h. oom.

Pondicherry

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Pondicherry	Asia.	India	B. Bengal	11 42 N.	79 58 E.	
Pontorion	Eu.	France	Eng. Chan.	48 33 N.	1 27 W.	
Ponoi	Eu.	Lapland	North Sea	67 5 N.	38 48 E.	
I. Ponza	Eu.	Italy	Medit. Sea	40 53 N.	13 09 E.	
Pool	Eu.	England	Eng. Channel	51 00 N.	1 50 W.	
Porto Port	Eu.	Portugal	Atl. Ocean	41 10 N.	8 22 W.	
Port Mahon	Eu.	Spain	Medit. Sea	39 51 N.	3 53 E.	
Portland	Eu.	England	Eng. Channel	50 30 N.	2 48 W.	8h. 15m.
Port l'Orient	Eu.	France	B. Biscay	47 47 N.	3 13 W.	
Porto Bello	Am.	New Spain	Carib. Sea	9 33 N.	79 45 W.	
Porto Praya	Africa	C. Verd	Atl. Ocean	14 54 N.	23 24 W.	11 00
I. Porto Rico	Am.	Antilles	Atl. Ocean	18 35 N.	65 58 W.	
{ E. point				18 29 N.	66 35 W.	
{ Port Rico				18 34 N.	67 51 W.	
{ W. point						
Port Royal	Am.	Jamaica	Atl. Ocean	17 59 N.	76 40 W.	
Port Royal	Am.	Martinico	Atl. Ocean	14 36 N.	61 4 W.	
I. Porto Sancto	Africa	Canaries	Atl. Ocean	32 58 N.	16 20 W.	
Portfall	Eu.	France	Eng. Channe	48 36 N.	4 43 W.	
Portsmouth. R. Aca.	Eu.	England	Eng. Channel	50 48 N.	1 01 W.	11 15
I. Portland	Eu.	Iceland	North Sea	63 22 N.	18 49 W.	
I. Portland	Asia	N. Zealand	Pacif. Ocean	39 25 N.	178 17 E.	
Prague	Eu.	Bohemia	Inland	50 4 N.	14 50 E.	
Praken	Asia	Joch. Chi.	Indian Ocean	17 15 N.	106 15 E.	
Prenau	Eu.	Livonia	Baltic Sea	58 26 N.	24 58 E.	
I. Princes	Africa	Guinea	Atl. Ocean	1 47 N.	6 39 E.	
C. Prior	Eu.	Spain	Atl. Ocean	43 29 N.	8 15 W.	
I. Providence	Am.	Bahama	Atl. Ocean	24 51 N.	77 01 W.	
I. Providence, or }	Am.	Mexico	Atl. Ocean	13 26 N.	80 42 W.	
St. Catherine						
Pudyoua	Asia	N. Caledonia	Pacif. Ocean	20 18 S.	164 46 E.	6 00
I. Pulo Condor	Asia	Cambaya	Indian Ocean	8 40 N.	107 25 E.	
I. Pylefiart	Asia	Friendly Is.	Pacif. Ocean	22 23 S.	175 44 W.	
Q						
Quaqua, or Ivory }	Africa	Guinea	Eth. Sea	5 00 N.	4 00 W.	
Coast						
Quebec	Am.	Canada	L. St. Lawr.	46 49 N.	71 10 W.	7 30
Queda	Asia	Malaya	B. Bengal	6 15 N.	100 12 E.	
Quelpert	Asia	Korea	Pacif. Ocean	33 32 N.	128 04 E.	
Quiloa	Africa	Zanguebar	Indian Ocean	9 30 S.	39 09 E.	
Quimper	Eu.	France	B. Biscay	47 58 N.	4 02 W.	
Quinzan	Asia	Coch. Chi.	Indian Ocean	12 52 N.	109 10 E.	
Quiraba Isles	Africa	Zanguebar	Indian Ocean	11 00 N.	41 39 E.	
C. Quiros	Asia	N. Hebrides	Pacif. Ocean	14 56 S.	167 25 E.	
Quito	Am.	Peru	Inland	0 13 S.	77 50 W.	
R						
C. Race	Am.	Newfoundl.	Atl. Ocean	46 40 N.	52 38 W.	
Ragusa	Eu.	Dalmatia	Medit. Sea	42 45 N.	20 00 E.	
Rajapour	Asia	India	Indian Ocean	17 19 N.	73 50 E.	
Raingate	Eu.	England	Downs	51 20 N.	1 22 E.	
Ramhead	Eu.	England	Eng. Channel	50 19 N.	4 15 W.	
C. Rafagata	Asia	Arabia	Indian Ocean	22 46 N.	58 48 E.	
Ravenna	Eu.	Italy	Medit. Sea	44 26 N.	12 21 E.	
C. Ray	Am.	Newfoundl.	Atl. Ocean	47 37 N.	59 8 W.	
I. Rhee	Eu.	France	B. Biscay	46 15 N.	1 28 W.	3 00
Regio	Eu.	Italy	Mediterran.	38 22 N.	16 37 E.	
C. Reikianesi	Eu.	Iceland	North Sea	63 55 N.	22 42 W.	
Cape Resolution	Am.	N. Main	Hudf. Straits	61 29 N.	65 10 W.	
Resolution Bay	Asia	Marquesas	Pacif. Ocean	9 55 S.	139 4 W.	
Resolution Island	Asia	Soc. Isles	Pacif. Ocean	17 23 S.	141 40 W.	
Resolution Port	Asia	N. Hebrides	Pacif. Ocean	19 32 S.	169 46 E.	
Revel	Eu.	Livonia	Baltic Sea	59 26 N.	24 24 E.	
Rhodes	Asia	Natolia	Archipelago	36 27 N.	28 36 E.	
{ Rhodes, N.						
{ end						
{ C. Tranquil,				35 55 N.	28 23 E.	
{ S. end						

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Riga	Eu.	Livonia	Baltic Sea	56 56 N.	23 44 E.	
Riprap, a sand	Eu.	England	Straits Dover	51 53 N.	1 25 E.	
Robin Hood's Bay	Eu.	England	Germ. Ocean	54 25 N.	0 08 W.	3h.com.
I. Rocca	Am.	Terra Firma	Atl. Ocean	11 21 N.	66 17 W.	
Rocheport	Eu.	France	B. Biscay	46 03 N.	0 54 W.	4 15
Rochel	Eu.	France	B. Biscay	46 10 N.	1 5 W.	3 45
Rocheater	Eu.	England	R. Medway	51 26 N.	0 30 E.	0 45
I. Rodrigue	Asia	Madagascar	Indian Ocean	19 41 S.	63 16 E.	
C. Romain	Am.	Terra Firma	Atl. Ocean	11 40 N.	69 05 W.	
Rome	Eu.	Italy	Medit. Sea	41 54 N.	12 34 E.	
I. Roncadore	Am.	Mexico	Atl. Ocean	13 30 N.	78 53 W.	
Rood Bay	Eu.	Greenland	North Ocean	79 53 N.	14 00 E.	
C. Roque	Am.	Brazil	Atl. Ocean	5 00 S.	35 43 W.	
I. Roquepiz	Africa	Madagascar	Indian Ocean	9 51 S.	64 30 E.	
G. Rofes	Eu.	Spain	Medit. Sea	42 10 N.	3 18 E.	
Rothock	Eu.	Germany	Baltic Sea	54 10 N.	12 50 E.	
I. Rotterdam	Asia	Friendly Is.	Pacif. Ocean	20 16 S.	174 25 W.	
Rotterdam	Eu.	D. Neth.	Germ. Ocean	51 56 N.	4 33 E.	3 00
Rouen	Eu.	France	R. Seine	49 27 N.	1 10 E.	1 15
C. Roxant	Eu.	Portugal	Atl. Ocean	38 45 N.	9 30 W.	
C. Roxo	Africa	Negroland	Atl. Ocean	11 42 N.	14 33 W.	
C. Rozier	Am.	Nova Scotia	G. St. Lawr.	48 55 N.	63 36 W.	
I. Rugen	Eu.	Germany	Baltic Sea	54 32 N.	14 30 E.	
I. Rum Key, or Samana	Am.	Bahama	Atl. Ocean	23 00 N.	74 20 W.	
R. Rupert	Am.	New Britain	Hudson's Bay	51 45 N.	78 40 W.	
C. Rufato	Africa	Barca	Medit. Sea	32 53 N.	20 41 E.	
Rust Isles	Eu.	Norway	North Sea	67 40 N.	10 25 E.	
Rye	Eu.	England	Eng. Channel	51 03 N.	0 45 E.	11 15
S						
C. Sable	Am.	Nova Scotia	Atl. Ocean	43 24 N.	65 35 W.	
I. Sable W. end	Am.	Nova Scotia	Atl. Ocean	44 09 N.	60 29 W.	
I. Saddle back	Am.	North Main	Hudf. Straits	62 07 N.	68 13 W.	10 00
Saffia	Africa	Barbary	Atl. Ocean	32 30 N.	8 50 W.	
I. Safanjal bahr.	Africa	Egypt	Red Sea	27 05 N.	34 40 E.	
I. Saintes	Eu.	France	B. Biscay	48 5 N.	5 0 W.	
B. Saldanna	Africa	Caffres	Atl. Ocean	32 35 S.	19 30 E.	
I. Sal	Africa	C. Verd	Atl. Ocean	16 38 N.	22 51 W.	
Salerno	Eu.	Italy	Medit. Sea	40 39 N.	14 48 E.	
I. Salini, Lipari Is.	Eu.	Italy	Medit. Sea	38 39 N.	15 24 E.	
I. Salisbury	Am.	N—Main	Hudson's Bay	63 29 N.	76 47 W.	
Sallee	Africa	Barbary	Atl. Ocean	33 58 N.	6 20 W.	
Solomon Isles	Asia		Pacif. Ocean	5 50 S.	171 05 W.	
Salonicha	Eu.	Turkey	Archipelago	40 41 N.	23 13 E.	
I. Salvages	Africa	Canaries	N. Atl. Ocean	30 00 N.	15 49 W.	
I. Salvages { Upper } { Lower }	Am.	North Main	Hudf. Straits	61 48 N. 62 32 1/2 N.	66 20 W. 70 48 1/2 W.	9 00 11 10
I. Samos	Asia	Natolia	Archipelago	37 46 N.	27 13 E.	
I. Sambelong	Asia	India	B. Bengal	7 10 N.	93 50 E.	
C. Sambrough	Am.	Nova Scotia	Atl. Ocean	44 33 N.	63 20 W.	
Sanganooda	Am.	Oonalaska	Pacif. Ocean	53 54 N.	166 17 W.	
Sandwich	Eu.	England	Downs	51 20 N.	1 20 E.	11 30
Sandwich Island	Asia	N. Hebrides	Pacif. Ocean	17 41 S.	168 38 E.	
Sandwich Harbour	Asia	Malicola	Pacif. Ocean	16 25 S.	167 58 E.	
Sandwich's Bay	Am.	S. Georgia	Atl. Ocean	54 42 S.	36 4 W.	
I. Sanguin	Asia	Philip. Isles	Pacif. Ocean	3 50 N.	122 30 E.	
I. Sanien	Eu.	Norway	North Ocean	69 30 N.	14 30 E.	
Santa Cruz	Africa	Barbary	Atl. Ocean	30 30 N.	9 35 W.	
Sardinia { N. limit S. pt. C. Tavo- laro Cagliari Oristagni }	Eu.	Italy	Medit. Sea	41 15 N. 38 54 N. 39 25 N. 39 53 N.	9 31 E. 9 15 E. 9 38 E. 9 01 E.	
Sarena	Am.	Chili	Pacif. Ocean	29 40 S.	71 15 W.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Saunders Isle	Am.	Sandwich L.	Atl. Ocean	58 00 S.	26 53 W.	
C. Saunders	Am.	S. Georgia	Atl. Ocean	54 6 S.	36 53 W.	
Scanderoun	Asia	Syria	Levant	36 35 N.	36 25 E.	
Scarborough head	Eu.	England	Germ. Ocean	54 18 N.	00 00	3h. 45m.
I. Scarpanto	Asia	Natolia	Archipelago	35 45 N.	27 40 E.	
I. Seatarie, N. E. pt.	Am.	Acadia	West. Ocean	46 01 N.	61 57 W.	
Scaw	Eu.	Denmark	Sound	57 34 N.	10 54 E.	
I. Schelling	Eu.	D. Neth.	Germ. Ocean	53 27 N.	5 30 E.	
Scio {	Asia	Natolia	Archipelago	38 38 N.	26 12 E.	
				38 24 N.	26 29 E.	
				38 08 N.	26 20 E.	
C. St. Nicholas						
Scio						
C. Blanco						
Scilly Isles	Eu.	England	St. Geo. Ch.	50 00 N.	6 45 W.	3 45
Scolt Head	Eu.	England	Germ. Ocean	53 00 N.	0 49 E.	6 20
Scots Settlement	Am.	Terra Firma	Caribbean Sea	8 45 N.	76 35 W.	
I. Sea	Eu.	Turkey	Archipelago	37 38 N.	24 53 E.	
Seames	Eu.	France	B. Biscay	48 00 N.	4 51 W.	
I. Sebaldes	Am.	Patagonia	S. Atl. Ocean	50 53 S.	59 35 W.	
C. Sebastian	Am.	California	Pacif. Ocean	43 00 N.	126 00 W.	
C. St. Sebastian	Africa	Madagascar	Indian Ocean	12 30 S.	46 30 E.	
St. Sebastian	Eu.	Spain	B. Biscay	43 16 N.	2 05 W.	
Port Segura	Am.	Brazil	Atl. Ocean	16 57 S.	39 45 W.	
R. Senegal	Africa	Negroland	Atl. Ocean	15 53 N.	16 36 W.	10 30
I. Seranilha	Am.	West Indies	Atl. Ocean	16 20 N.	79 40 W.	
Serdze Kamen	Asia	Ischutskit	Beering's St.	67 3 N.	171 49 W.	
I. Serigo	Eu.	Turkey	Archipelago	36 09 N.	23 24 E.	
Seringapatam	Asia	India	Caiver	12 32 N.	76 52 E.	
I. Sertes	Africa	Canaries	Atl. Ocean	32 35 N.	16 20 W.	
R. Seftos	Africa	Guinea	Atl. Ocean	5 48 N.	8 13 W.	
Seven Capes	Africa	Barbary	Medit. Sea	37 30 N.	6 15 W.	
Seven Stones, or Isles	Eu.	England	St. Geo. Ch.	50 10 N.	6 40 W.	4 30
R. Severn, Ent.	Eu.	England	St. Geo. Ch.	51 41 N.	3 05 W.	6 0
R. Severn	Am.	New Wales	Hudson's Bay	56 12 N.	88 57 W.	
R. Seyn, Ent.	Eu.	France	Eng. Channel	49 36 N.	0 30 E.	9 00
Seynhead	Eu.	France	Eng. Channel	49 44 N.	0 34 E.	
Sheerness	Eu.	England	R. Thames	51 25 N.	0 50 E.	0 00
Shepherd's Isles	Asia	N. Hebrides	Pacif. Ocean	17 00 S.	168 47 E.	
Siam	Asia	India	Bay Siam	14 18 N.	100 55 E.	
R. Siam, Ent.	Asia	India	Bay Siam	13 15 N.	100 47 E.	
Siara	Am.	Brazil	Atl. Ocean	3 18 S.	39 50 W.	
Sicily {	Eu.	Italy	Medit. Sea	38 10 N.	15 58 E.	
				37 22 N.	15 21 E.	
				37 04 N.	15 31 E.	
				36 35 N.	15 22 E.	
				37 11 N.	14 07 E.	
				37 51 N.	21 43 E.	
W. end, C. Bocca				38 10 N.	13 43 E.	
Palermo						
Sierra Leona	Africa	Guinea	Atl. Ocean	8 30 N.	12 07 W.	8 15
Sillabar Road	Asia	I. Sumatra	Indian Ocean	4 00 S.	102 50 E.	
Str. Sincapore	Asia	Malacca	Indian Ocean	1 00 N.	104 30 E.	
R. Sinda, or Indus, }	Asia	India	Indian Ocean	24 30 N.	63 10 E.	
mouth				25 45 N.	62 40 E.	
Po. Shabak	Africa	Abyssinia	Red Sea	18 58 N.	38 24 E.	
Shark, or Seahorse }	Am.	New Wales	Hudson's Bay	64 05 N.	82 12 W.	
point						
Shields	Eu.	England	Germ. Ocean	55 02 N.	1 20 W.	
Shelvoek's Isle	Am.	California	Pacif. Ocean	23 15 N.	117 35 W.	
Shillocks	Eu.	Ireland	West Ocean	51 30 N.	11 05 W.	5 0
I. Shetland, limits	Eu.	Scotland	West. Ocean	60 47 N.	0 10 W.	
Shoreham	Eu.	England	Eng. Channel	59 54 N.	1 31 W.	3 00
I. Sky {	Eu.	Scotland	West. Ocean	50 55 N.	00 17 E.	10 30
N. point				57 50 N.	6 30 W.	
S. point				57 15 N.	6 16 W.	5 30
Sleepers Isles				60 00 N.		
Great Sleeper }	Am.	New Britain	Hudson's Bay	58 35 N.	81 30 W.	
				60 10 N.	82 00 W.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Sline Head	Eu.	Ireland	West. Ocean	53 20 N.	2 15 W.	
R. Slude	Am.	New Britain	Hudson's Bay	53 24 N.	78 50 W.	
Sluyce	Eu.	D. Neth.	Germ. Ocean	51 19 N.	3 50 E.	
C. Smith	Am.	Labradore	Hudson's Bay	60 48 N.	80 55 W.	
Smyrna	Asia	Natolia	Archipelago	38 28 N.	27 25 E.	
I. Socatora	Africa	Anian	Indian Ocean	12 15 N.	52 55 E.	
C. Solomon	Eu.	I. Candia	Medit. Sea	34 37 N.	27 06 E.	
R. Somme	Eu.	France	Eng. Channel	50 18 N.	1 40 E.	11h oom.
Sound Royal	Eu.	Iceland	North Ocean	66 22 N.	15 15 W.	
Southampton	Eu.	England	Eng. Channel	50 55 N.	1 00 W.	0 00
C. Southampton	Am.	New Wales	Hudson's Bay	61 54 N.	86 14 W.	
South Cape	Asia	Diemen's la.	Pacif. Ocean	43 42 S.	147 03 E.	
C. Spartivento	Eu.	Italy	Medit. Sea	37 50 N.	16 41 E.	
C. Spartel	Africa	Barbary	Atl. Ocean	35 46 N.	5 53 W.	
I. Spirito Sancto	Am.	Brasil	Atl. Ocean	20 24 S.	39 55 W.	
Spurn	Eu.	England	Germ. Ocean	53 35 N.	0 30 E.	5 15
I. Stampalia	Asia	Natolia	Archipelago	36 25 N.	26 55 E.	
I. Stancho	Asia	Natolia	Archipelago	36 50 N.	27 30 E.	
Start point	Eu.	England	Eng. Channel	50 14 N.	3 39 W.	6 45
I. States {	Am.	Patagonia	Atl. Ocean	54 45 S.	60 35 W.	
				55 08 S.	60 45 W.	
Stavenger	Eu.	Norway	West. Ocean	58 47 N.	6 45 E.	
C. Stephens	Asia	N. Zealand	Pacif. Ocean	40 36 S.	174 05 E.	
C. Stephens	Am.		Beering's Str.	63 33 N.	162 12 W.	
P. Stephens	Asia	N. Holland	Pacif. Ocean	32 45 N.	152 17 E.	
Stetin	Eu.	Germany	Baltic Sea	53 36 N.	15 25 E.	
C. Stillo	Eu.	Italy	Medit. Sea	38 23 N.	17 07 E.	
Port Steven	Am.	Chili	Pacif. Ocean	46 50 S.	82 36 W.	
Stockholm	Eu.	Sweden	Baltic Sea	59 22 N.	18 12 E.	
Stockton	Eu.	England	Germ. Ocean	54 33 N.	1 15 W.	5 15
Stralsund	Eu.	Germany	Baltic Sea	54 23 N.	14 10 E.	
Strangford Bay	Eu.	Ireland	Irish Sea	54 23 N.	5 40 W.	10 30
I. Stromboli	Eu.	Italy	Medit. Sea	38 42 N.	15 48 E.	
Stromness	Eu.	Orkneys	North Sea	58 56 N.	3 26 W.	
B. Success	Am.	T. del Fuego	Atl. Ocean	54 50 S.	65 20 W.	
C. Success	Am.	T. del Fuego	Atl. Ocean	55 1 S.	65 22 W.	
Suez Town	Africa	Egypt	Red Sea	29 50 N.	33 27 E.	
Sukadana	Asia	I. Borneo	Indian Ocean	1 00 S.	110 40 E.	
I. Sumatra {	Asia	India	Indian Ocean	5 15 N.	95 55 E.	
				5 07 S.	106 20 E.	
Sunderland	Eu.	England	Germ. Ocean	54 55 N.	1 00 W.	3 30
Str. Sunda	Asia	Siam	Indian Ocean	6 10 S.	105 35 E.	
Surinam	Am.	Terra Firma	Atl. Ocean	6 30 N.	55 30 W.	
Surat	Asia	India	Indian Ocean	21 10 N.	72 28 E.	
I. Surroy	Eu.	Lapland	North Ocean	71 00 N.	22 00 E.	
Swaken	Africa	Abyssinia	Red Sea	19 30 N.	37 38 E.	
Swally Road	Asia	India	Arabian Sea	21 55 N.	72 00 E.	
Swansey	Eu.	Wales	St. Geo. Cha.	51 40 N.	4 25 W.	
Sweetnose	Eu.	Lapland	North Ocean	68 08 N.	34 42 E.	
Sicilly Island	Asia	N. Holland	Indian Ocean	43 55 S.	147 13 E.	
Swin, a sand	Eu.	England	Ent. Thames	51 37 N.	1 12 E.	12 00
Syracuse	Eu.	I. Sicily	Medit. Sea	37 04 N.	15 31 E.	
Syriam	Asia	Pegu	B. Bengal	16 00 N.	96 40 E.	
T.						
Tadousac Fort	Am.	Canada	R. St. Lawr.	48 00 N.	67 35 W.	
I. Tamarica	Am.	Brasil	Atl. Ocean	7 56 S.	35 05 W.	
Tamarin Town	Africa	I. Socatora	Indian Ocean	12 30 N.	53 14 E.	9 00
B. Tanassar	Asia	Malacca	B. Bengal	12 00 N.	98 48 E.	
I. Tandoxima	Asia	Japan	Pacif. Ocean	30 30 N.	130 40 E.	
Tangier	Africa	Barbary	Atl. Ocean	35 55 N.	5 45 W.	
Tanna	Asia	N. Hebrides	Pacif. Ocean	19 32 S.	169 45 E.	3 00
Taoukaa	Asia	Society Isles	Pacif. Ocean	14 31 S.	145 4 W.	
Tarento	Eu.	Italy	Medit. Sea	40 43 N.	17 31 E.	
Tasman's Head	Asia	N. Holland	Indian Ocean	43 33 S.	147 36 E.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Taffacorta	Africa	I. Palma	Atl. Ocean	28 38 N.	0 53 W.	
C. Tat'nam	Am.	New Wales	Hudson's Bay	57 35 N.	91 30 W.	
R. Tees, mouth	Eu.	England	Germ. Ocean	54 36 N.	0 52 W.	3h.oom.
Tegoantepec	Am.	Mexico	Pacif. Ocean	14 45 N.	96 23 W.	
Tellichery	Asia	India	Malabar Coast	11 42 N.	75 30 E.	
C. Telling	Eu.	Ireland	West. Ocean	54 40 N.	10 07 W.	
I. Tenedos	Asia	Natolia	Archipelago	39 57 N.	26 14 E.	
I. Teneriff (Peak)	Africa	Canaries	Atl. Ocean	28 13 N.	16 24 W.	3 00
C. Tenes	Africa	Barbary	Medit. Sea	36 26 N.	1 53 E.	
I. Tercera	Eu.	Azores	Atl. Ocean	38 45 N.	27 01 W.	
Terra Nieva	Am.	N. Main	Hudf. Straits	62 4 N.	67 2 W.	9 50
Tervere	Eu.	D. Neth.	Germ. Ocean	51 38 N.	3 35 E.	0 45
Tetuan	Africa	Barbary	Medit. Sea	35 27 N.	4 50 W.	
I. Texel	Eu.	D. Neth.	Germ. Ocean	53 10 N.	4 59 E.	7 30
C. St. Thadæus	Asia	Siberia	North Ocean	62 10 N.	175 05 E.	
R. Thames, mouth	Eu.	England	Germ. Ocean	51 28 N.	1 10 E.	1 30
C. St. Thomas	Africa	Caffres	Atl. Ocean	24 54 S.	15 25 E.	
I. St. Thomas	Africa	Guinea	Atl. Ocean	00 00	1 00 E.	
St. Thomas	Asia	India	B. Bengal	13 00 N.	80 00 E.	
I. St. Thomas	Am.	Virgin Isles	Atl. Ocean	18 22 N.	64 46 W.	
C. Three Points	Am.	Terra Firma	Atl. Ocean	10 51 N.	62 41 W.	
C. Three Points	Africa	Guinea	Atl. Ocean	4 48 N.	1 21 W.	
South Thule	Am.	Sandwich Is.	Atl. Ocean	59 34 S.	27 45 W.	
I. Tidore	Asia	Molucca Is.	Indian Ocean	0 35 N.	126 40 E.	
I. Timor { NE. pt. { SW. pt.	Asia	Molucca Is.	Indian Ocean	8 20 S.	127 40 E.	
				10 23 S.	124 05 E.	
Tinmouth	Eu.	England	Germ. Ocean	55 03 N.	1 17 W.	3 00
I. Tino	Eu.	Turkey	Archipelago	37 33 N.	25 43 E.	
I. Tobago	Am.	Caribbee	Atl. Ocean	11 15 N.	60 27 W.	
Tobolski	Asia	Siberia	Inland	58 12 N.	68 20 E.	
B. Todos Sanctos	Am.	Brazil	Atl. Ocean	13 05 S.	38 45 W.	
Tonquin	Asia	India	Pacif. Ocean	20 50 N.	105 55 E.	
Toniberg	Eu.	Norway	Sound	58 50 N.	10 05 E.	
Topsham	Eu.	England	Eng. Channel	50 37 N.	3 27 W.	6 00
Torbay	Eu.	England	Eng. Channel	50 34 N.	3 36 W.	5 15
Tornea	Eu.	Sweden	G. Bothnia	65 51 N.	24 16 E.	
R. Tortosa	Eu.	Spain	Medit. Sea	40 47 N.	1 03 E.	
I. Tortola	Am.	Atl. Ocean	Atl. Ocean	18 24 N.	65 00 W.	
I. Tory	Eu.	Ireland	West. Ocean	55 09 N.	8 30 W.	5 30
Toulon	Eu.	France	Medit. Sea	43 07 N.	6 02 E.	
C. Trefalga	Eu.	Spain	Atl. Ocean	36 08 N.	5 58 W.	
I. Tremiti	Eu.	Italy	Medit. Sea	42 09 N.	15 40 E.	
C. de Tres forcas	Africa	Barbary	Medit. Sea	35 30 N.	2 11 W.	
I. Traillies	Asia	India	Indian Ocean	19 30 S.	101 25 E.	
I. Trinity	Am.	Brazil	Atl. Ocean	20 25 S.	23 35 W.	
I. Trinidad, E. pt.	Am.	Terra Firma	Atl. Ocean	10 38 N.	60 27 W.	
Trinity Bay, Ent.	Am.	Newfoundl.	Atl. Ocean	48 30 N.	52 35 W.	
Trieste	Eu.	Carniola	Adriat. Sea	45 51 N.	14 03 E.	
Trinquemali	Asia	I. Ceylon	Indian Ocean	8 50 N.	83 24 E.	
Tripoli	Asia	Syria	Levant	34 53 N.	36 07 E.	
Tripoly	Africa	Barbary	Medit. Sea	32 54 N.	13 10 E.	
G. Triste	Am.	Terra Firma	Atl. Ocean	10 19 N.	67 41 W.	
I. Tristian d'Acunha	Africa	Caffres	S. Atl. Ocean	37 12 S.	13 23 W.	
II. Tromsund	Eu.	Lapland	North Ocean	70 20 N.	19 00 E.	
Truxilla	Am.	Peru	Pacif. Ocean	8 00 S.	78 35 W.	
Tunder	Eu.	Denmark	West. Ocean	55 00 N.	9 35 E.	
Tunis	Africa	Barbary	Medit. Sea	36 47 N.	10 16 E.	
Turin	Eu.	Italy	R. Po	45 05 N.	7 45 E.	
I. Turks	Am.	Bahama	Atl. Ocean	21 18 N.	71 05 W.	
Turtle Island	Asia	—	Pacif. Ocean	19 49 S.	177 52 W.	
Tschukotskoi Nofs	Asia	Kamtschat.	Beering's Str.	64 15 N.	173 26 W.	
Tyrnaw	Eu.	Hungary	Danube	48 23 N.	17 39 E.	
V.						
Valencia	Eu.	Spain	Medit. Sea	39 30 N.	0 40 W.	
St. Valery	Eu.	France	Eng. Channel	50 11 N.	1 42 E.	10 30

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Valona	Eu.	Turkey	Medit. Sea	40 55 N	21 15 E.	
Valparaiso	Am.	Chili	Pacif. Ocean	33 03 S.	72 14 W.	
Van Diemen's land	Asia	N. Holland	Indian Ocean	43 38 S.	146 27 E.	
Vannes	Eu.	France	B. Biscay	47 39 N.	2 41 W.	3h. 45m.
C. Vela	Am.	Terra Firma	Atl. Ocean	12 15 N.	71 20 W.	
P. Venus	Asia	Otaheite	Pacif. Ocean	17 29 S.	149 31 W.	10 38
Venice	Eu.	Italy	Medit. Sea	45 27 N.	12 9 E.	
Vera Cruz	Am.	New Spain	G. Mexico	19 12 N.	97 25 W.	
C. Verd	Africa	Negroland	Atl. Ocean	14 45 N.	17 28 W.	
Ufa	Eu.	Russia	Inland	54 43 N.	55 59 E.	
Uhma	Eu.	Sweden	G. Bothnia	63 45 N.	21 10 E.	
Vicegapatam	Asia	India	B. Bengal	17 30 N.	84 02 E.	
C. Victory	Am.	Patagonia	Pacif. Ocean	52 15 S.	74 28 W.	
Vienna	Eu.	Germany	R. Danube	48 13 N.	16 28 E.	
Vigo	Eu.	Spain	Atl. Ocean	42 14 N.	8 23 W.	
Uliateah	Eu.	Society Isles	Pacif. Ocean	16 45 S.	151 26 W.	
B. St. Vincent	Am.	Paraguay	Atl. Ocean	23 55 S.	45 11 W.	
C. St. Vincent	Eu.	Portugal	Atl. Ocean	37 01 N.	8 58 W.	
I. St. Vincent	Africa	C. Verd	Atl. Ocean	17 47 N.	24 44 W.	
I. St. Vincent	Am.	Caribbee	Atl. Ocean	13 05 N.	61 05 W.	
R. St. Vincent	Africa	Guinea	Eth. Ocean	4 50 N.	7 41 W.	
C. Virgins	Am.	Patagonia	Atl. Ocean	52 23 S.	67 50 W.	
I. Virgins	Am.	Antil. Isle	Atl. Ocean	18 18 N.	64 14 W.	
Virgin Rocks	Am.	Newfoundl.	Atl. Ocean	46 30 N.	51 30 W.	
Umba	Eu.	Russia	Inland	66 40 N.	34 15 E.	
C. Volo	Eu.	Turkey	Archipelago	39 07 N.	23 23 E.	
R. Voltas	Africa	Guinea	Atl. Ocean	5 52 N.	1 10 E.	
C. Voltas	Africa	Caffres	Atl. Ocean	28 04 S.	16 18 E.	
Upfal	Eu.	Sweden	R. Sala	59 52 N.	17 47 E.	
Uraniberg	Eu.	Denmark	Baltic Sea	55 54 N.	12 57 E.	
I. Ushant	Eu.	France	Eng. Channel	48 30 N.	5 0 W.	4 30
I. Ustica	Eu.	Italy	Medit. Sea	38 43 N.	13 33 E.	
I. Vulcano	Eu.	Italy	Medit. Sea	38 29 N.	15 33 E.	
W.						
C. Prince of Wales	Am.		Beering's Str.	65 45 N.	168 12 W.	
Prin. Wales's Isles	Asia	New Guinea	Endeavour St.	10 26 S.	141 00 E.	
R. Wager	Am.	New Wales	Hudson's Bay	69 28 N.	87 25 W.	6 00
Wallis's Isle	Asia		Pacif. Ocean	13 18 S.	176 20 W.	
C. Walfingham	Am.	New Britain	Hudson's Str.	62 39 N.	77 48 W.	12 00
Wardhus	Eu.	Lapland	North Ocean	70 23 N.	51 12 E.	
Warsaw	Eu.	Poland	R. Vistula	52 14 N.	21 5 E.	
Waterford	Eu.	Ireland	St. Geo. Ch.	52 07 N.	7 42 W.	6 30
Watling Isle	Am.	Bahama	Atl. Ocean	23 42 N.	74 22 W.	
Wells	Eu.	England	Germ. Ocean	53 07 N.	1 00 E.	6 00
Western Isles	Asia	Diemen's Is.	Pacif. Ocean	43 36 S.	147 00 E.	
Western { S. point	Eu.	Scotland	West. Ocean	56 46 N.	7 40 W.	
Isles { N. point				58 35 N.	6 37 W.	
If. Westmania	Eu.	Iceland	West. Ocean	63 55 N.	17 30 W.	
If. Westrol	Eu.	Lapland	North Ocean	69 15 N.	24 00 E.	
Wexford	Eu.	Ireland	St. Geo. Ch.	52 13 N.	6 56 W.	
Weymouth	Eu.	England	Eng. Channel	52 40 N.	2 34 W.	7 20
Whale's Back	Eu.	Iceland	West. Ocean	63 44 N.	17 05 W.	
Whale's Head	Eu.	Greenland	North Ocean	77 18 N.	21 30 E.	
Whale Rock	Eu.	Azores	Atl. Ocean	38 50 N.	24 41 W.	
Whitby	Eu.	England	Germ. Ocean	54 30 N.	0 50 W.	3 00
Whitehaven	Eu.	England	Irish Sea	54 25 N.	3 15 W.	
Whitsuntide I.	Asia	N. Hebrides	Pacif. Ocean	16 44 S.	168 25 E.	
Wicklow	Eu.	Ireland	St. Geo. Ch.	52 50 N.	6 30 W.	
Willis's Isles	Am.	S. Georgia	Atl. Ocean	54 00 S.	38 25 W.	
Windaw	Eu.	Courland	Baltic Sea	57 08 N.	22 20 E.	
I. Wight { N. end	Eu.	England	Eng. Channel	50 47 N.	1 11 W.	
{ S. end				50 34 N.	1 10 W.	
{ E. end				50 41 N.	1 00 W.	0 00
{ W. end				50 41 N.	1 23 W.	
If. Prince William	Asia		Pacif. Ocean	16 45 S.	177 55 E.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
William Henry I.	Asia	Society Isles	Pacif. Ocean	19 00 S.	141 6 E.	
Winchelsea	Eu.	England	Eng. Channel	50 58 N.	0 50 E.	oh. 45 m.
Wintertonefs	Eu.	England	Germ. Ocean	53 02 N.	1 22 E.	9 00
Wisthuy in I. Gotland	Eu.	Sweden	Baltic Sea	57 40 N.	19 50 E.	
C. Wrath	Eu.	Scotland	West. Ocean	58 40 N.	4 50 E.	
Wypoung	Eu.	Finland	G. Finland	60 55 N.	30 20 E.	
Y.						
Yamboa	Asia	Arabia	Red Sea	24 25 N.	38 54 E.	
Yarmouth	Eu.	England	Germ. Ocean	52 55 N.	1 40 E.	9 45
Yas de Amber	Africa	Zanguebar	Indian Ocean	0 00	47 15 E.	
Yellow River	Asia	China	Pacif. Ocean	34 06 N.	120 10 E.	
Ylo	Am.	Peru	Pacif. Ocean	17 36 S.	71 08 W.	
Cape York	Asia	N. Holland	Endeavour St.	10 41 S.	141 39 E.	
York Fort	Am.	New Wales	Hudson's Bay	57 02 N.	92 12 W.	9 10
York, New	Am.	N. England	Atl. Ocean	40 43 N.	74 04 W.	3 0
Youghall	Eu.	Ireland	St. Geo. Ch.	51 46 N.	8 06 W.	4 30
Z.						
Zacatula	Am.	Mexico	Pacif. Ocean	17 10 N.	105 00 W.	
Zachee	Am.	Antilles	Atl. Ocean	18 24 N.	67 52 W.	
I. Zant	Eu.	Italy	Adriatic Sea	37 50 N.	21 30 E.	
I. Zanzibar	Africa	Zanguebar	Indian Ocean	6 55 S.	40 10 E.	
Zaro	Eu.	Dalmatia	Medit. Sea	44 15 N.	16 65 E.	
N. Zealand	Asia	—	Pacif. Ocean	34 28 S.	172 44 E.	
land						
NW. point						
S. point						
Zenan city	Asia	Arabia	Inland	47 20 S.	167 50 E.	
Zuric Sea	Eu.	D. Neth.	Germ. Ocean	16 20 N.	47 44 E.	3 00

Besides the times of high-water in the preceding Table, the following times serve for coasts of considerable extent, and will serve nearly for the places on those coasts.

Finmark, or NNW. coast of Lapland, 1 h. 30 m. Jutland Isles oh. om.
 Friesland coast 7 h. 30 m. Zealand coast 1 h. 30 m.
 Flanders coast oh. om. Picardy and Normandy coasts 10 h. 30 m.
 Biscay, Gallician, and Portugal coasts 3 h. 00 m.
 Irish W. coast 3 h. 00 m. Irish S. coast 5 h. 15 m.
 Africa W. coast 3 h. 0 m. America W. coast 3 h. 0 m.
 America E. coast 4 h. 30 m.

END OF BOOK VI. AND OF VOL. I.

